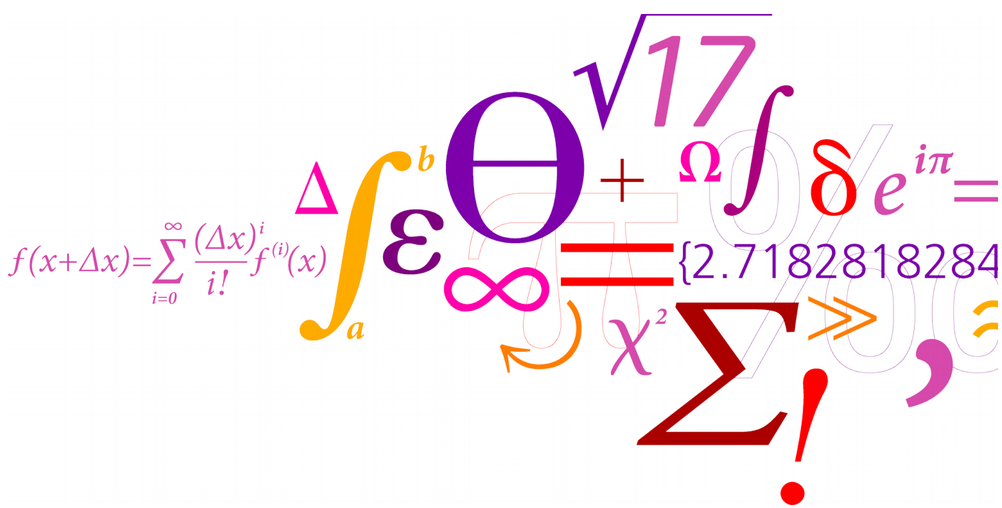
Statistical Models and Methods;
Thermal performance and Occupant behavior

REBUS WP2B
Saint-Gobain, May 2017

Henrik Madsen

www.henrikmadsen.org



DTU Compute
Department of Applied Mathematics and Computer Science

George Box:

All models are wrong – but some are useful

Modeling made simple

Suppose we have a time series of data:

$$\{X_t\} = X_1, X_2, ... , X_t, ...$$

The purpose of any modeling is to find a nonlinear function $h(\{X_t\})$ such that

$$h(\{X_t\}) = \varepsilon_t$$

Where $\{\varepsilon_t\}$ is white noise – ie **no autocorrelation**

Thermal performance characterization using time series data - statistical guidelines

IEA EBC Annex 58

Henrik Madsen and Peder Bacher [1]
DTU Compute, Technical University of Denmark
Lyngby, Denmark

Geert Bauwens, An-Heleen Deconinck, Glenn Reynders and Staf Roels
KU Leuven, Civil Engineering Department, Building Physics Section
Heverlee, Belgium

Eline Himpe
Ghent University, Department of Architecture and Urban Planning, Building Physics Group
Gent, Belgium

Guillaume Lethé
BBRI, Belgian Building Research Institute
Brussels, Belgium

November 28, 2016

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Methods in Annex 58 Guidelines

- Linear regression
  (steady state approach)

- ARX model
  (dynamical, linear, time-invariant)

- Grey-box model (RC-network model +)
  (dynamical, linear or nonlinear, time-varying)

The Annex 58 Guidelines contains recipes as well as examples are in R (open source stat package)

Guidelines from Annex 58

Static and dynamic conditions: estimate the Heat Loss Coefficient (HLC) and gA-value from 'simple' data:

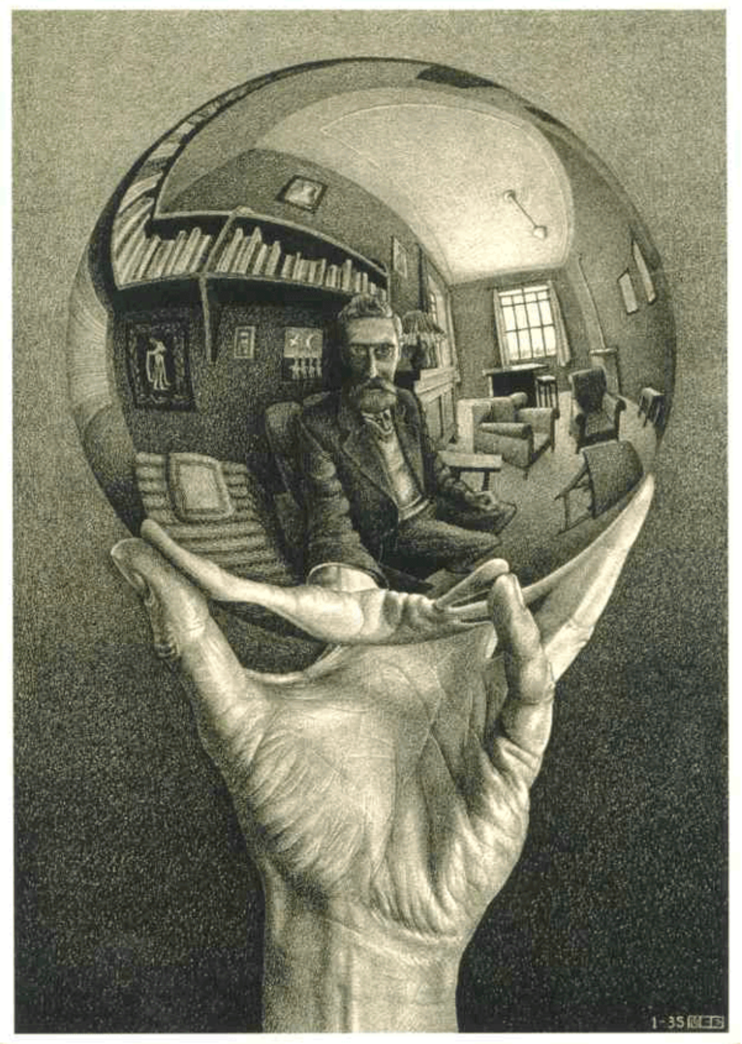
- Constant indoor temperature
- *Model input*: ambient temperature and global radiation (wind not included in guideline models)
- *Model output*: heat load

Grey-box models for detailed building behavior characterization:

- Varying indoor temperature (turn the heating on/off)
- *Model input*: ambient temperature, global radiation, wind
- *Model output*: indoor air temperature

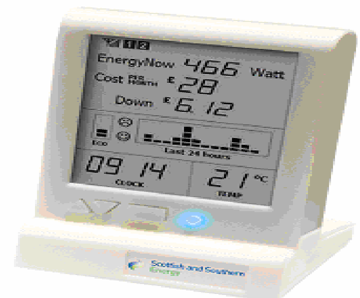
Procedures (recipes) for model selection and validation, with examples in R

Contents



1. A single sensor (a smart meter)
2. Several sensors (and grey-box modelling)
3. Special sensors (model for occupant behavior)

Part 1
A single sensor (smart meter)



- Smart Meters and data splitting
- Smart Meters and Thermal Characteristics
    - Problem setting
    - Simple tool

Case Study No. 1

Split of total readings into space heating and domestic hot water using data from smart meters

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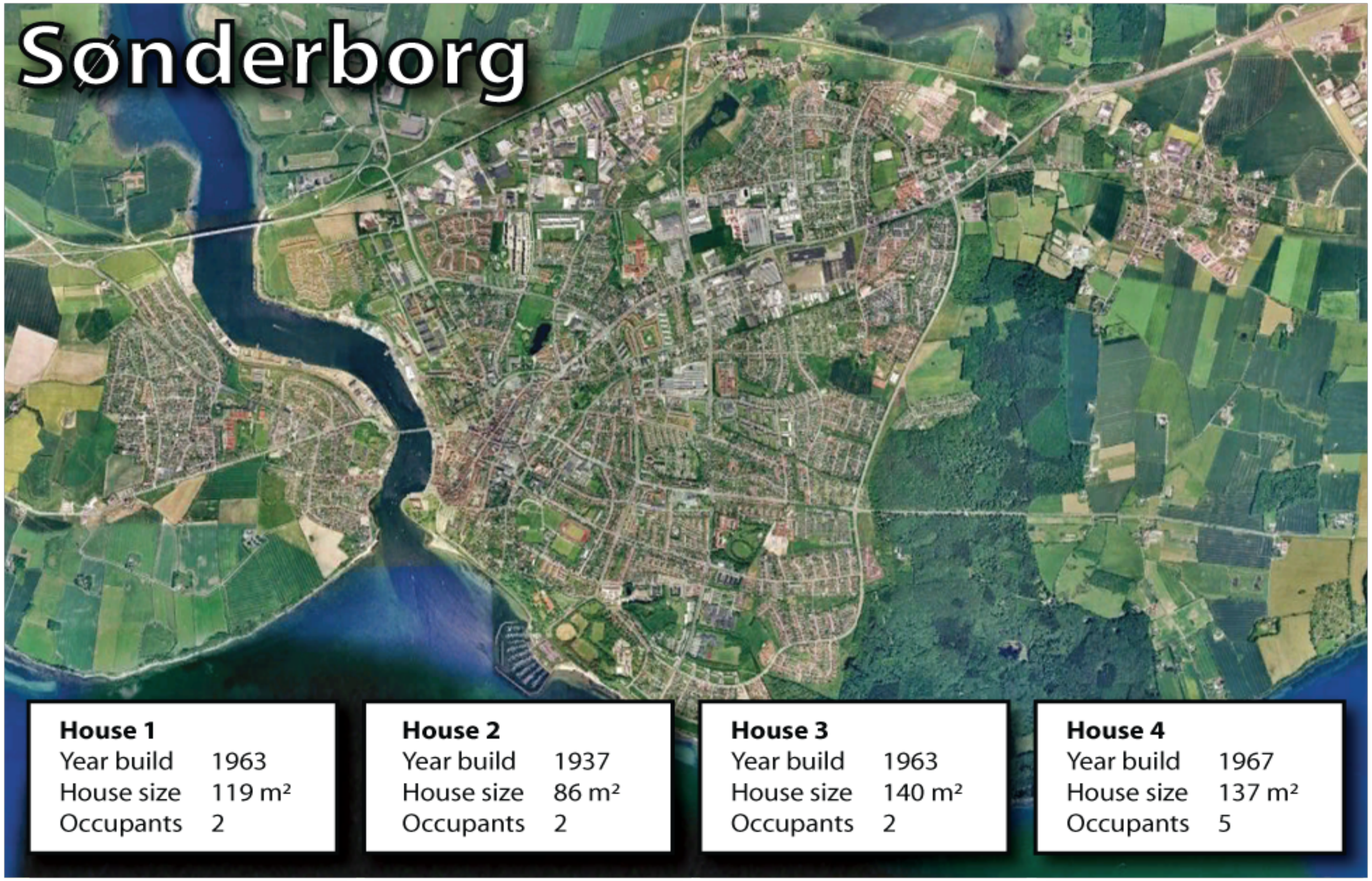
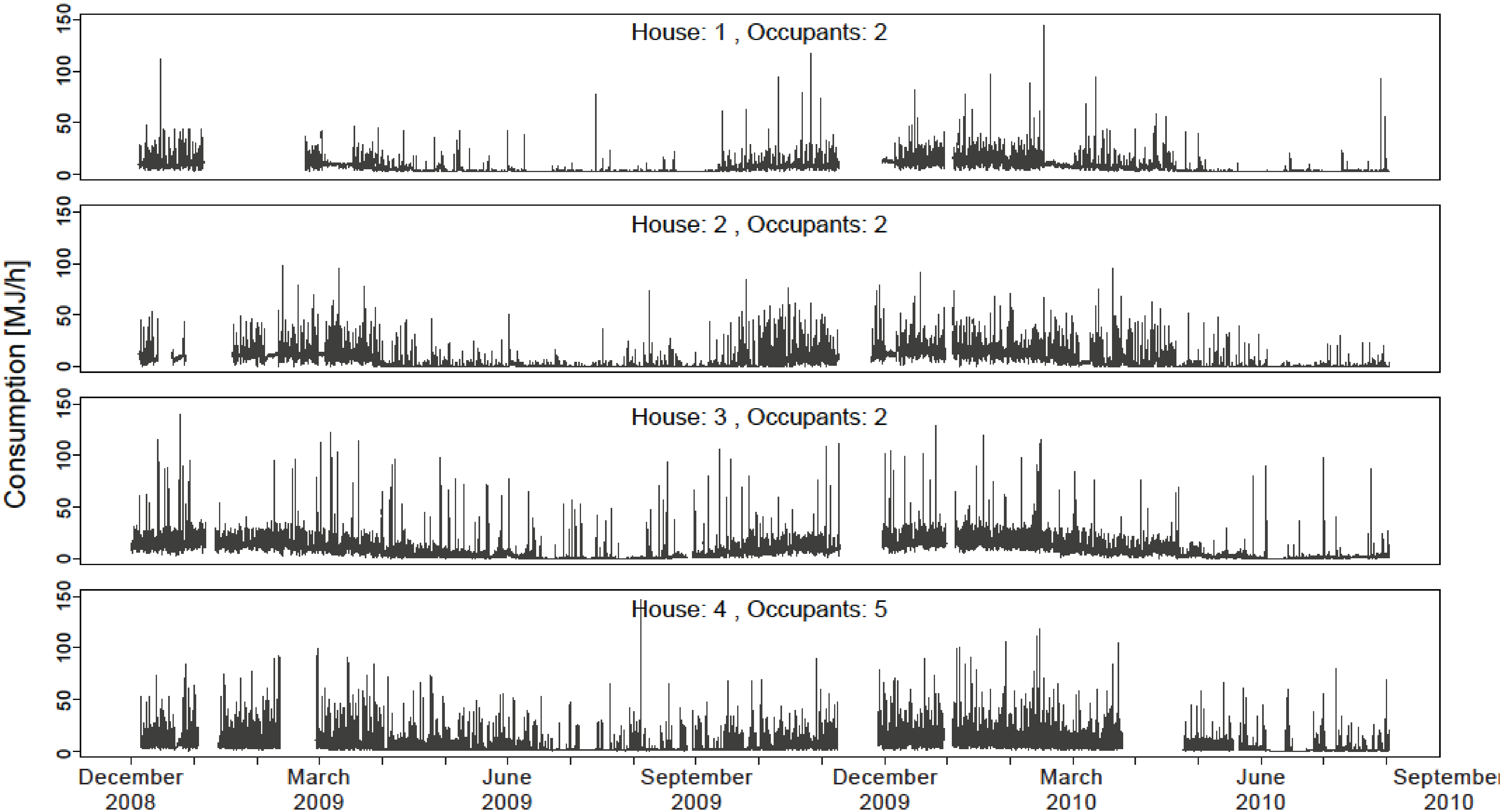
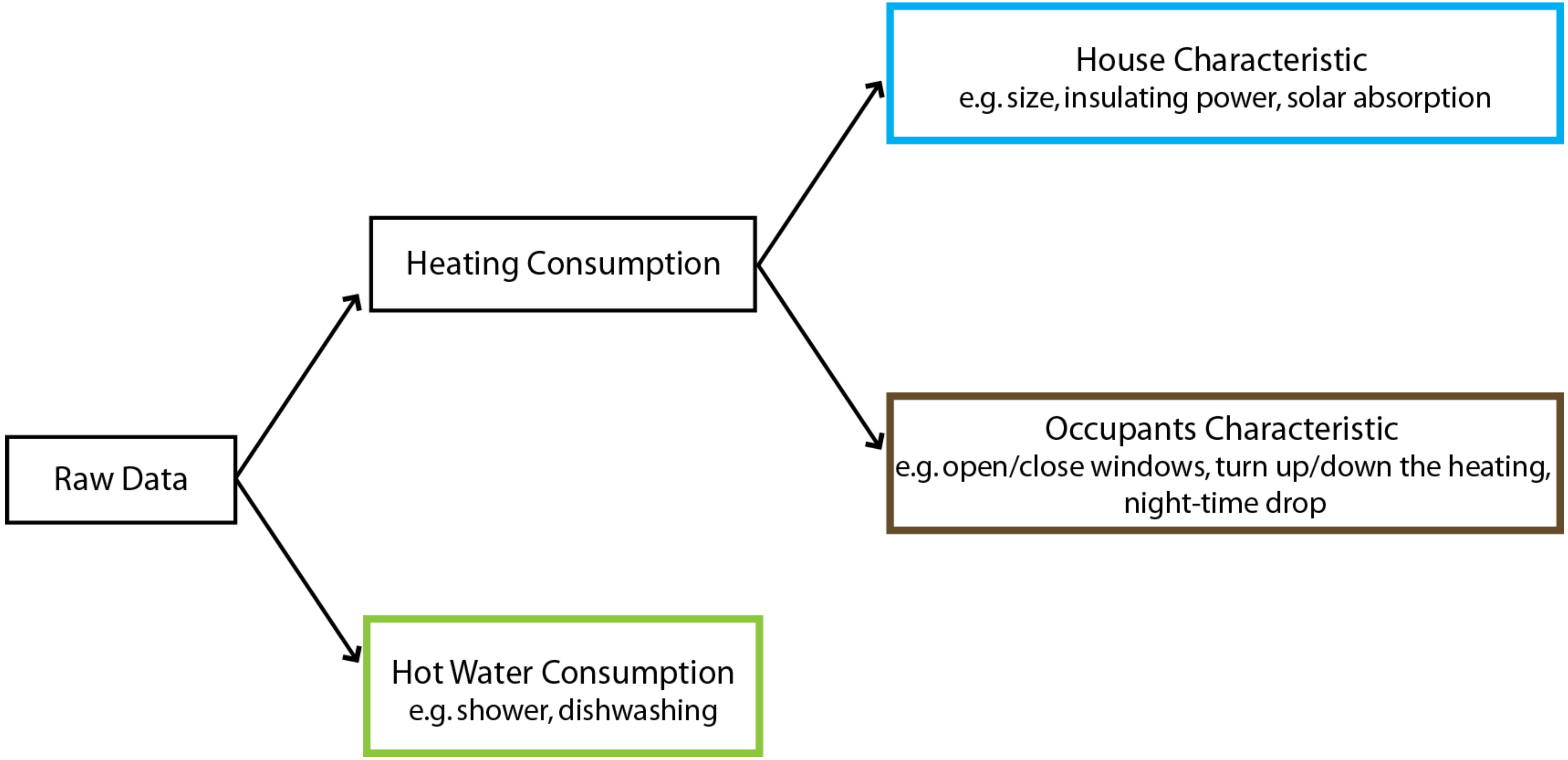
Data

- 10 min averages from a number of houses

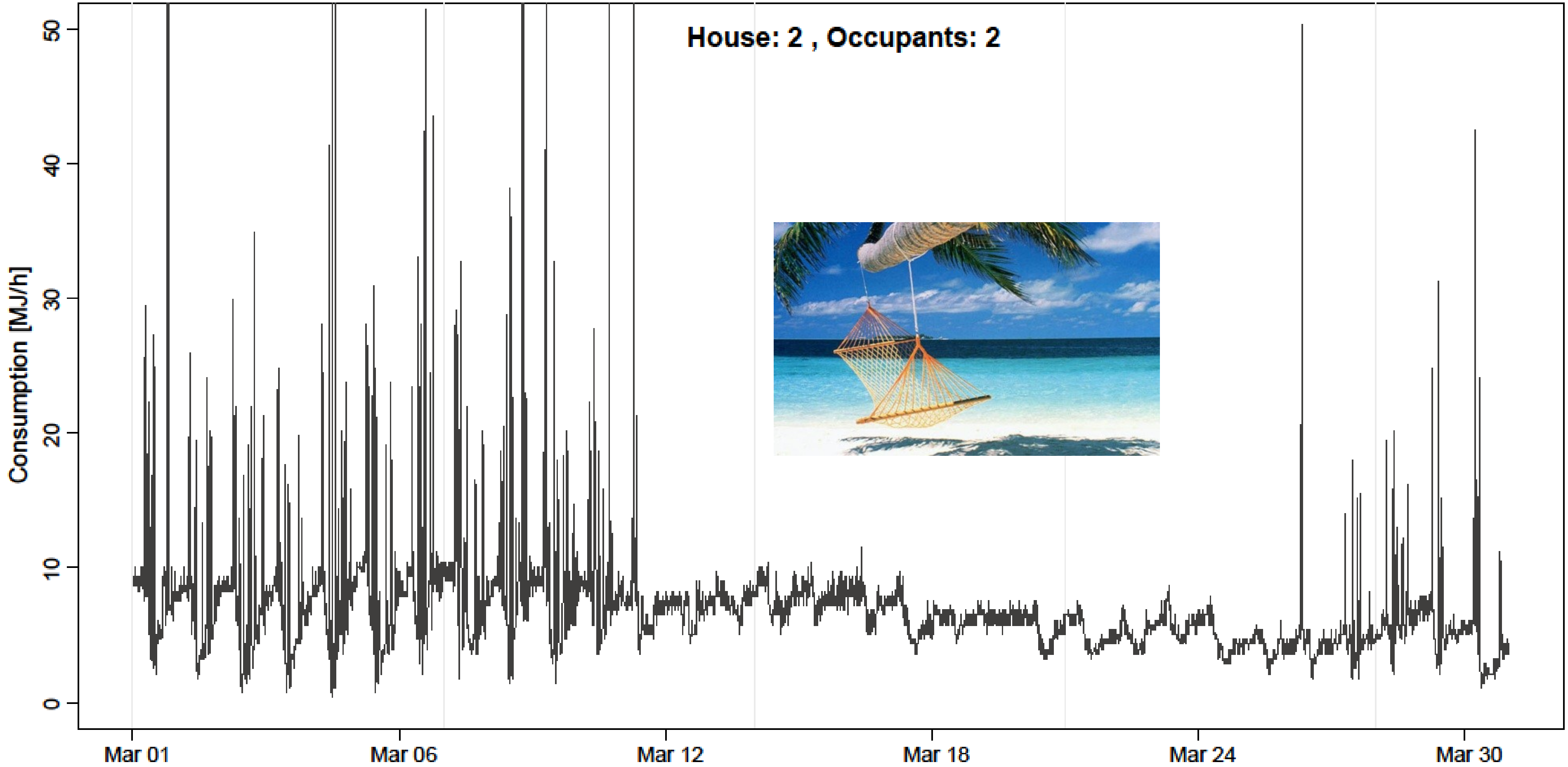



House: 1 , Occupants: 2
House: 2 , Occupants: 2
House: 3 , Occupants: 2
House: 4 , Occupants: 5
Consumption [MJ/h]
0
50
100
150
December 2008
March 2009
June 2009
September 2009
December 2009
March 2010
June 2010
September 2010

Data separation principle



Holiday period



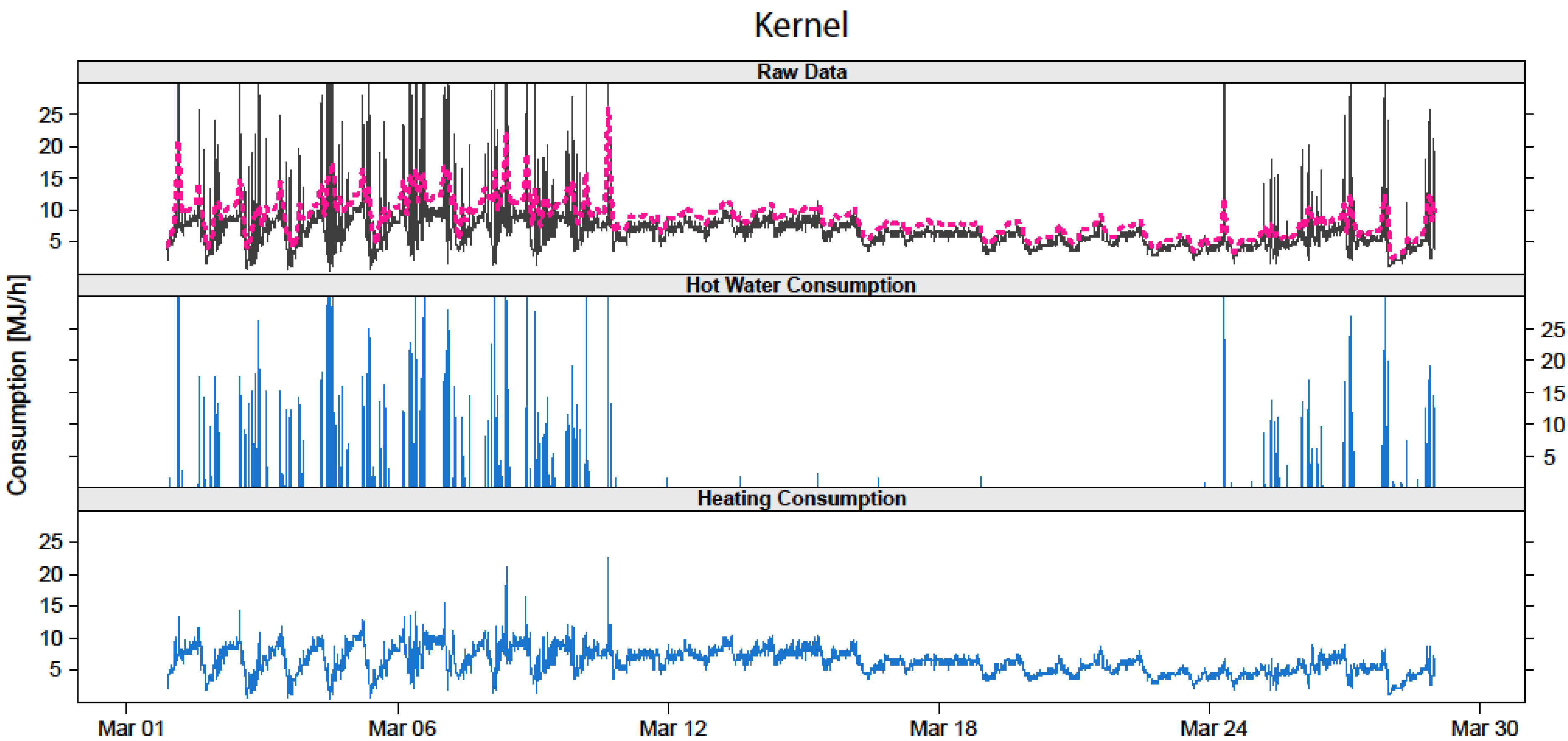
Non-parametric regression

$$\hat{g}(x) = \frac{\sum_{s=1}^{N} Y_s k\{\frac{x-X_s}{h}\}}{\sum_{s=1}^{N} k\{\frac{x-X_s}{h}\}}$$

$$k(u) = \frac{1}{2\pi} \exp\{-\frac{u^2}{2}\}$$

Weighted average

Every spike above $1.25 \cdot \hat{g}(x)$ Is regarded as hot water use.


Kernel
Raw Data
Hot Water Consumption
Heating Consumption
Consumption [MJ/h]
25
20
15
10
5
Mar 01
Mar 06
Mar 12
Mar 18
Mar 24
Mar 30

Robust Polynomial Kernel

To improve the kernel method

Rewrite the kernel smoother to a Least Square Problem

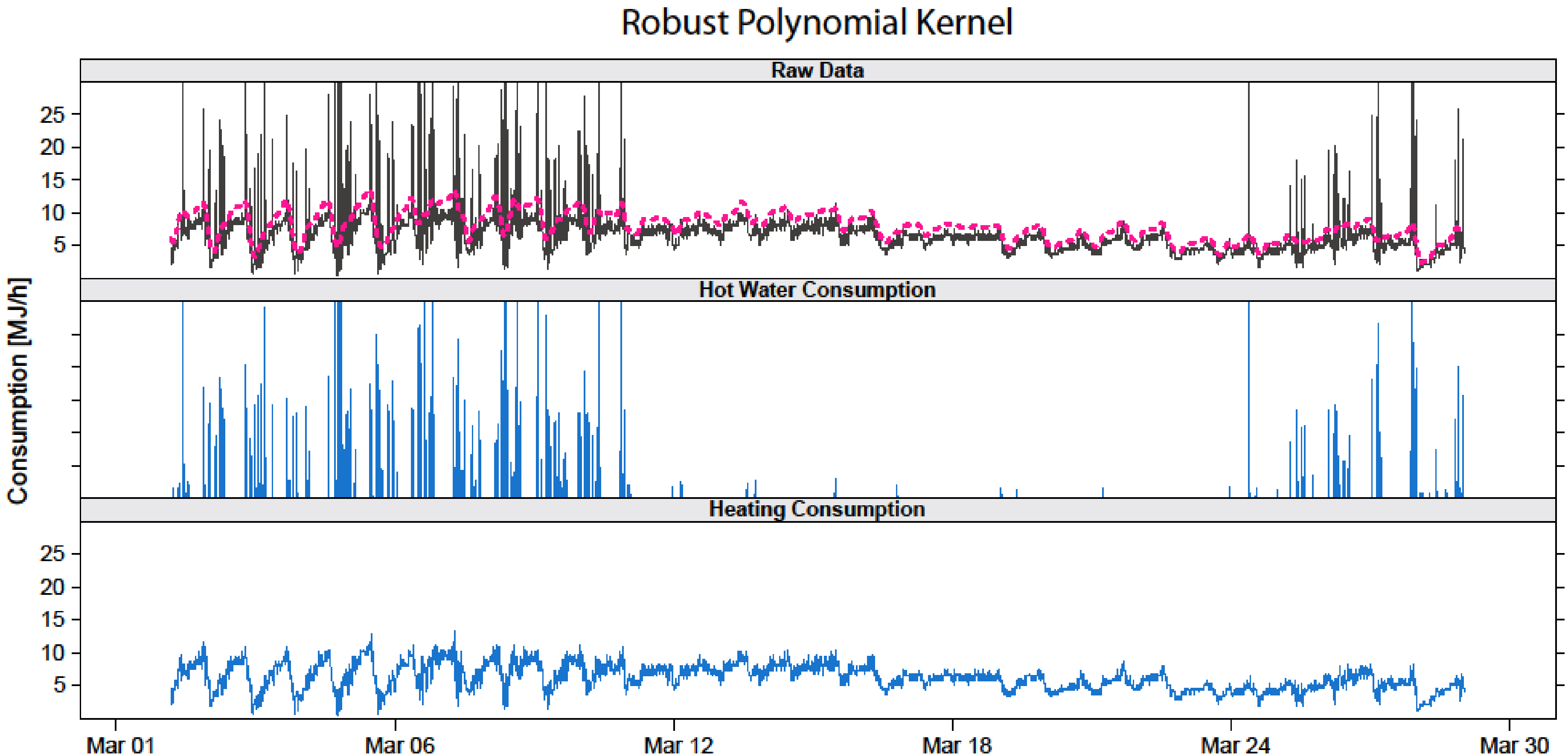
$$\arg\min_{\theta} \frac{1}{N} \sum_{s=1}^{N} w_s(x) \left(Y_s - \theta\right)^2 \qquad w_s(x) = \frac{k\{x - X_s\}}{\frac{1}{N}\sum_{s=1}^{N} k\{x - X_s\}}$$

Make the method robust by replacing $\left(Y_s - \theta\right)^2$ with

$$\rho_{\text{Huber}}(\varepsilon) = \begin{cases} \frac{1}{2\gamma}\varepsilon^2 & \text{if } |\varepsilon| \leq \gamma \\ |\varepsilon| - \frac{1}{2}\gamma & \text{if } |\varepsilon| > \gamma \end{cases} \qquad \varepsilon_s = Y_s - \theta$$

Make the method polynomial by replacing θ with

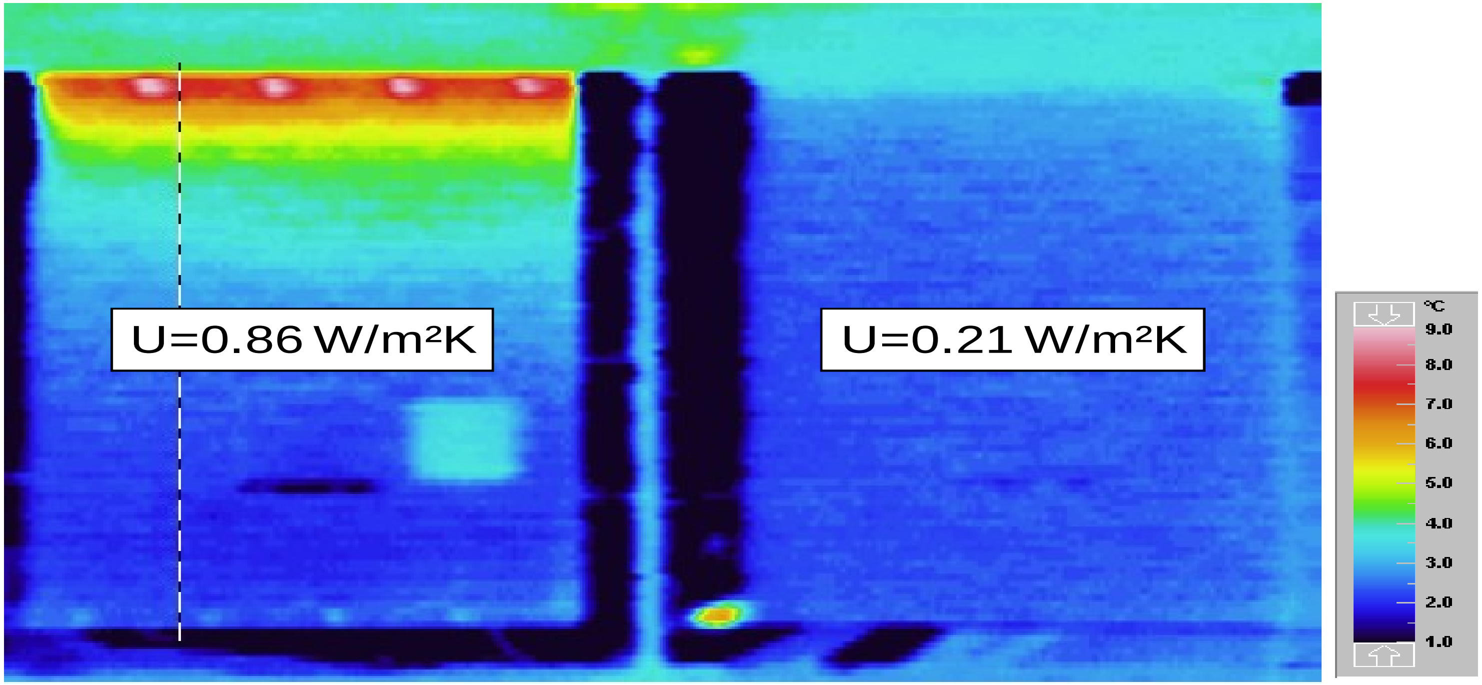
$$P_s = \theta_0 + \theta_1(X_t - x) + \theta_2(X_t - x)^2$$


Robust Polynomial Kernel
Raw Data
Hot Water Consumption
Heating Consumption
Consumption [MJ/h]
25
20
15
10
5
Mar 01
Mar 06
Mar 12
Mar 18
Mar 24
Mar 30

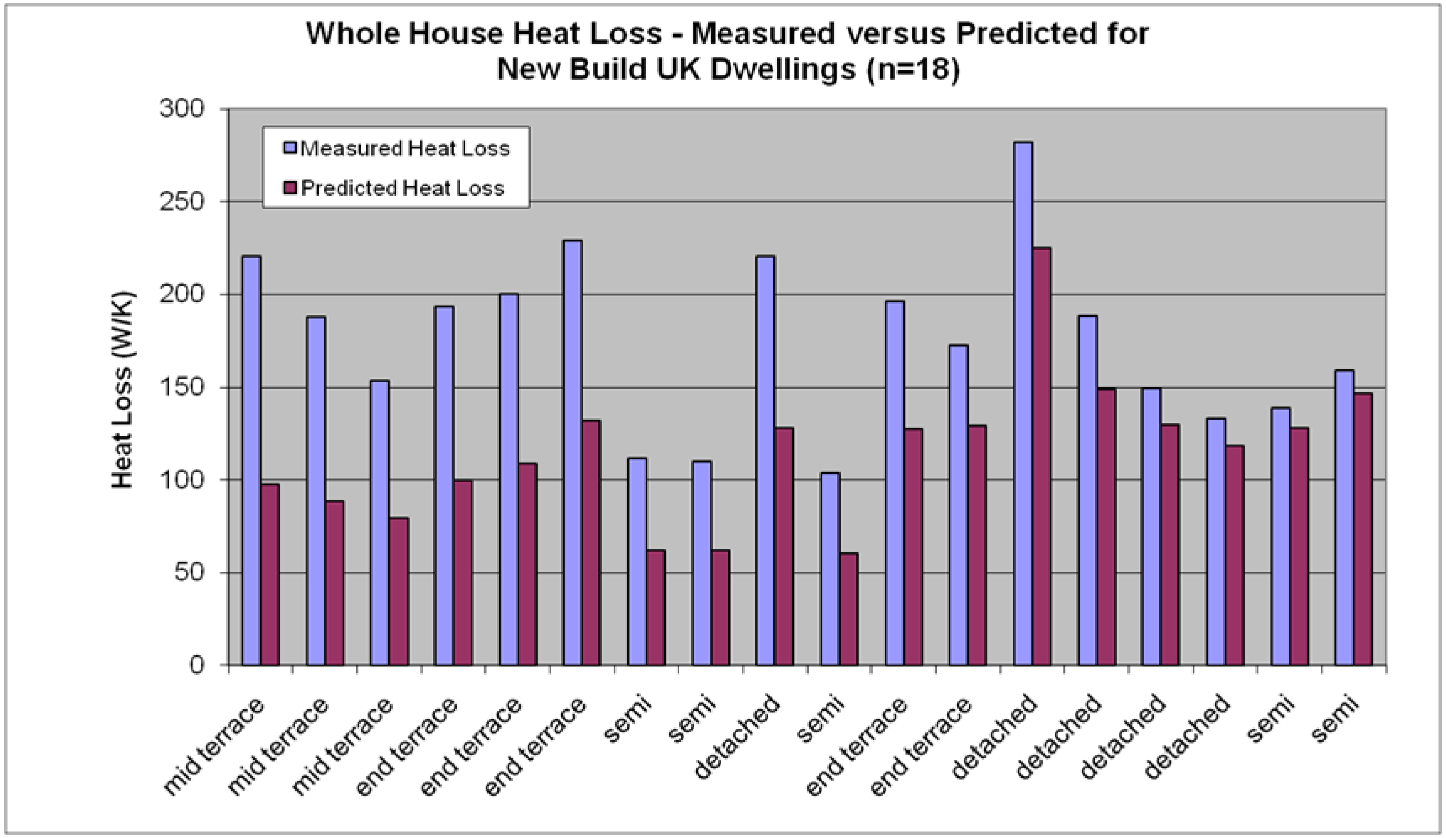
Case Study No. 2

Identification of Thermal Performance using Smart Meter Data

Example



Examples (2)



Measured versus predicted energy consumption for different dwellings

Characterization Smart Meter Data

- Energy labelling
- Estimation of UA and gA values
- Estimation of energy signature
- Estimation of dynamic characteristics
- Estimation of time constants

Simple estimation of UA-values

- Consider the following model (t=day No.) estimated by kernel-smoothing:

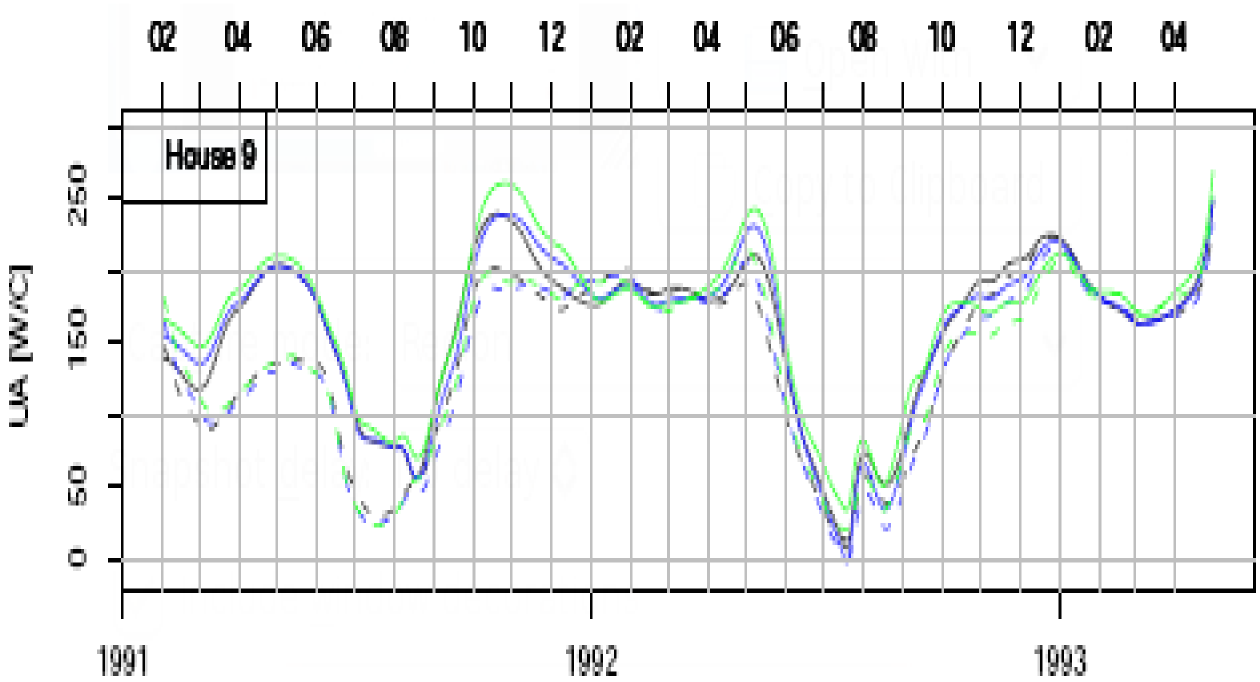
$$Q_t = Q_0(t) + c_0(t)(T_{i,t} - T_{a,t}) + c_1(t)(T_{i,t-1} - T_{a,t-1}) \qquad (1)$$

- The estimated UA-value is

$$\hat{UA}(t) = \hat{c}_o(t) + \hat{c}_1(t) \qquad (2)$$

- With more involved (but similar models) also gA and wA values can be stimated

Estimated UA-values



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Results

	UA W/°C	σ_{UA}	gA^{max} W	wA_E^{max} W/°C	wA_S^{max} W/°C	wA_W^{max} W/°C	T_i °C	σ_{T_i}
4218598	211.8	10.4	597.0	11.0	3.3	8.9	23.6	1.1
4381449	228.2	12.6	1012.3	29.8	42.8	39.7	19.4	1.0
4711160	155.4	6.3	518.8	14.5	4.4	9.1	22.5	0.9
4836681	155.3	8.1	591.0	39.5	28.0	21.4	23.5	1.1
4836722	236.0	17.7	1578.3	4.3	3.3	18.9	23.5	1.6
4986050	159.6	10.7	715.7	10.2	7.5	7.2	20.8	1.4
5069878	144.8	10.4	87.6	3.7	1.6	17.3	21.8	1.5
5069913	207.8	9.0	962.5	3.7	8.6	10.6	22.6	0.9
5107720	189.4	15.4	657.7	41.4	29.4	16.5	21.0	1.6
.	.	.	.	.	.	.	.	.

Based on measurements from the heating season **2009/2010** your typical indoor temperature during the heating season has been estimated to **24** ^oC. If this is not correct you can change it here $\boxed{24}$ ^oC.

If your house has been left empty in longer periods with a partly reduced heat supply you have the possibility of specifying the periods in this $\boxed{\text{calendar}}$.

According to BBR the area of your house is **155** m^2 and from **1971**.

Based on BBR information it is assumed that **you do not use any supplementary heat supply**. If this is not correct you can specify the type and frequency of use here:

- Wood burning stove used $\boxed{0}$ times per week in cold periods.
- Solar heating $\boxed{\text{y/n}}$, approximate size of solar panel $\boxed{0} \times \boxed{0}$ meters.

Based on the indoor temperature **24** ^oC, the use of a wood burning stove **0** times per week, and **no** solar heating installed, the response of your house to climate is estimated as:

- The response to outdoor temperature is estimated to **200** $W/^oC$ **which given the size and age of your house is expectable**[a].
- On a windy day the above value is estimated to increase with **60** $W/^oC$ **when the wind blows from easterly directions. This response to wind is relatively high and indicates a problem related to the air sealing on the eastern side of the house.**
- On a sunny day during the heating season the house is estimated to receive **800** W as an average over 24 hours. **This value is quite expectable.**

[a]Many kind of different recommendations can be given here.

Perspectives for using Smart Meters

- Reliable Energy Signature.
- Energy Labelling
- Time Constants (eg for night set-back)
- Proposals for Energy Savings:
  - Replace the windows?
  - Put more insulation on the roof?
  - Is the house too untight?
  - ......
- Optimized Control
- Integration of Solar and Wind Power using DSM

Part 2
Several sensors

- Introduction to Grey-Box Modelling (a continuous-discrete state space models)
- A model for the thermal characteristics of a small office building
- Models for control

Introduction to Grey-Box modelling

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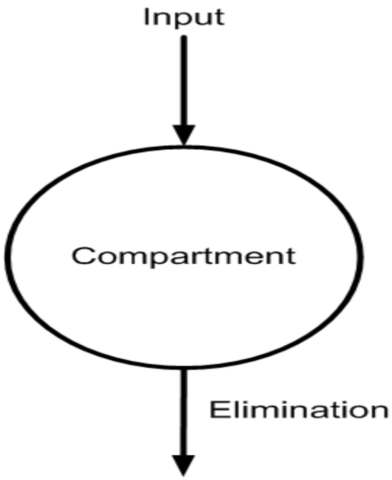
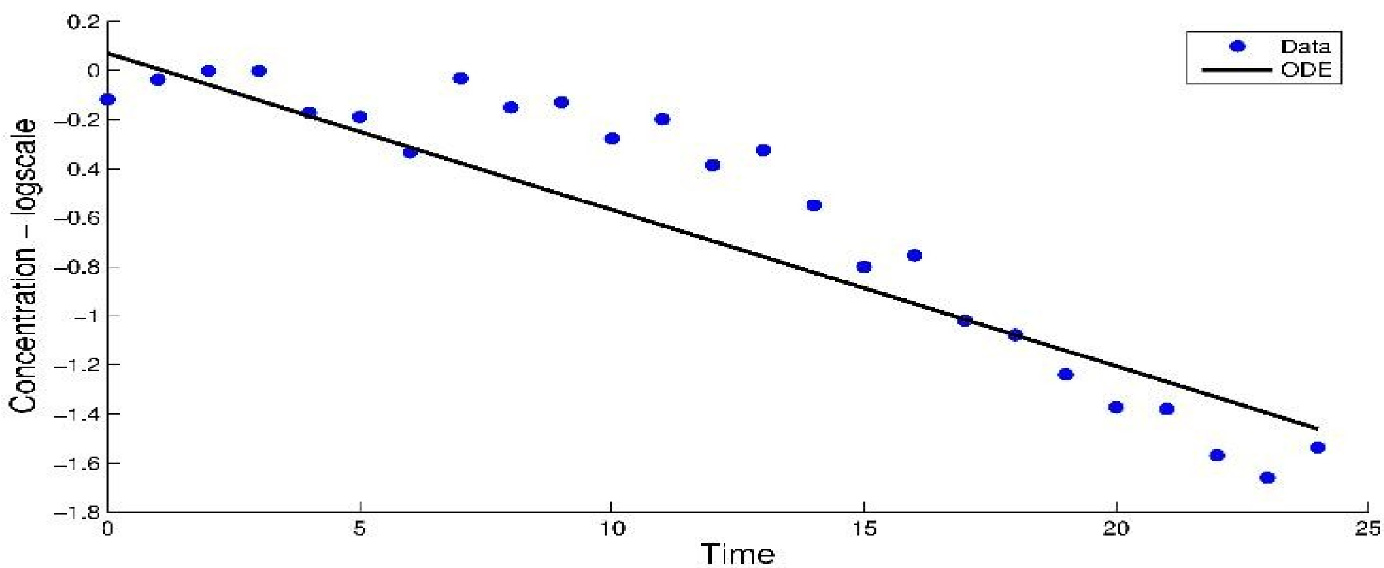
Traditional Dynamical Model

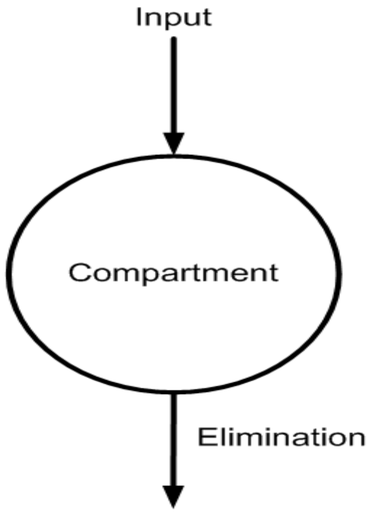


- Ordinary Differential Equation:

$$\begin{aligned} dA &= -KAdt \\ Y &= A + \epsilon \end{aligned}$$

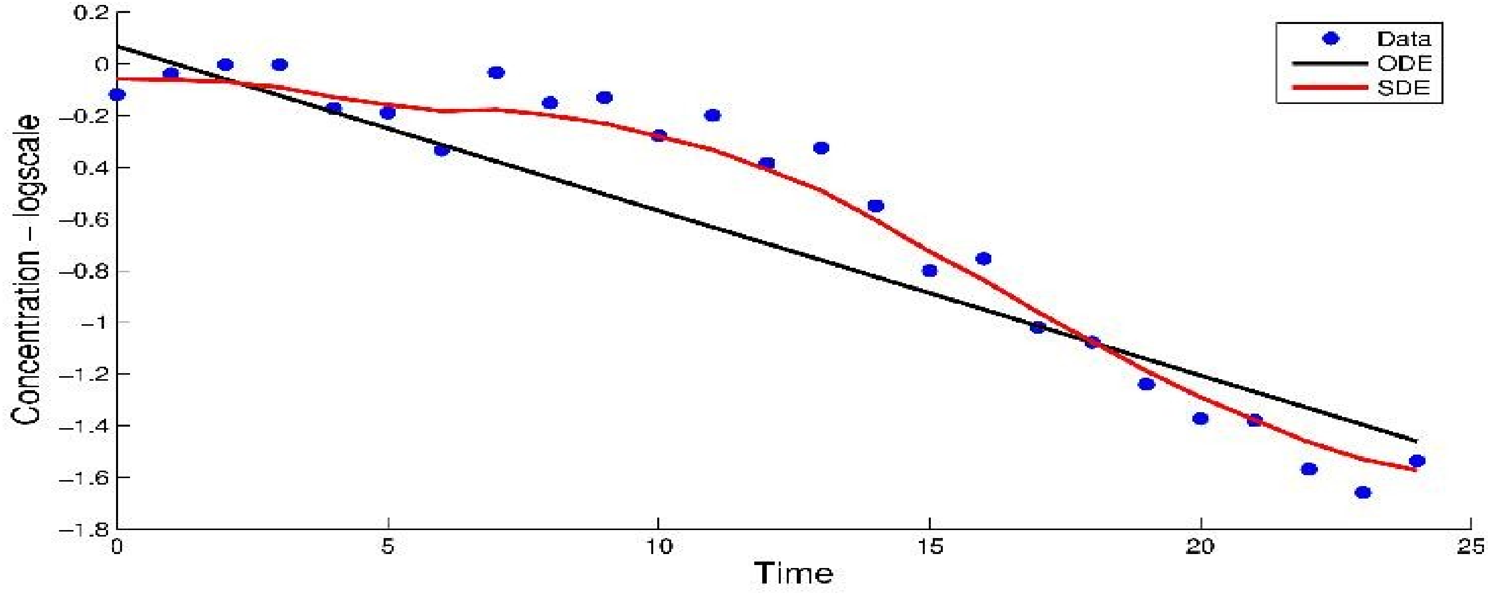


Stochastic Dynamical Model



- Stochastic Differential Equation:

$$
\begin{aligned}
dA &= -KA\,dt + \sigma\,dw \\
Y &= A + e
\end{aligned}
$$



The grey box model

Drift term

Diffusion term

$$dX_t = f(X_t, u_t, t, \theta)dt + \sigma(X_t, u_t, t, \theta)d\omega_t$$ System equation

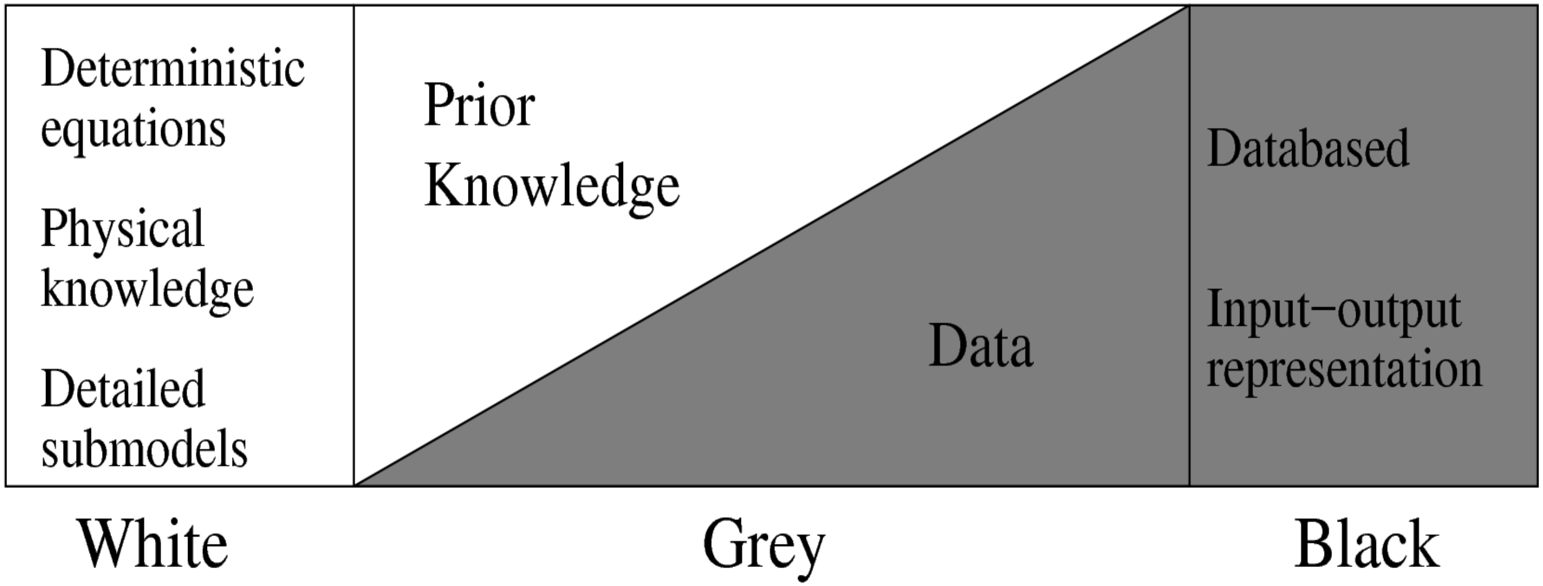
$$Y_k = h(X_k, u_k, t_k, \theta) + e_k,$$ Observation equation

Observation noise

Notation:

X_t:	State variables
u_t:	Input variables
θ:	Parameters
Y_k:	Output variables
t:	Time
ω_t:	Standard Wiener process
e_k:	White noise process with $N(\mathbf{0}, S)$

Grey-box modelling concept



- Combines prior physical knowledge with information in data
- Equations and parameters are physically interpretable

Forecasting and Simulation

Grey-Box models are well suited for ...

- One-step forecasts
- K-step forecasts
- Simulations
- Control
- … of both *observed* and *hidden* states.

- **It provides a framework for pinpointing model deficiencies – like:**
  - Time-tracking of unexplained variations in e.g. parameters
  - Missing (differential) equations
  - Missing functional relations
  - Lack of proper description of the uncertainty

Grey-Box Modelling

- Bridges the gap between physical and statistical modelling
- Provides methods for model identification
- Provides methods for model validation
- Provides methods for pinpointing model deficiencies
- Enables methods for a reliable description of the uncertainties, which implies that the same model can be used for k-step forecasting, simulation and control

Grey-Box Modelling

- Bridges the gap between physical and statistical modelling
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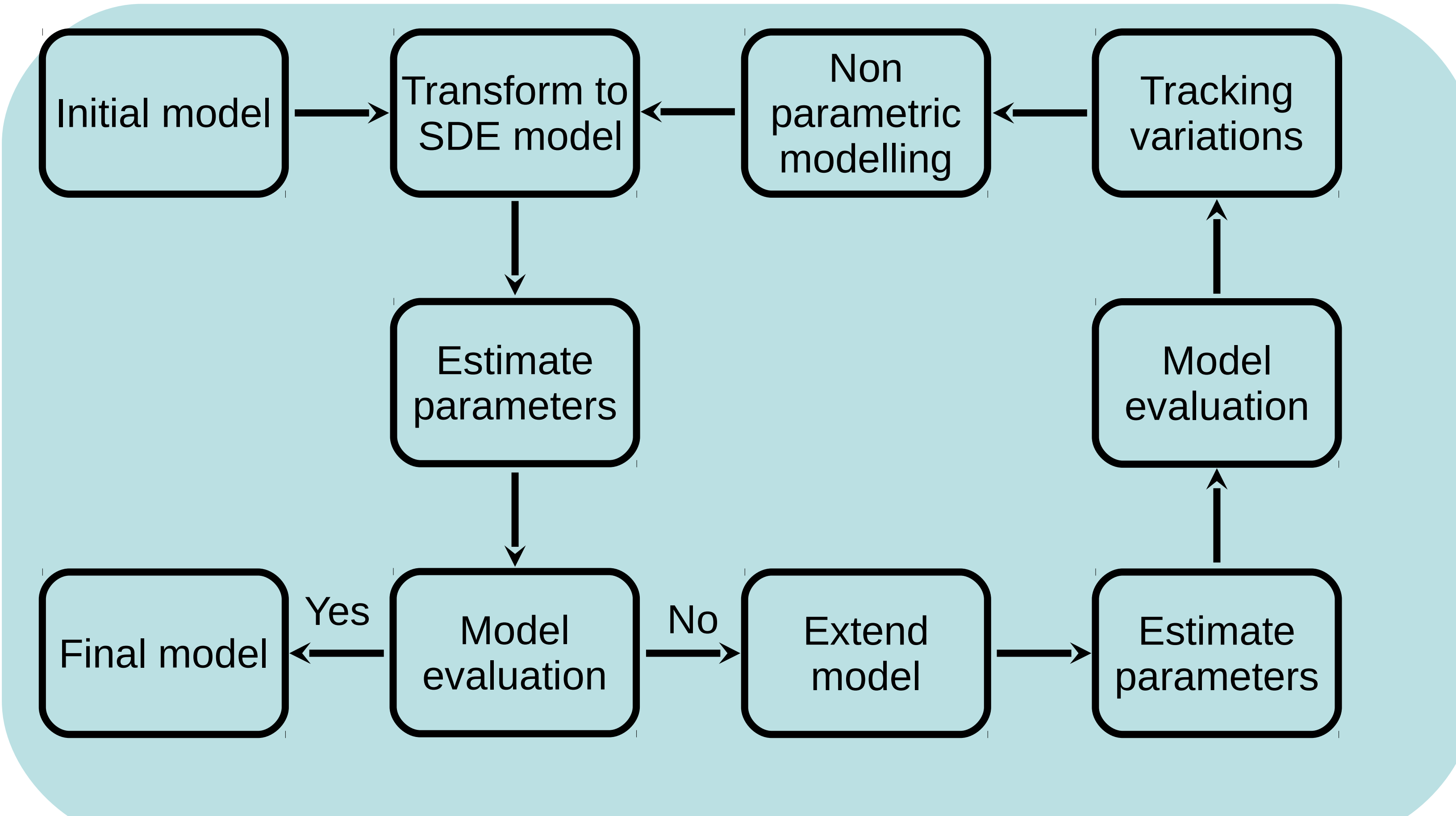
Grey box model building framework



A method for systematic improvement of stochastic grey box models. Computers a

Case study

Model for the thermal characteristics of a small office building

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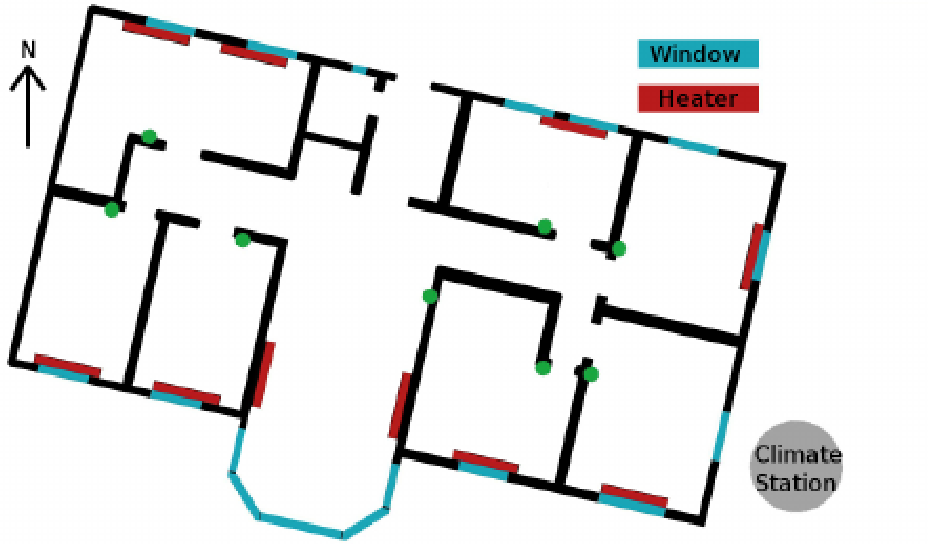
TEST CASE: ONE FLOORED 120 M^2 BUILDING

Objective

Find the best model describing the heat dynamics of this building ([1], [4])



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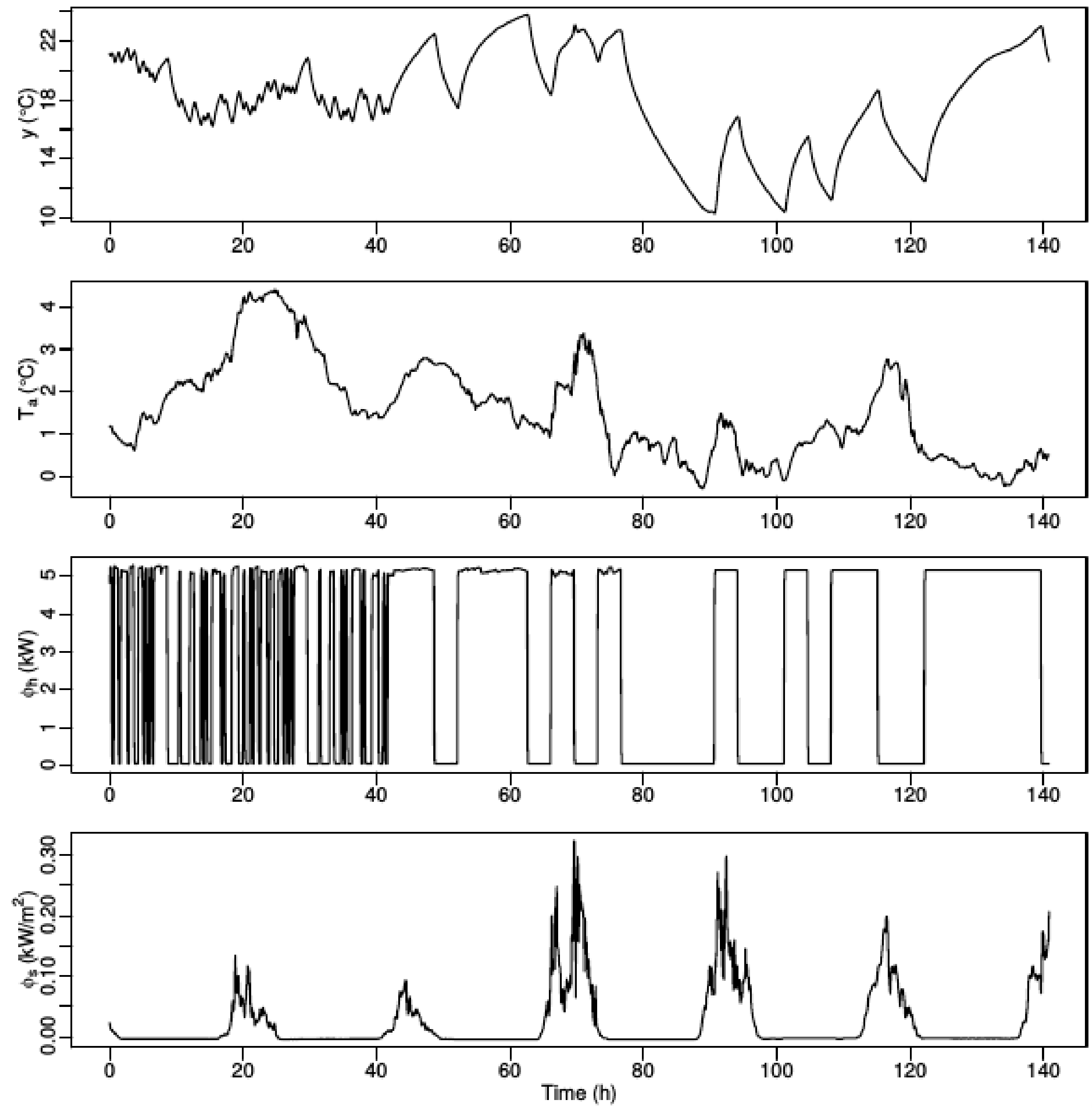
DATA

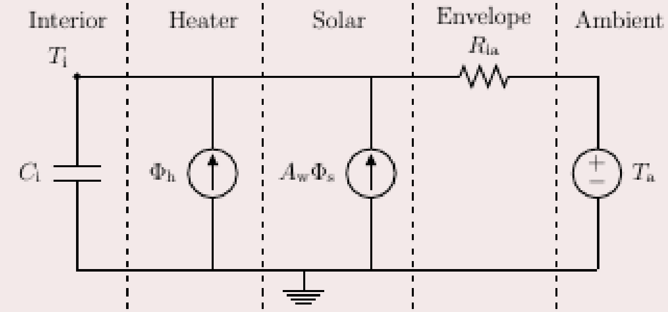
Measurements of:

- y_t Indoor air temperature
- T_a Ambient temperature
- Φ_h Heat input
- Φ_s Global irradiance

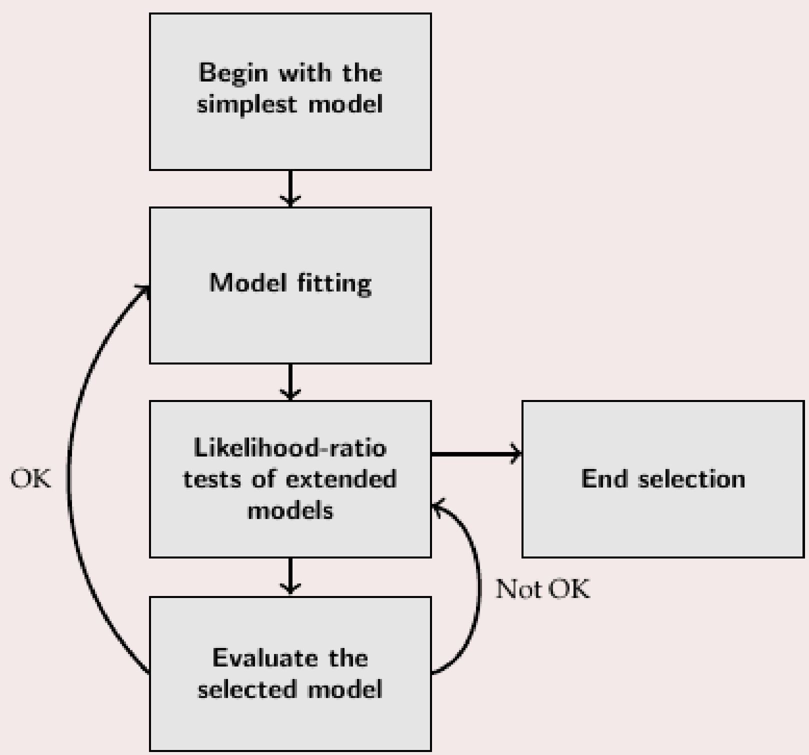


SELECTION PROCEDURE

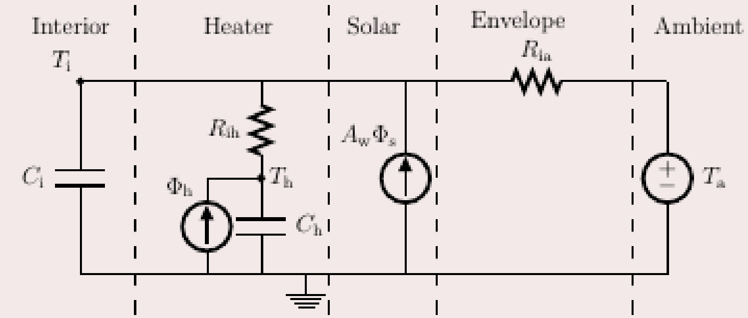
Simplest model



Iterative procedure using statistical tests

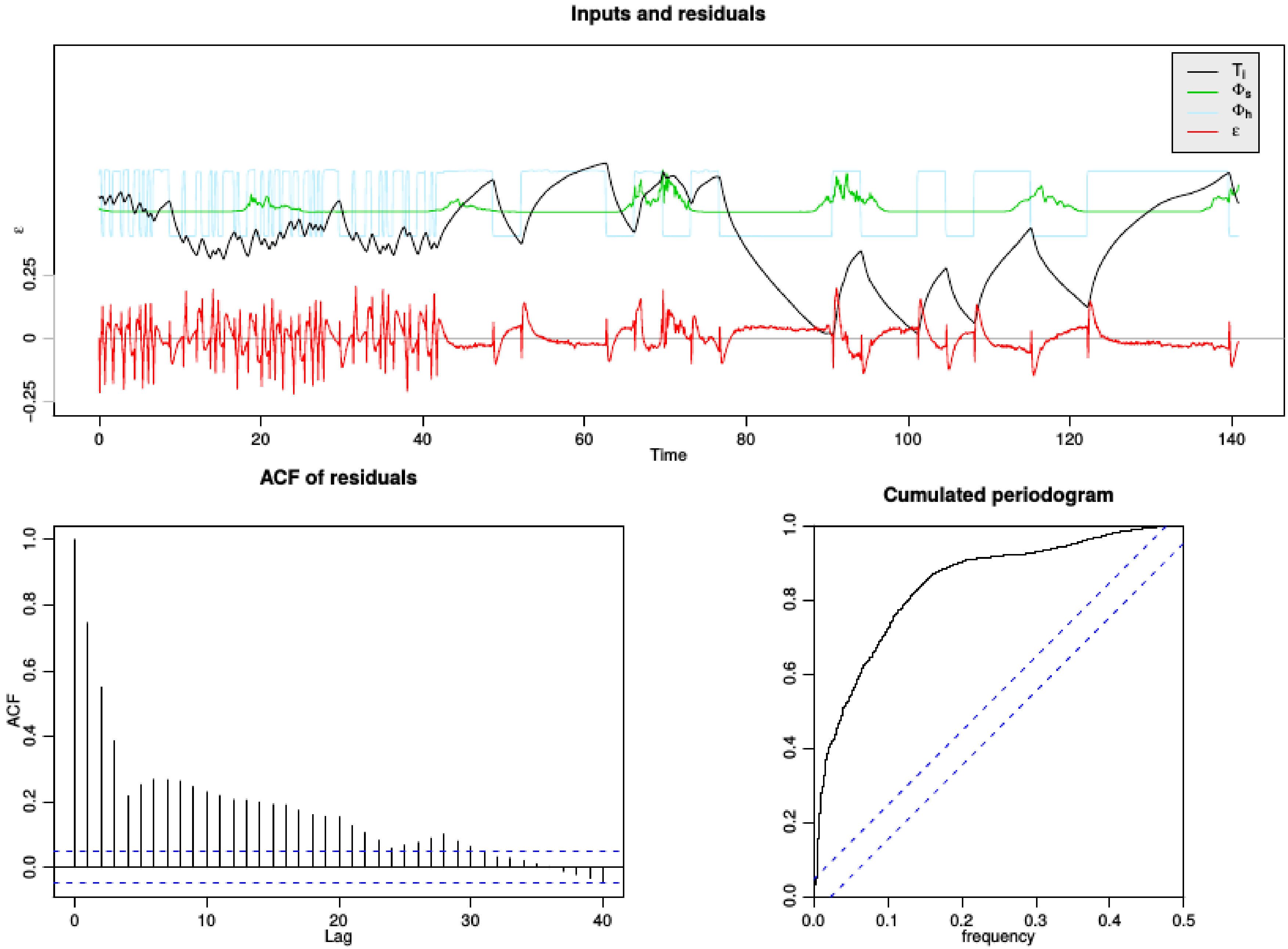


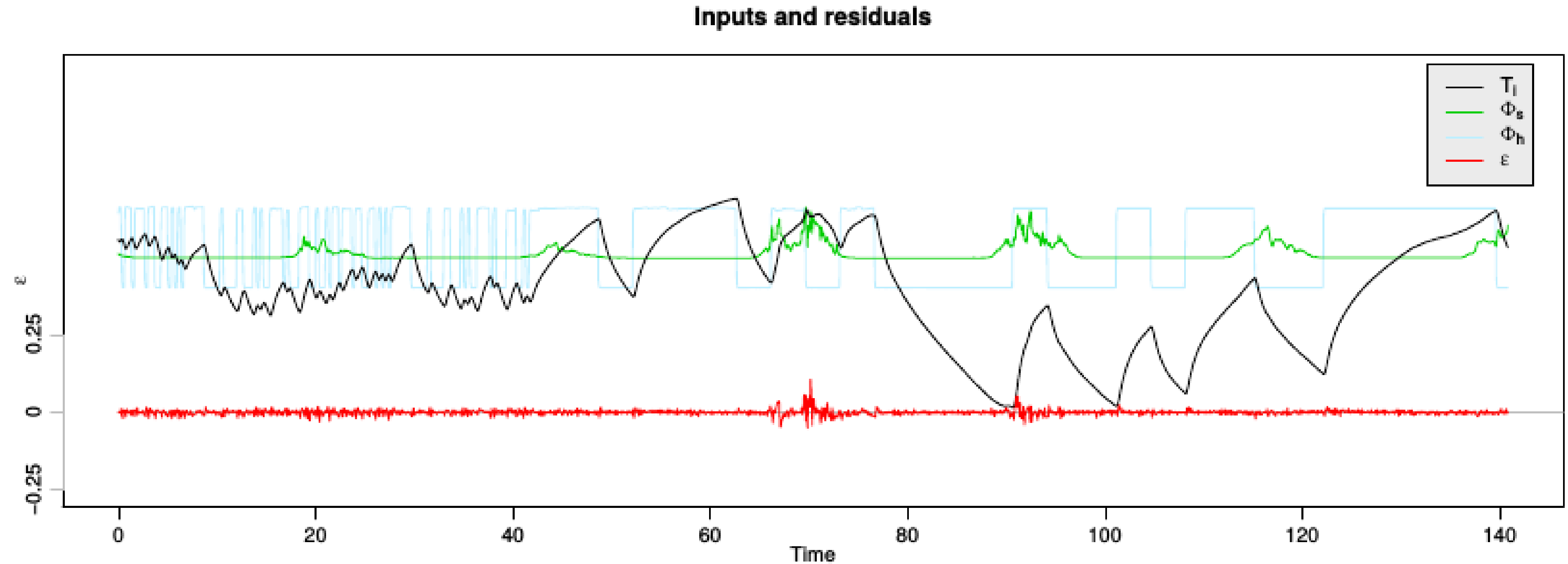
First extension: heater part

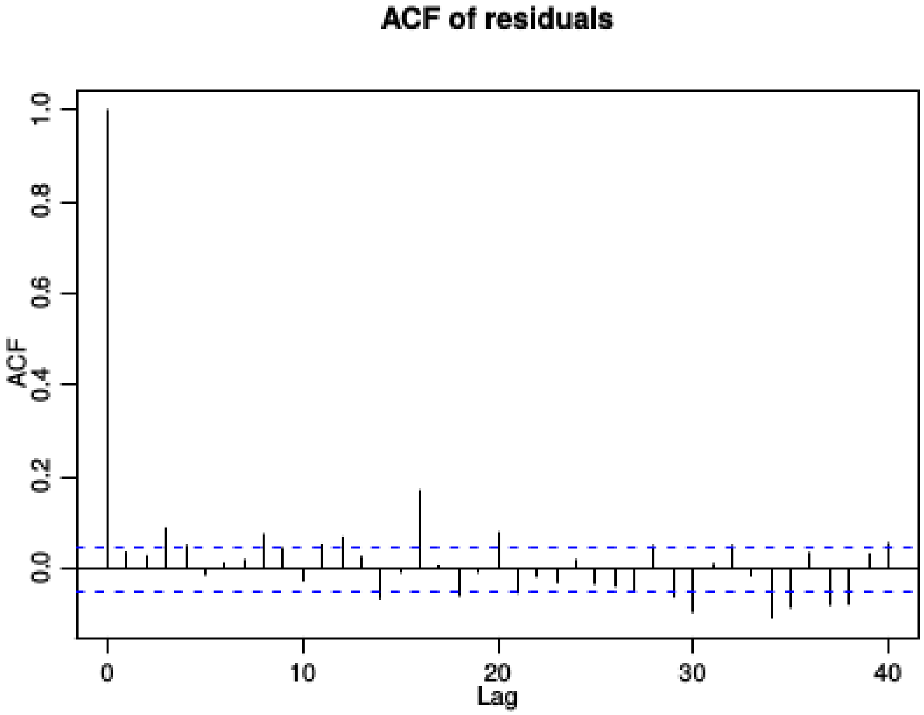


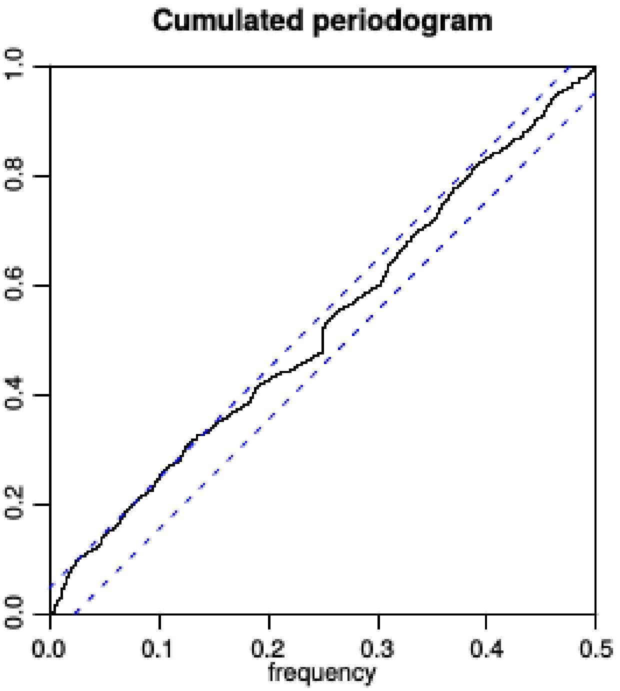
Start	$Model_{\mathrm{Ti}}$			
$l(\theta;\mathcal{Y}_N)$	2482.6			
m	6			
1	$Model_{\mathrm{TiTe}}$	$Model_{\mathrm{TiTm}}$	$Model_{\mathrm{TiTs}}$	$Model_{\mathrm{TiTh}}$
$l(\theta;\mathcal{Y}_N)$	3628.0	3639.4	3884.4	3911.1
m	10	10	10	10
2	…			

EVALUATE THE SIMPLEST MODEL









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Grey-box modelling

Continuous time models *(grey-box: stochastic state-space model)*

$$\text{States} = \text{Fun}_1(\text{States}, \text{Inputs}) + \text{Fun}_2(\text{Inputs}) \cdot \text{SystemError}$$
$$\text{Measurements} = \text{Fun}_3(\text{States}, \text{Inputs}) + \text{Fun}_4(\text{Inputs}) \cdot \text{MeasurementError}$$

- Used for buildings (single- and multi-zone), walls, systems (hot water tank, integrated PV, heat pumpts, heat exchanger, solar collectors, ...)
- Formulate the model based on physical knowledge
- Maximum likelihood estimation
  (we have the entire statistical framework available)
- Description of the system noise is part of the model provides some very useful possibilities
  (e.g. control the weight of data in the estimation depending on input signals)
- Software, for example our R package `CTSM-R` [1]

[1] `http://ctsm.info`

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Case study

Models for Smart Control (Ex: Control of Heat Pumps)

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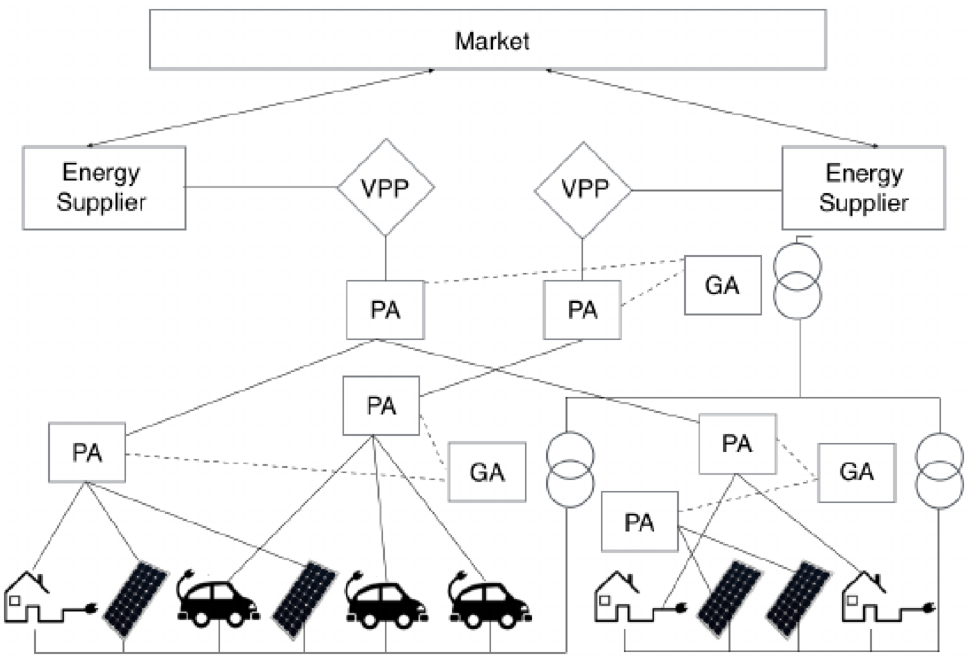
Existing Markets - Challenges

- Dynamics
- Stochasticity
- Nonlinearities
- Many power related services (voltage, frequency, balancing, spinning reserve, congestion, ...)
- Speed / problem size
- Characterization of flexibility
- Requirements on user installations

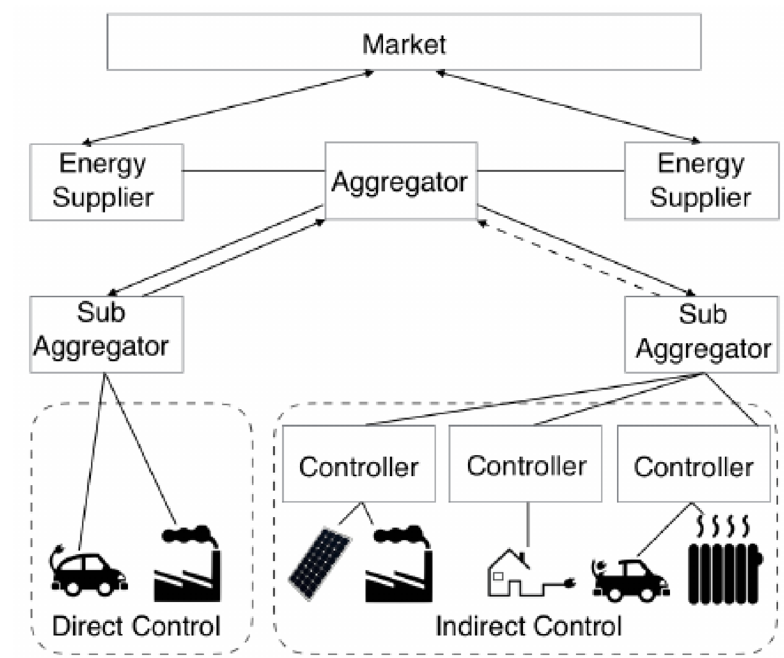
Different possibilities can be investigated for the coordination of the flexible resources:

Market- based approach



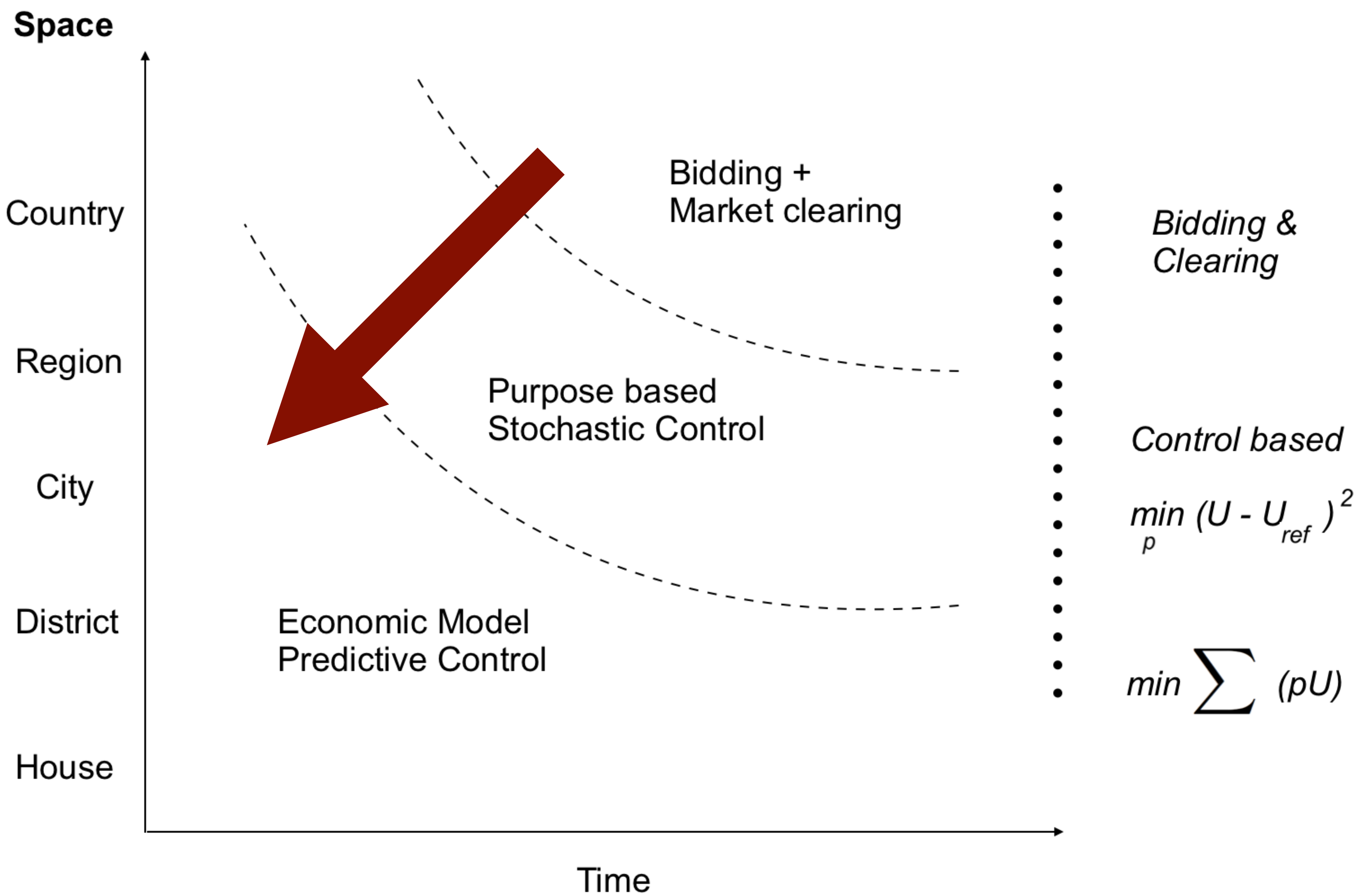
Market operation is intended all the way down to the prosumers' level.

Control- based approach



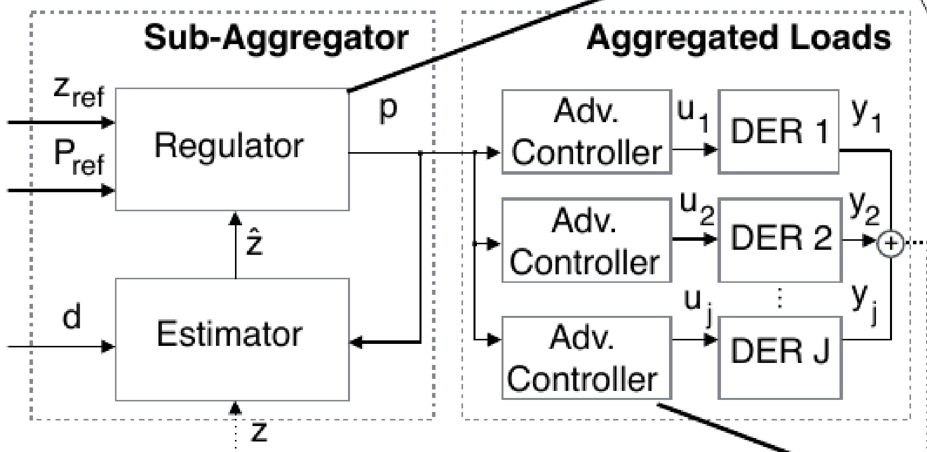
Control problem is formulated at the prosumers' level.

Suggested 'Market' Setup (Smart-Energy OS)



Proposed methodology
Control-based methodology

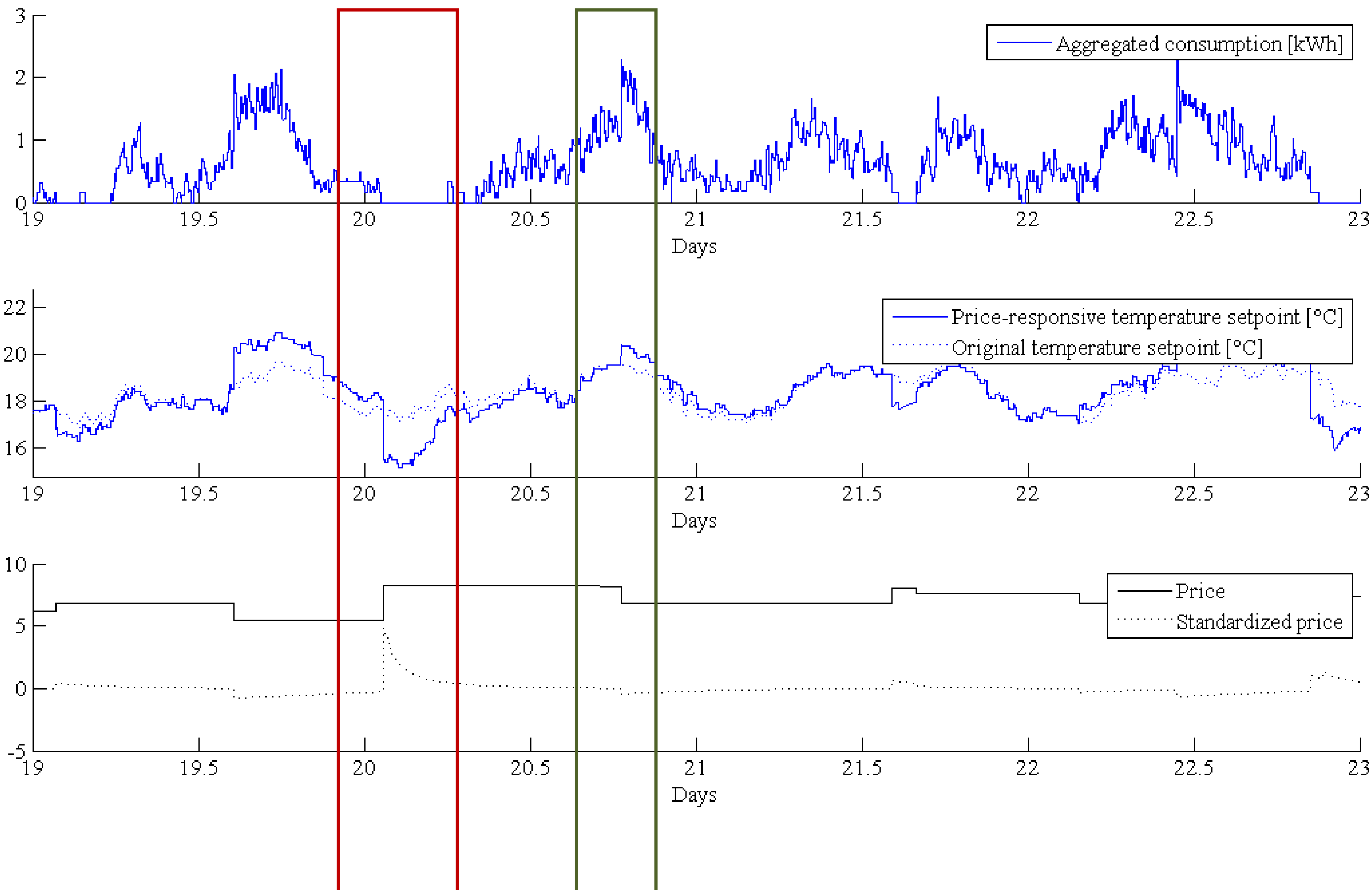
$$
\begin{aligned}
\min_{p} \quad & \mathrm{E}[\sum_{k=0}^{N} w_{j,k}||\hat{z}_k - z_{ref,k}|| + \mu||p_k - p_{ref,k}||] \\
\text{s.t.} \quad & \hat{z}_{k+1} = f(p_k)
\end{aligned}
$$



$$
\begin{aligned}
\min_{u} \quad & \mathrm{E}[\sum_{k=0}^{N}\sum_{j=1}^{J} \phi_j(x_{j,k}, u_{j,k}, p_k)] \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k, \\
& y_k = Cx_k, \\
& y_k^{min} \leq y_k \leq y_k^{max}, \\
& u_k^{min} \leq u_k \leq u_k^{max}
\end{aligned}
$$

We adopt a control-based approach where the **price** becomes the driver to **manipulate** the behaviour of a certain pool flexible prosumers.

Aggregation (over 20 houses)



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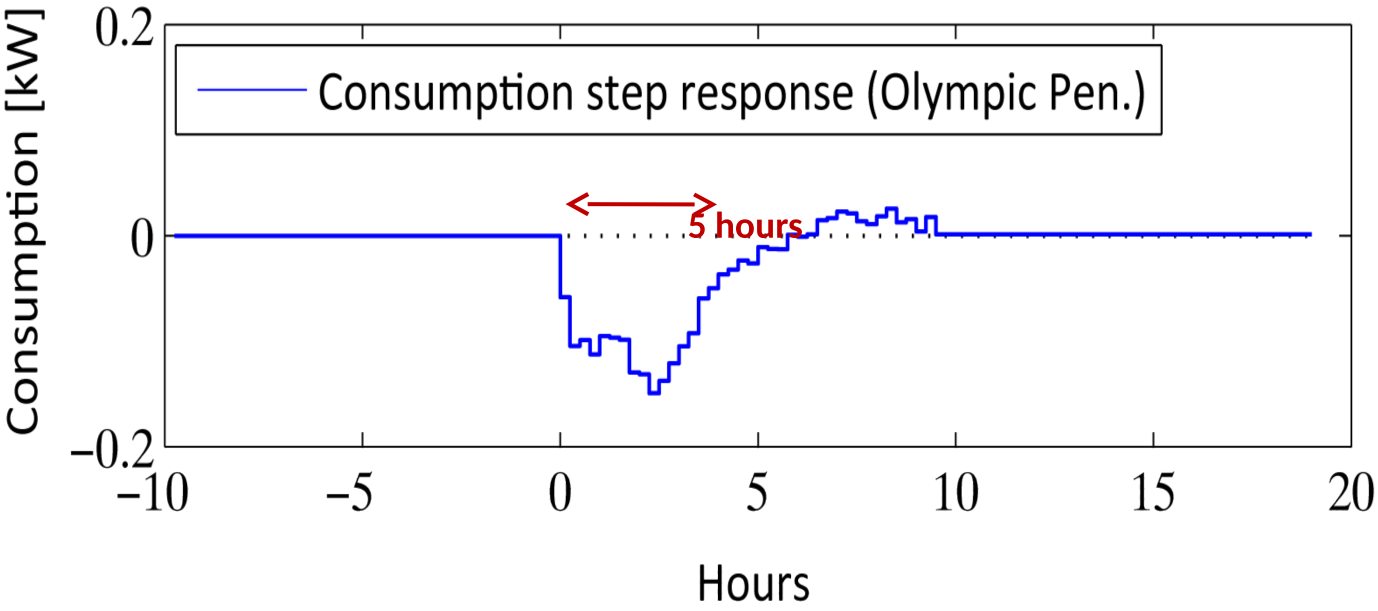
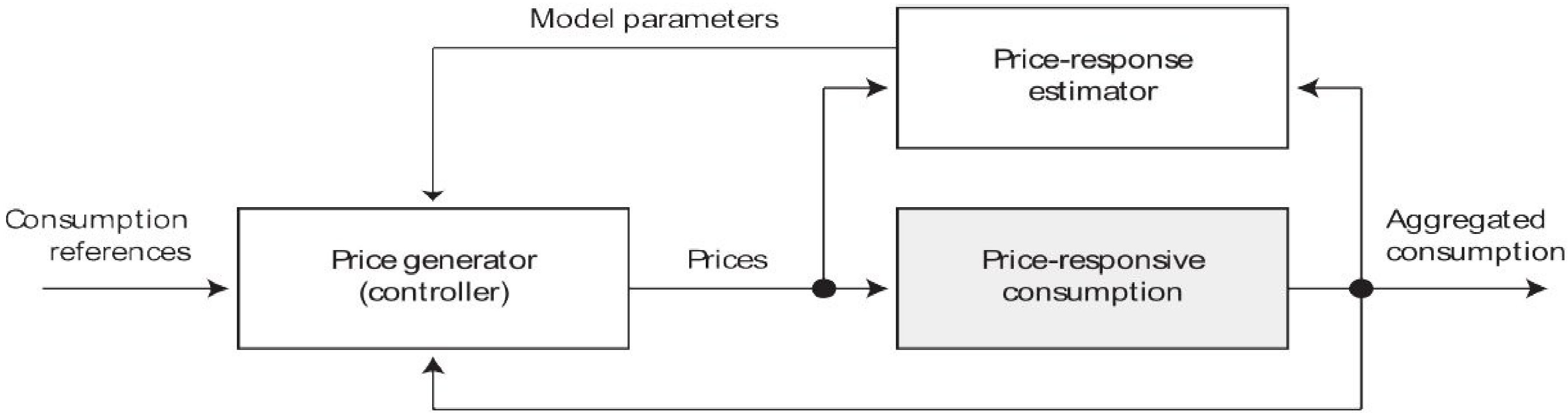
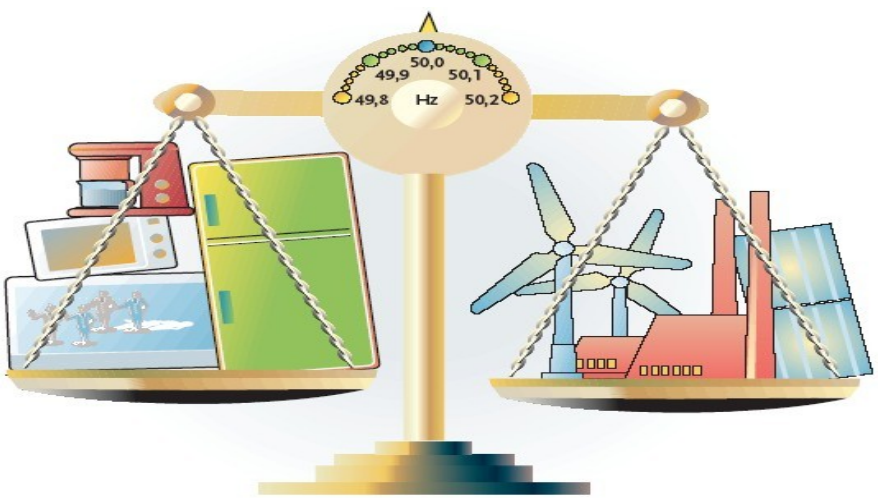
Response on Price Step Change



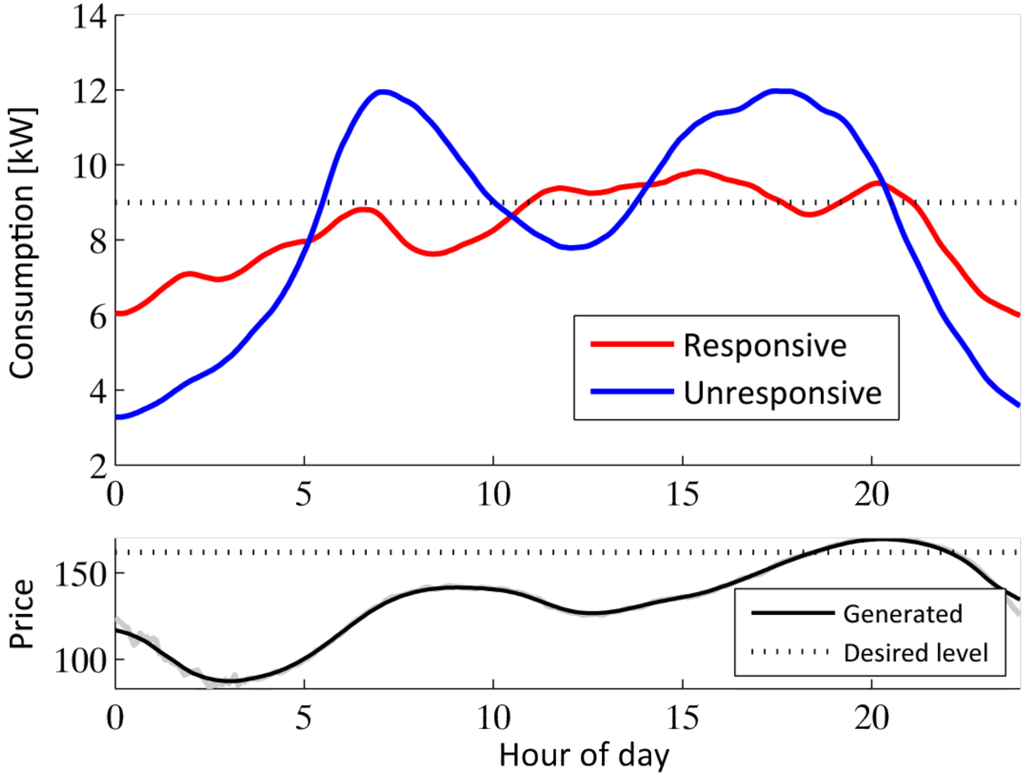
Control of Power Consumption





Control performance

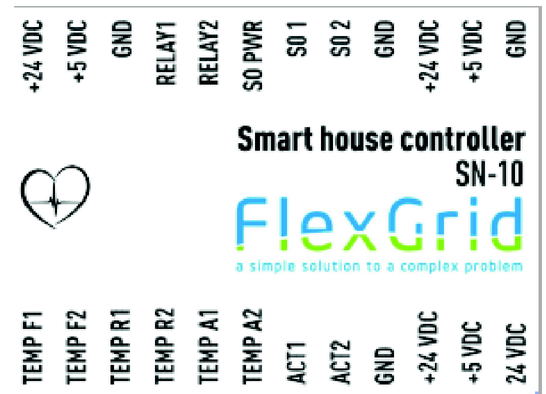
- Considerable **reduction in peak consumption**

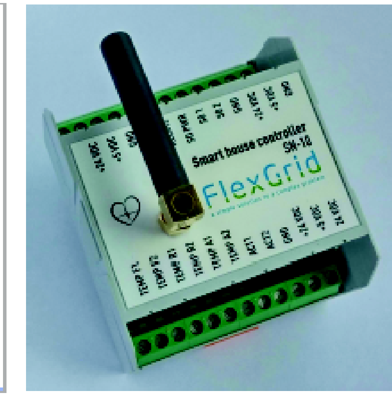


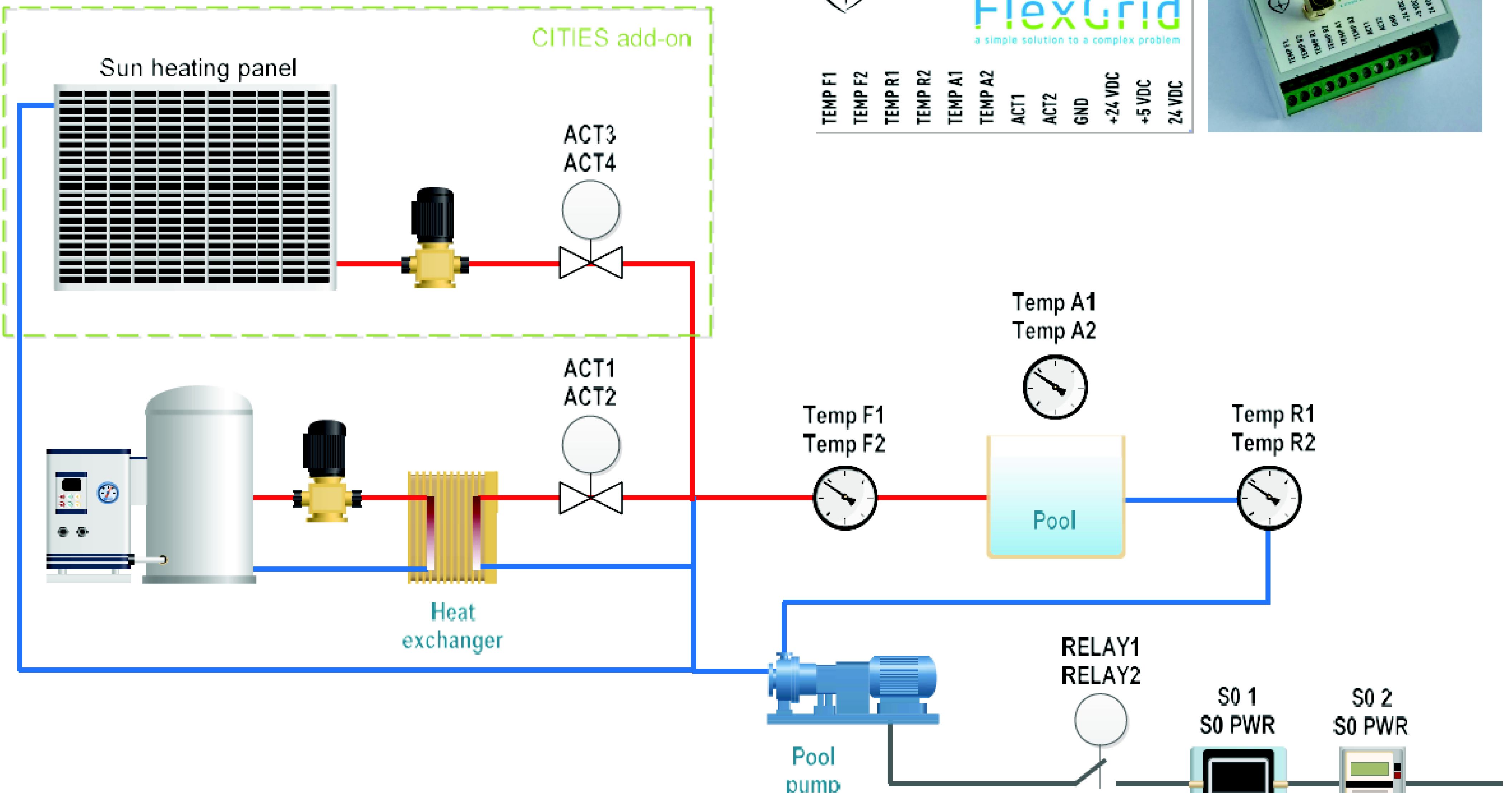
Smart Control of Houses with a Pool

PilotB SN-10 signal overview

revision 1.0 (CITIES add-on)







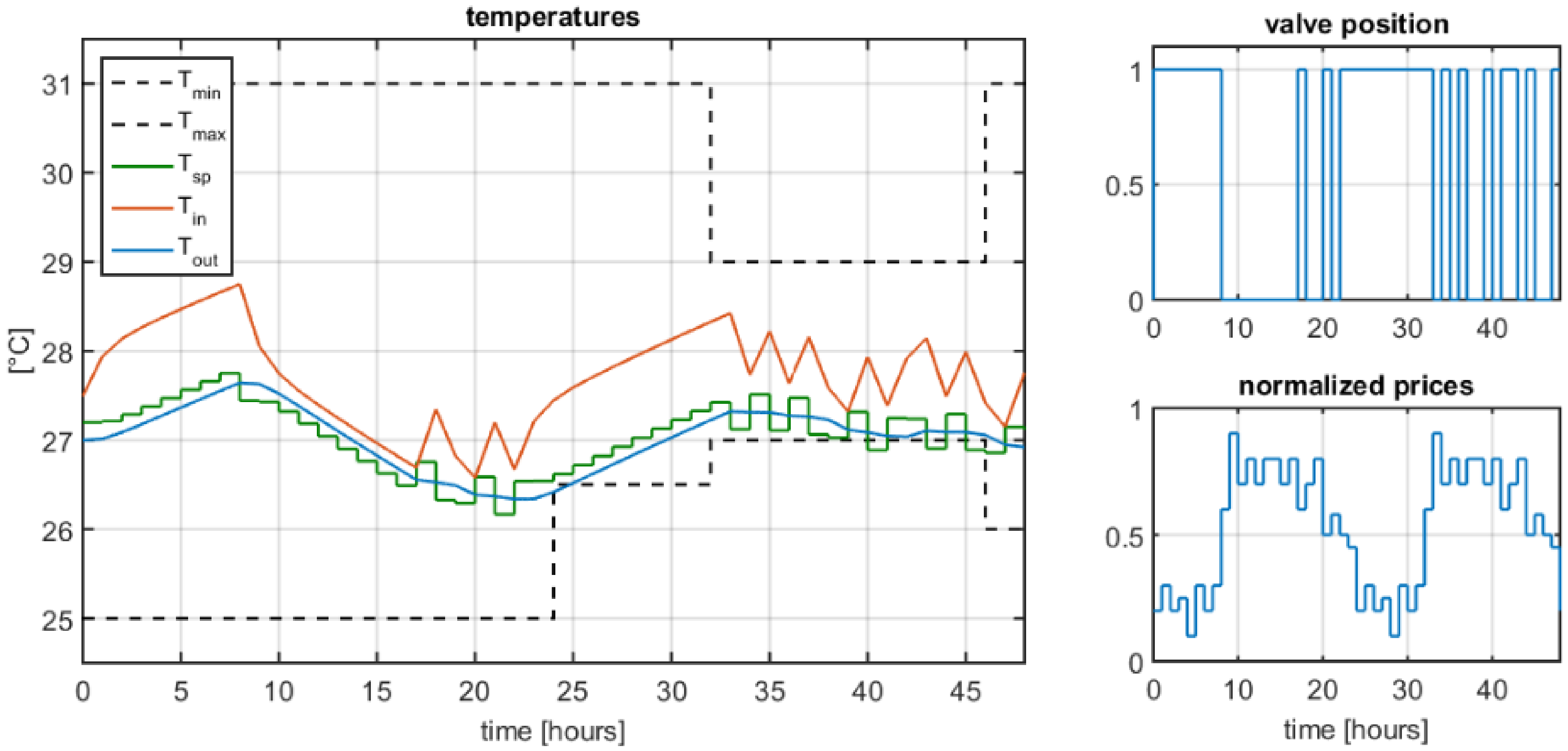
3.2 Optimization problem

The MPC controller solves the following mixed integer linear optimization problem:

$$
\begin{aligned}
&\min_{u} \quad \sum_{k=0}^{N-1} c_k u_k && (3.2a)\\
&s.t. \quad x_{k+1} = A_d(T_o, w, T_a)x_k + B_d(T_o, w, T_a)u_k && (3.2b)\\
&\qquad\quad y_k = C_d(T_o, w, T_a)x_k && (3.2c)\\
&\qquad\quad u_k \in \{0, 1\} && (3.2d)\\
&\qquad\quad y_{min} \le y_k \le y_{max} && (3.2e)
\end{aligned}
$$

where (3.2b) and (3.2c) is discretized state-space model of (2.6); u_k is the valve position (1 - open; 0 - closed); $y_k = [T_{in,k}\ T_{out,k}]^T$; N is the predictive horizon; c_k is the electricity price.

MPC Results


temperatures
T_{min}
T_{max}
T_{sp}
T_{in}
T_{out}
[°C]
31
30
29
28
27
26
25
0
5
10
15
20
25
30
35
40
45
time [hours]
valve position
1
0.5
0
0
10
20
30
40
normalized prices
1
0.5
0
0
10
20
30
40
time [hours]

Part 3
Special Data (eg Non-Gaussian)

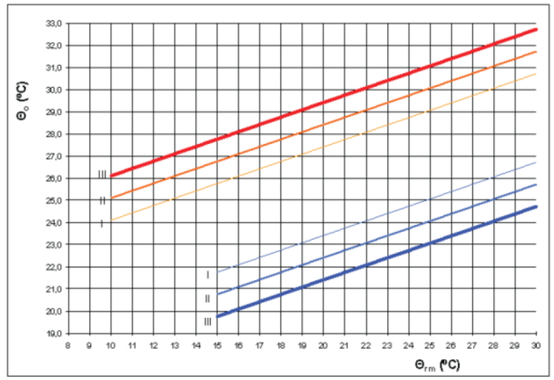
Identification of Occupant Behavior

- Use of CO_2 measurements to model occupant behavior in summer houses

Today’s situation for indoor climate in buildings

Design

Regulations
Specifications
Designers
experience
Standards



Percent Persons Dissatisfied

Temperature
Air Quality
Humidity
Draft
Radiation
symmetry

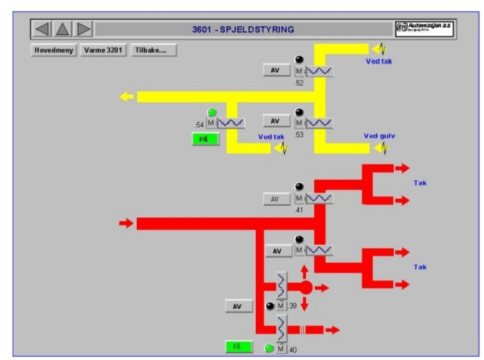
Designed based on assumptions
Focus on in-direct parameters to user satisfaction
Little knowledge of true user preferences

Service

User

Telephone contact

Janitor (subjective assessment)



Adjustments to indoor climate

No info of total user satisfaction
Subjective responses and actions
Building controlled after in-direct parameters to user satisfaction
No systematic logging or learning

Digital revolution

Google

facebook

When we use their product, they gather information on how we use it, our preferences etc.
Crucial information regarding how the product can be further developed

The «Internet Of Things» (IoT) and sensor technology enables us to do the same for buildings. But how?

Accuracy of the parameters used in building automation

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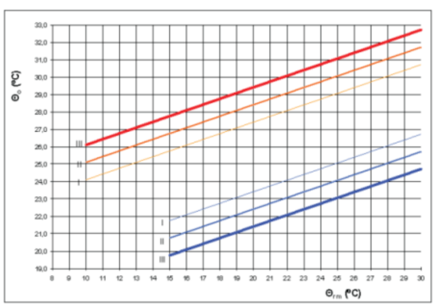


Ole Fanger

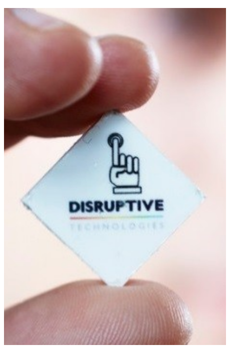
Vs.

+

Building automation can be made more accurate to the user's needs.
Rather than using theoretical values and «ad-hoc» user feedback, Smart Technology enables use of Big Data from sensors and continous user feedback.

Collect data on three levels

Physical environment	Sensed environment	Total user satisfaction



Skanska Comfort Control App

Please submit your feedback:

Cooler | Warmer

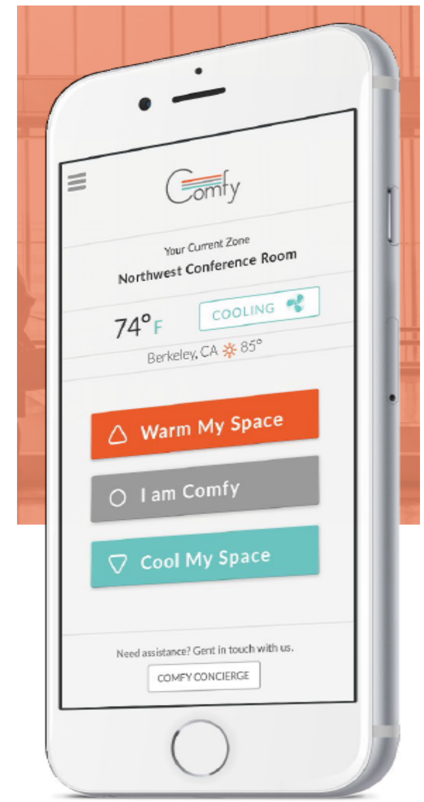
Bad air | Draft

Smartphone or room tablet

Measurement of user satisfaction at entrance door

By using a simple system for data collection via existing rooom automation systems, new smart sensors, smart phones with IoT and cloud computing we can achieve a high degree of accuracy for the automation system. Collecting data about the indoor environment and user at the same time

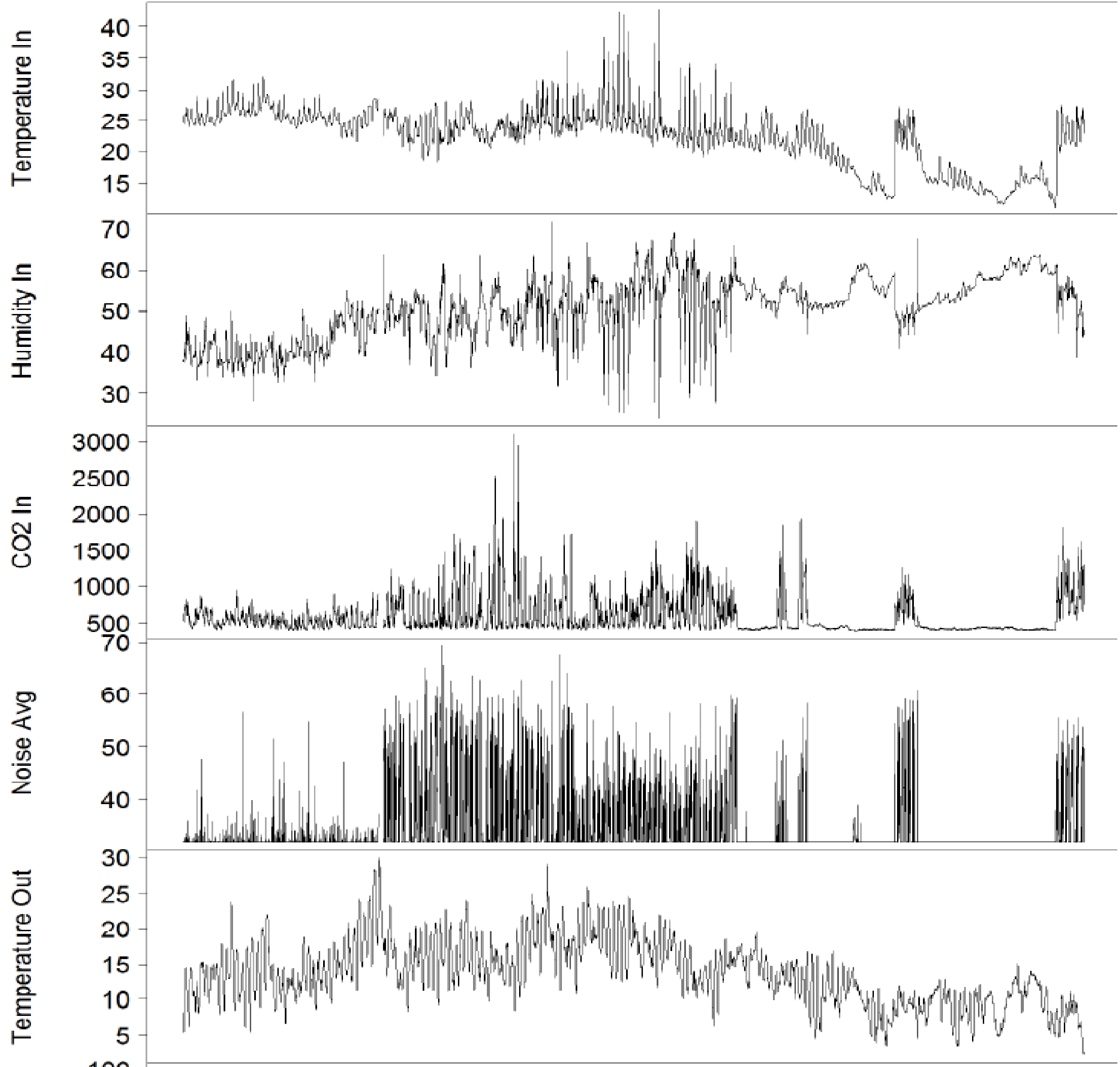
Example:



Existing product/service enabling the user to give feedback regarding thermal comfort via smartphone app. The system uses this feedback to find the best temperature profile regarding economy and user satisfaction for the building. Claims to give 20% reduction in energy costs as well as improved user satisfaction.
Could REBUS do the same, using the system to keep our customers happy and at the same time collect data helping us innovate and improve our buildings?

Summer houses represent a special challenge

- Large variation in the number of people present in the house
- Power Grids in summer house areas represent a special problem for some DSOs
- Time series of CO_2 measurements are the key to the classification



Temperature In
40
35
30
25
20
15
Humidity In
70
60
50
40
30
CO2 In
3000
2500
2000
1500
1000
500
Noise Avg
70
60
50
40
Temperature Out
30
25
20
15
10
5

The Model Space

$$\boldsymbol{\theta} \sim f\left(\beta_{\text{fixed}}, t, \cdots\right) + g\left(U_{\text{random}}, t, \cdots\right) \tag{1a}$$

$$d\boldsymbol{X}_t \sim \text{Dynamical model}\left(\boldsymbol{\theta}\right) \tag{1b}$$

$$\begin{aligned} Y_t^{(1)} &= \text{Electrical consumption} \\ Y_t^{(2)} &= \text{Noise (indoor)} \\ Y_t^{(3)} &= \text{CO}_2 \text{ (indoor)} \\ &\vdots \end{aligned} \tag{1c}$$

- $\boldsymbol{\theta}$ parameter vector for population/hierarchical model
  - Time, weather, demographics
- $d\boldsymbol{X}_t$ state vector described by some dynamical model depending on $\boldsymbol{\theta}$
  - People, consumption, windows
- Y's: Observed measurements related to occupancy behavior, including measurements inside and outside the building and smart metering data

Hidden Markov Model

First Order Markov Property

$$p(X_t|X_{t-1}) = p(X_t|\mathcal{X}^{(t-1)}), \quad t \in \mathbb{N} \tag{2}$$

$$p(Y_t|X_t) = p(Y_t|\mathcal{X}^{(t)}, \mathcal{Y}^{(t-1)}), \quad t \in \mathbb{N} \tag{3}$$

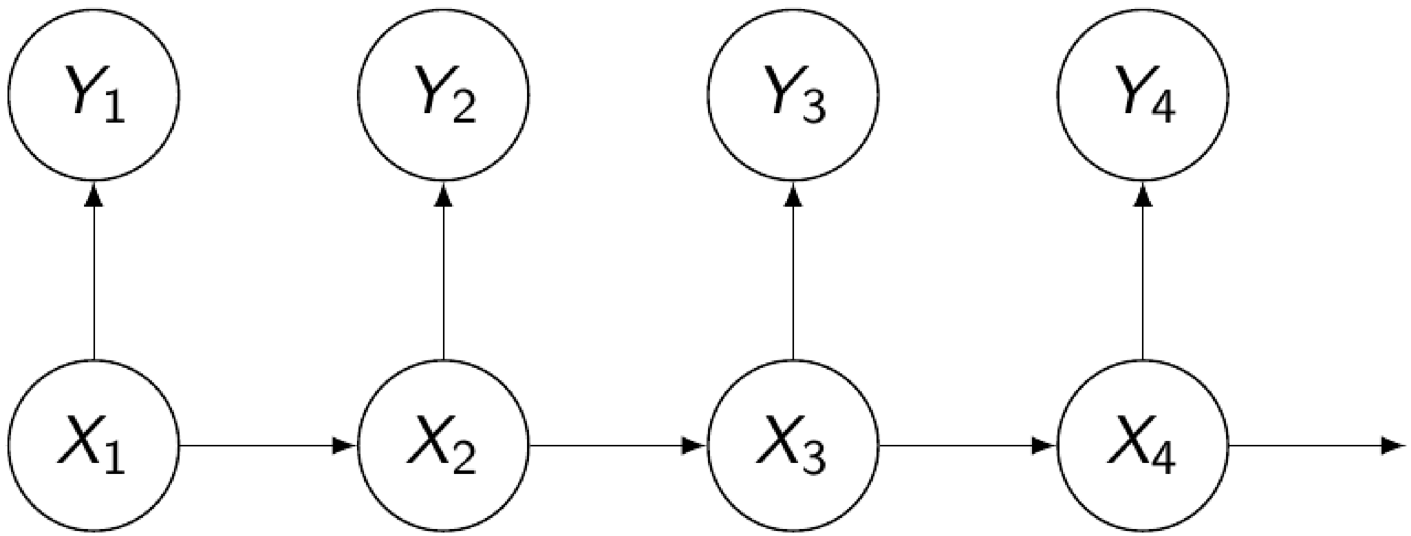


Figure: Directed graph of basic HMM. The index denotes time.

Markov Chains

Discrete state vector at time t, X_t, with m states.

Transition probability

$$p(X_t = j | X_{t-s} = i) \tag{4}$$

One-step transition probability

$$\gamma_{ij,t} = p(X_t = j | X_{t-1} = i) \tag{5}$$

One-step transition probability matrix from time $t-1$ to t

$$\mathbf{\Gamma}_t = \begin{pmatrix} \gamma_{11,t} & \cdots & \gamma_{1m,t} \\ \vdots & \ddots & \vdots \\ \gamma_{m1,t} & \cdots & \gamma_{mm,t} \end{pmatrix} \tag{6}$$

where the row must sum to 1.

Homogen Hidden Markov Model

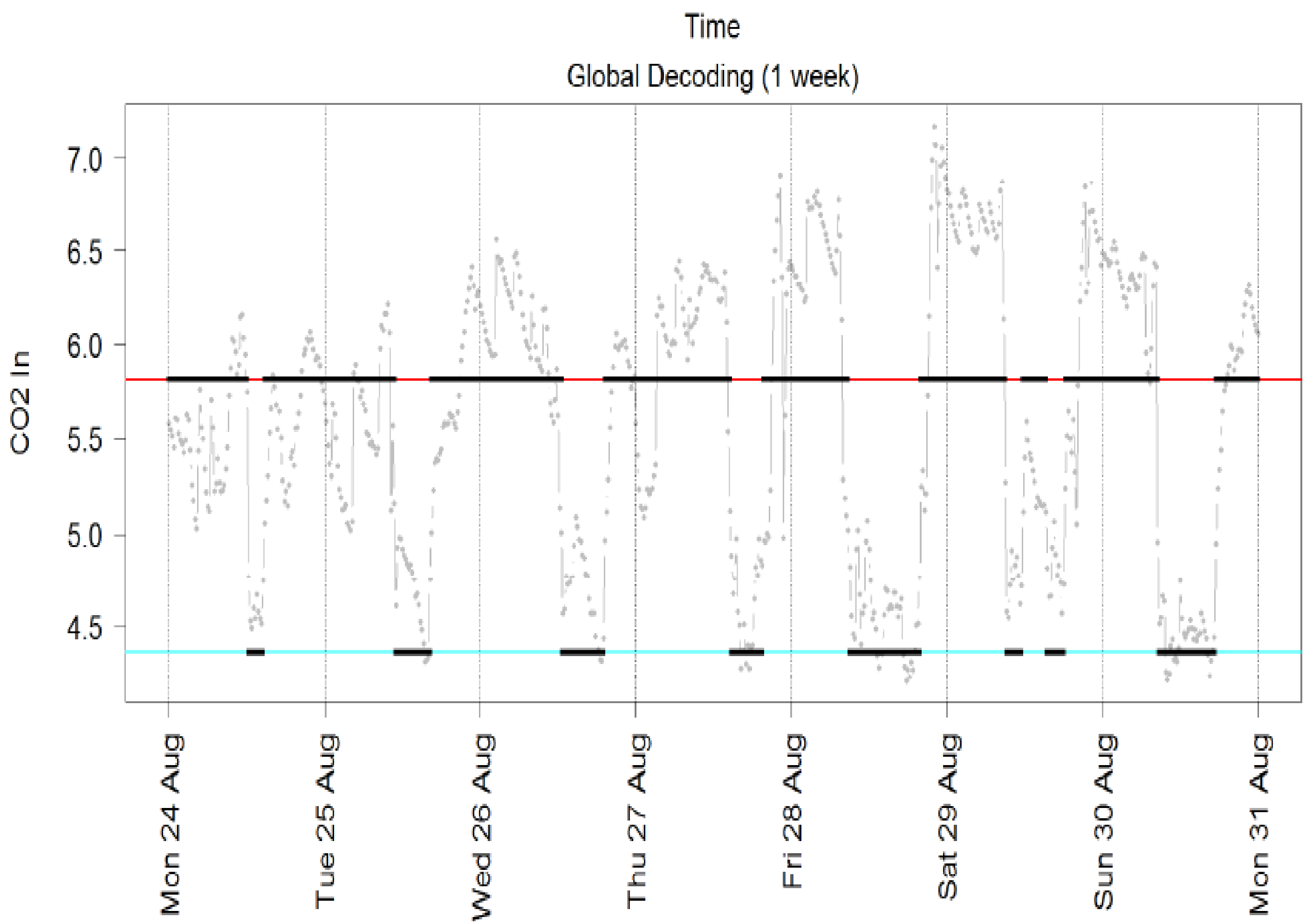
Setting

$$
\begin{aligned}
y_t &= h(CO_{2,t}) \\
p(x_t|x_{t-1}) &\sim \Gamma \\
p(y_t|x_t) &\sim \mathcal{N}\left(\mu_i, \sigma_i^2\right) \text{ for } i = 1, 2, \cdots, m
\end{aligned}
$$

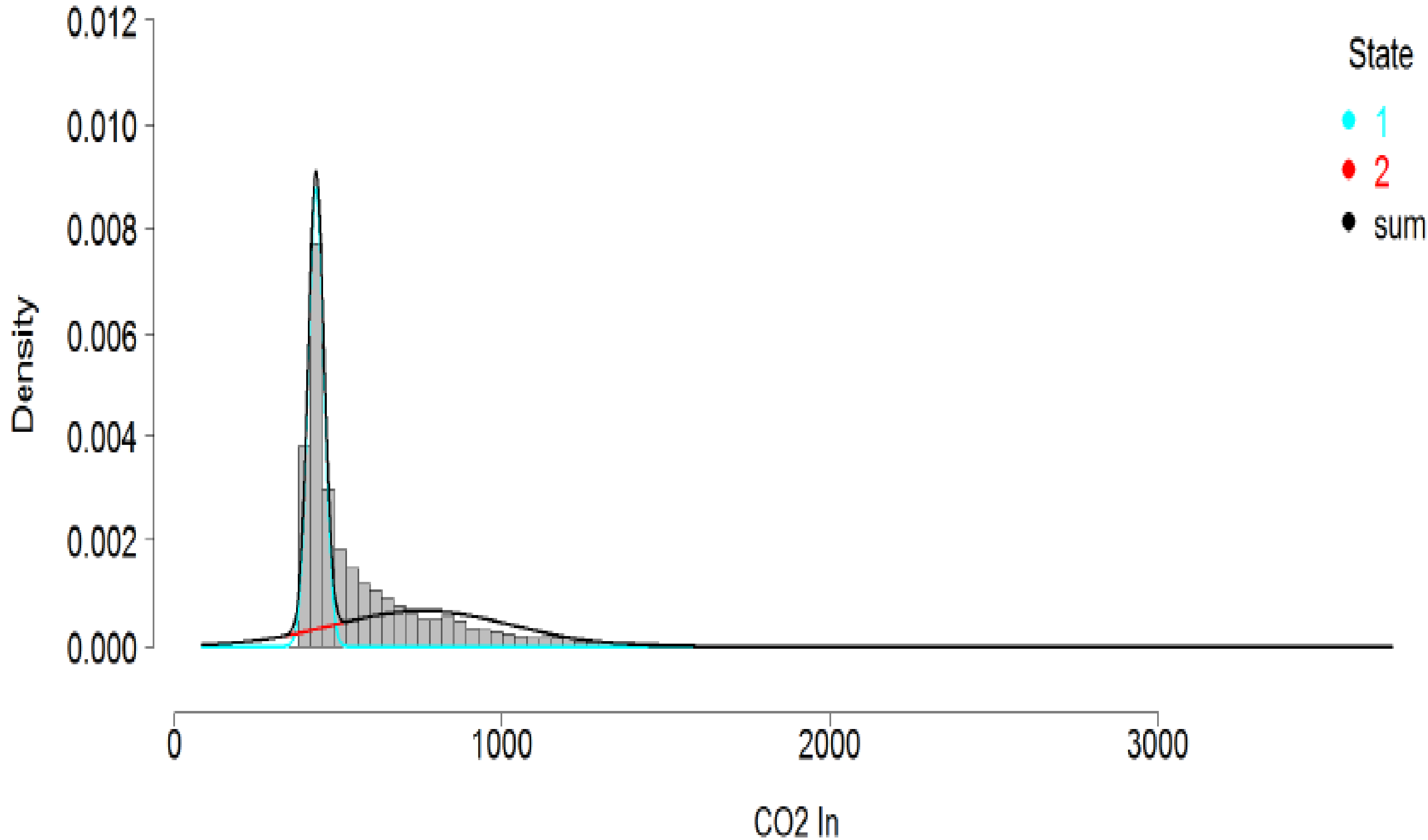
Note that there is no time dependence in the transition probabilities in the homogen case.

Table 8.4: Comparison of univariate (log transformed CO_2) homogen HMMs for 2 to 5 states.

	$\mathcal{L}$	p	AIC	BIC
2 states	-9378	6	18768	18814
3 states	-4292	12	8609	8701
4 states	-800	20	1640	1795
5 states	2181	30	-4303	-4071



Time
Global Decoding (1 week)
CO2 In
7.0
6.5
6.0
5.5
5.0
4.5
Mon 24 Aug
Tue 25 Aug
Wed 26 Aug
Thu 27 Aug
Fri 28 Aug
Sat 29 Aug
Sun 30 Aug
Mon 31 Aug


State
1
2
sum
Density
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
1000
2000
3000
CO2 In

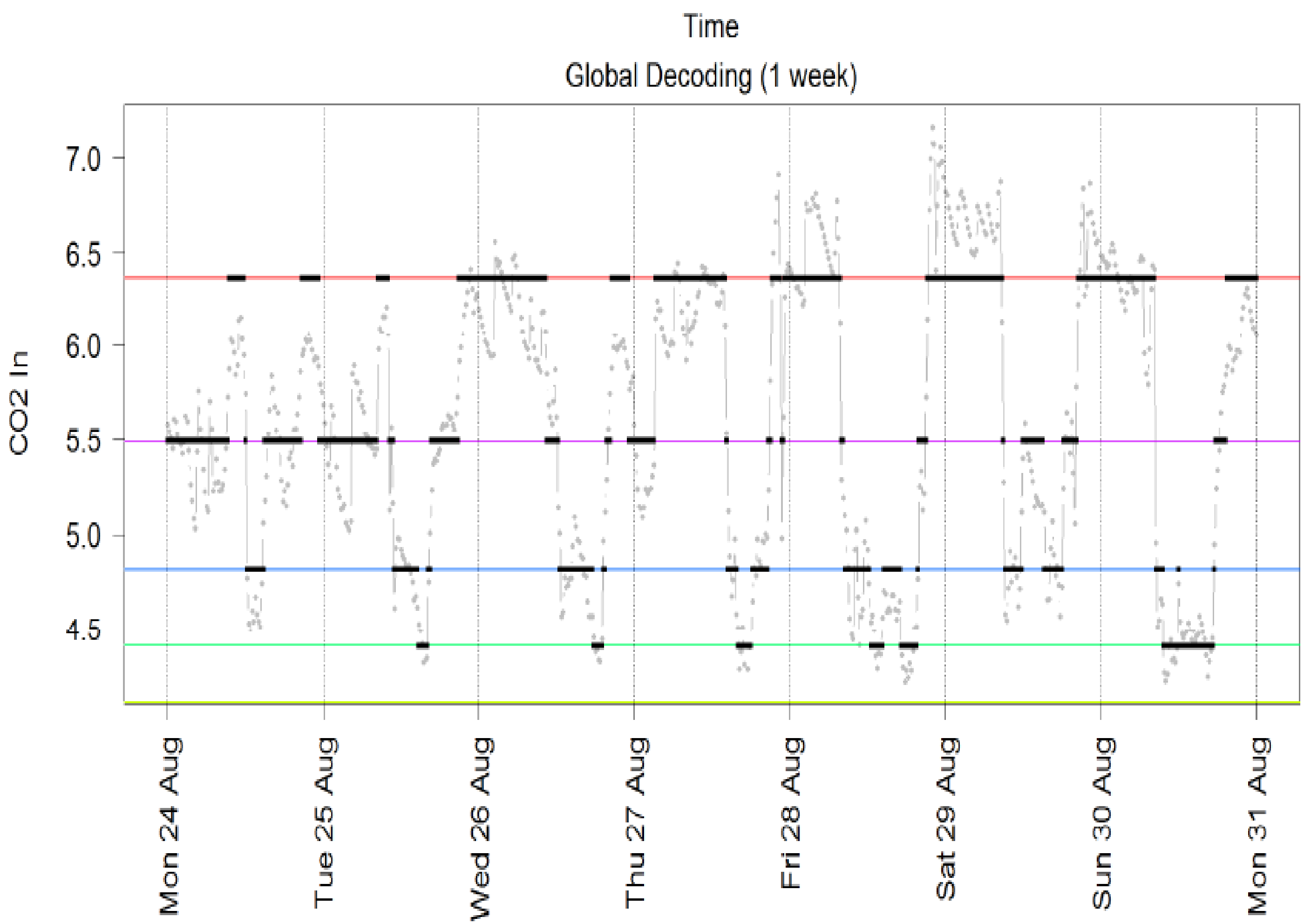
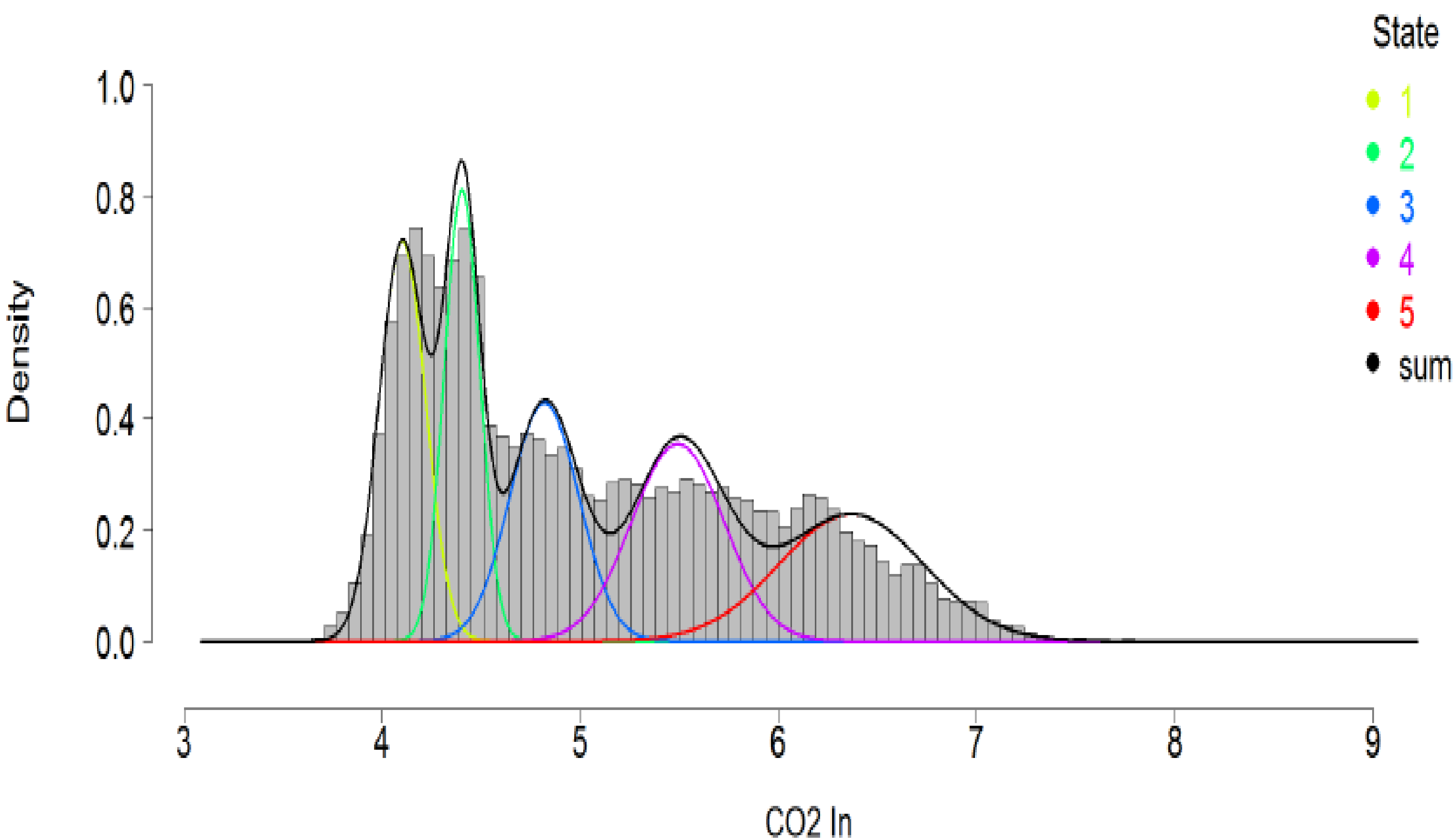
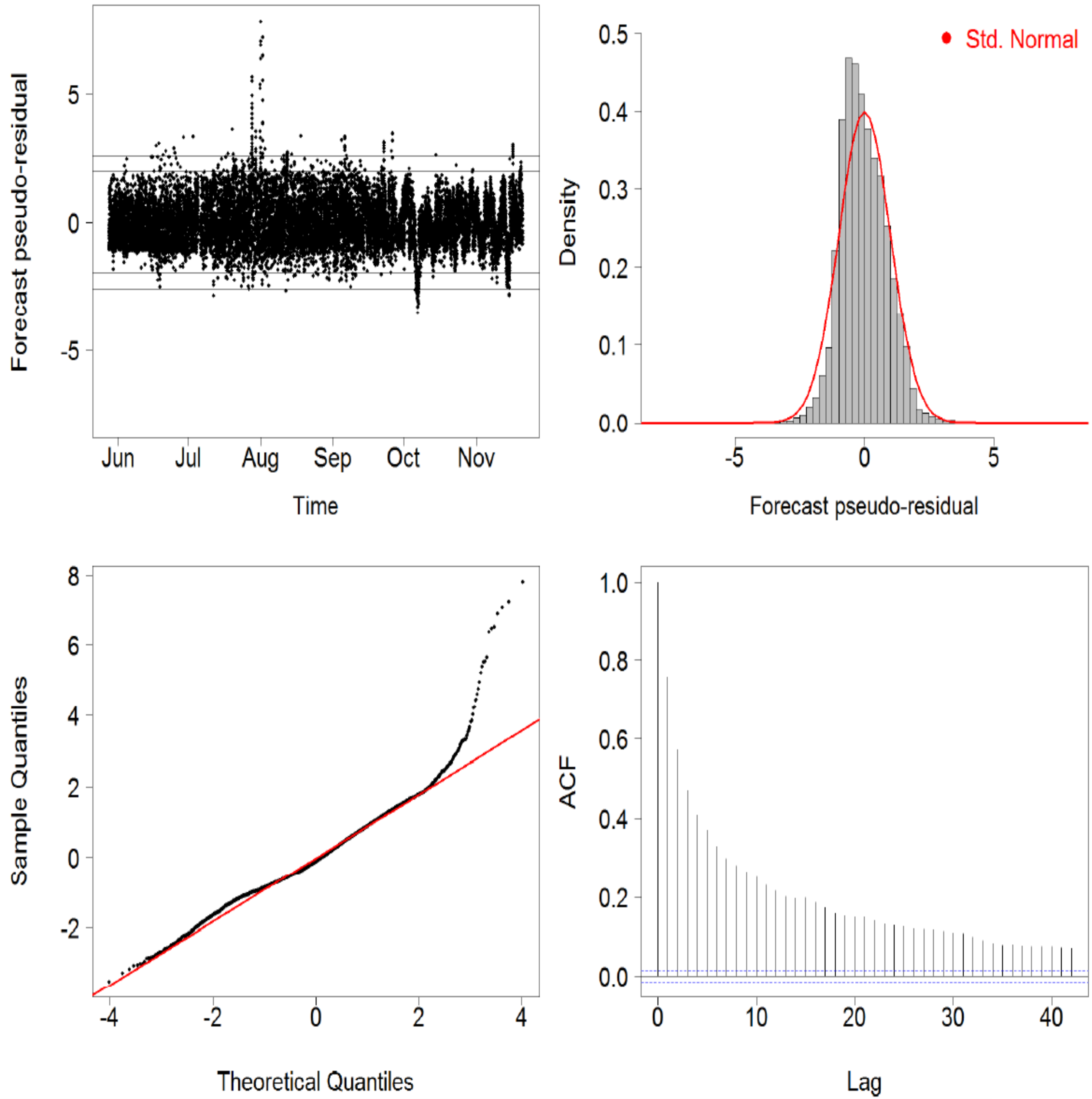


Figure 8.7: Global Decoding of the HMM ($\log CO_2$) with 5 states.



State
1
2
3
4
5
sum
1.0
0.8
0.6
0.4
0.2
0.0
Density
3
4
5
6
7
8
9
CO2 In

CO2 In


Forecast pseudo-residual
5
0
-5
Jun
Jul
Aug
Sep
Oct
Nov
Time
Std. Normal
Density
0.5
0.4
0.3
0.2
0.1
0.0
-5
0
5
Forecast pseudo-residual
Sample Quantiles
8
6
4
2
0
-2
-4
-2
0
2
4
Theoretical Quantiles
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
Lag

Inhomogen Hidden Markov Model

Setting

$$
\begin{aligned}
y_t &= h(CO_{2,t}) \\
p(x_t|x_{t-1}) &\sim \Gamma_t \\
p(y_t|x_t) &\sim \mathcal{N}\left(\mu_i, \sigma_i^2\right) \text{ for } i = 1, 2, \cdots, m
\end{aligned}
$$

Note that there is time dependence in the transition probabilities in the inhomogen case.

Inhomogen Markov-switching with auto-dependent observations

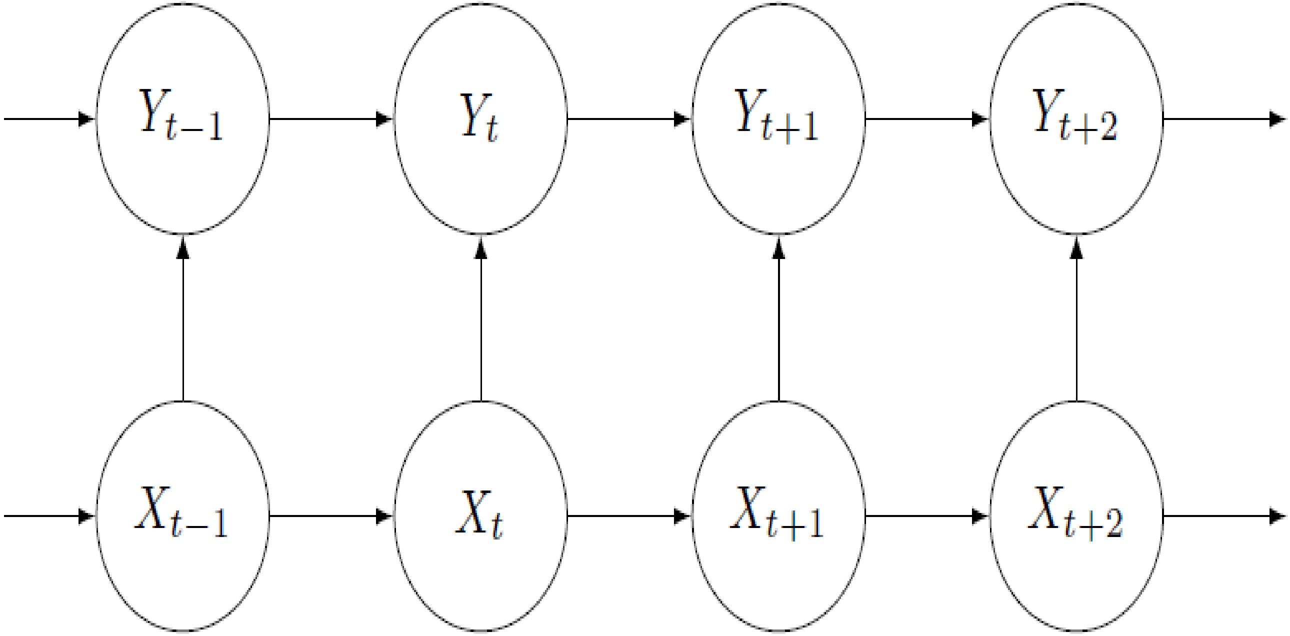


Figure 8.10: Directed graph of Markov switching AR(1).

Inhomogen Markov-switching AR(1)

Setting

$$
\begin{aligned}
y_t &= h(CO_{2,t}) \\
p(x_t | x_{t-1}) &\sim \Gamma_t \\
p(y_t | x_t, y_{t-1}) &\sim \mathcal{N}\left(c_i + \phi_i y_{t-1}\ ,\ \sigma_i^2\right) \text{ for } i = 1, 2, \cdots, m
\end{aligned}
$$

Note that there is time dependence in the transition probabilities in the inhomogen case.

Interpretation of the states

- State 1: Absence or sleeping
- State 2: Long term absence
- State 3: Outdoor interaction
- State 4: Presence (high activity)
- State 5: Presence (long term, low activity)

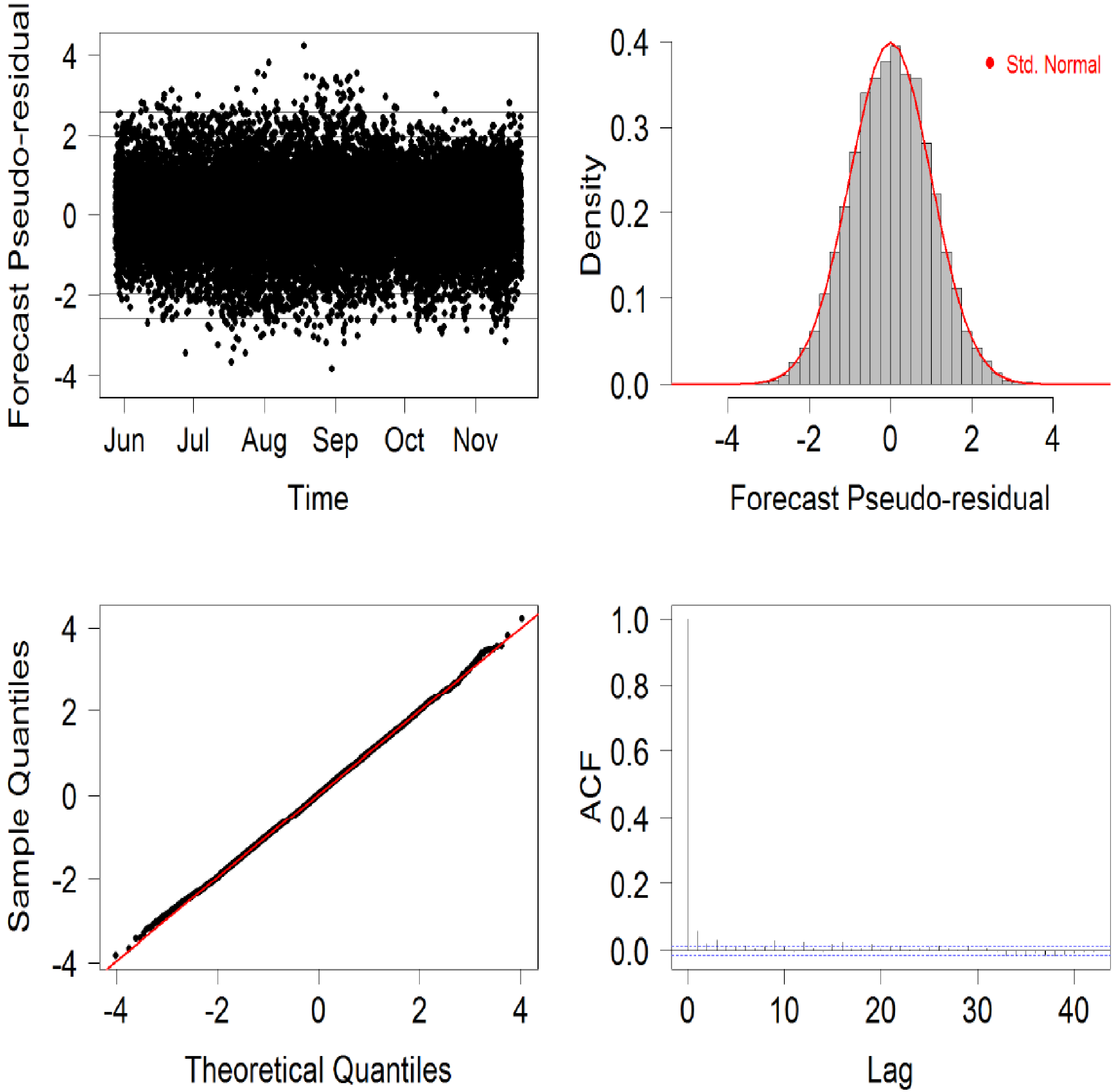
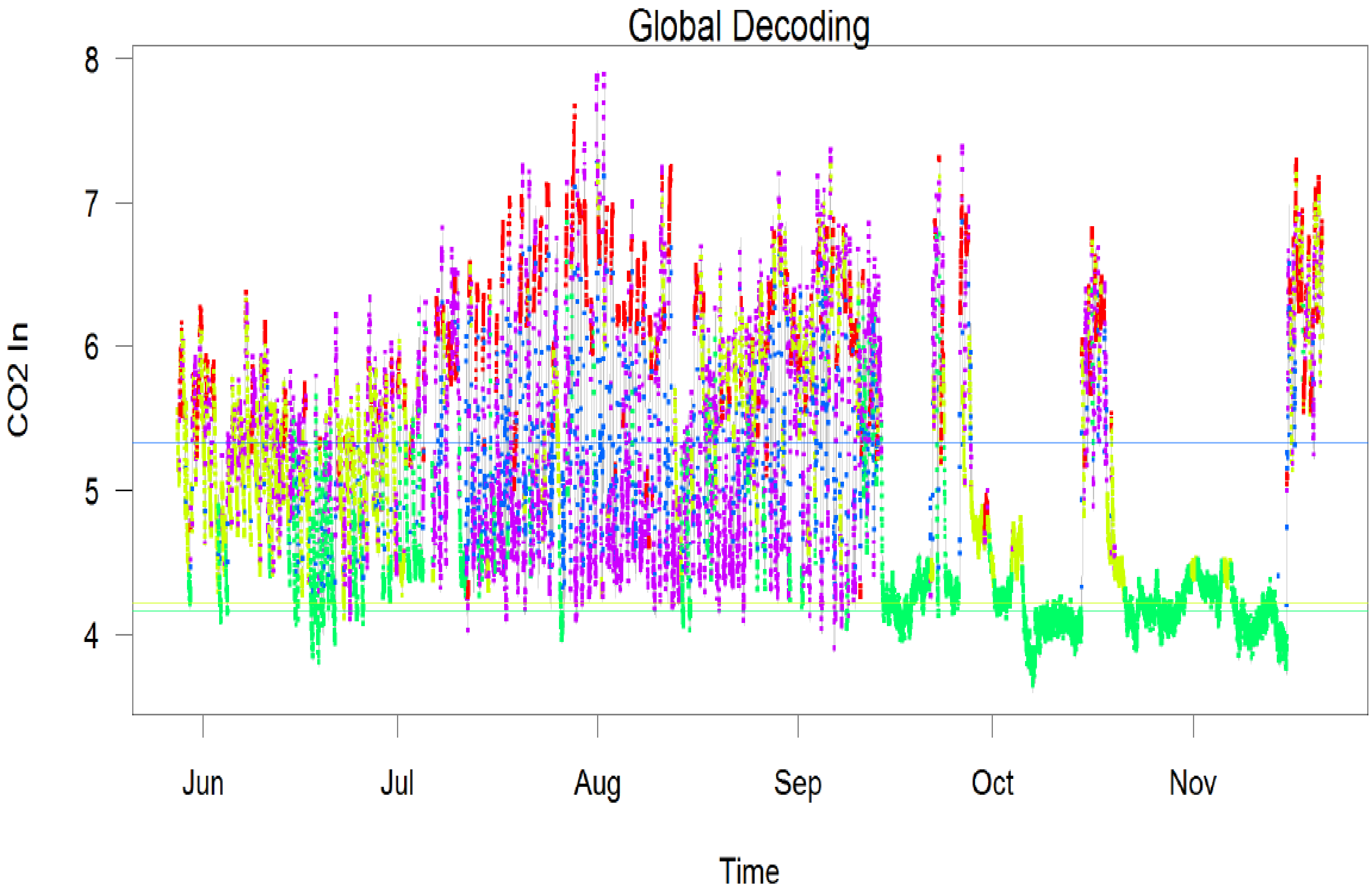


Figure 8.11: Model diagnostics of the final model.


Global Decoding
CO2 In
8
7
6
5
4
Jun
Jul
Aug
Sep
Oct
Nov
Time

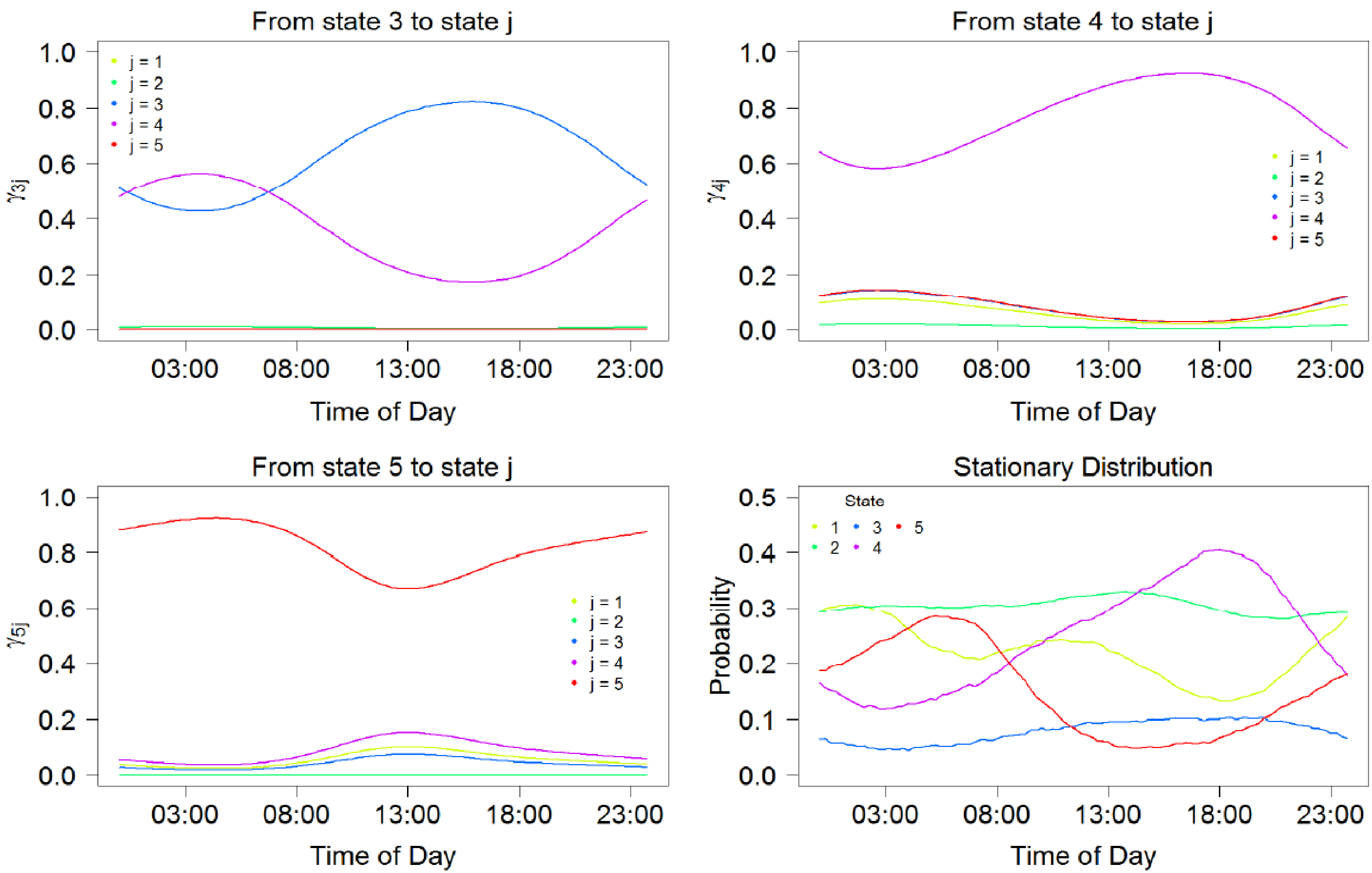


Figure 8.16: Transition probabilities over the day of the final model. The lower right plot is the stationary distribution.

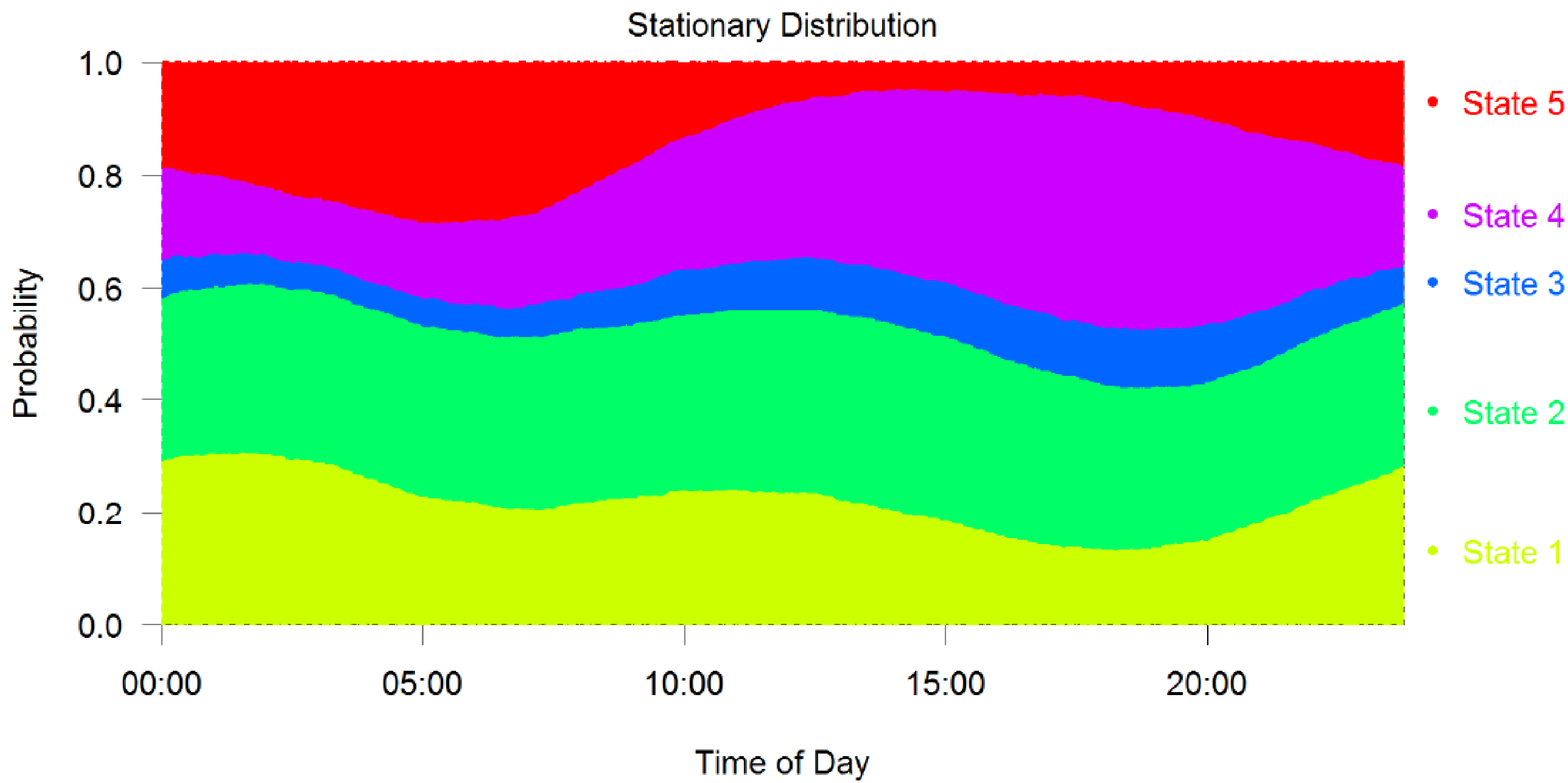


Figure 8.17: Profile of the states over the course of the day. I.e. Stacked stationary probabilities over the course of the day of the final model.

Some conclusions:

That the low activity state 5 is not very likely from 10 am to 11 pm.
The high activity is seen in the late afternoon.

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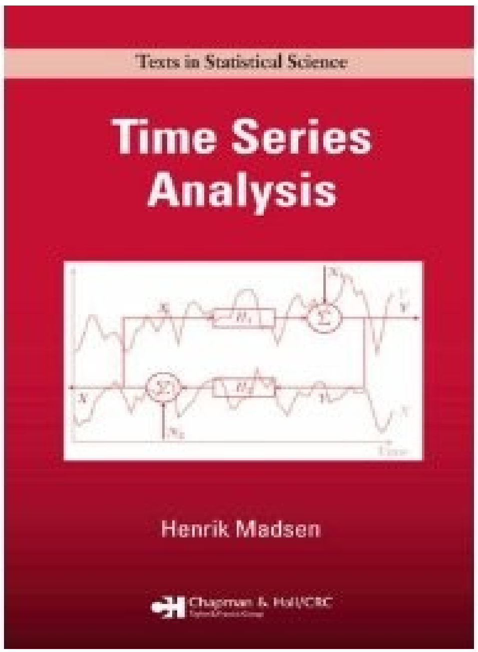
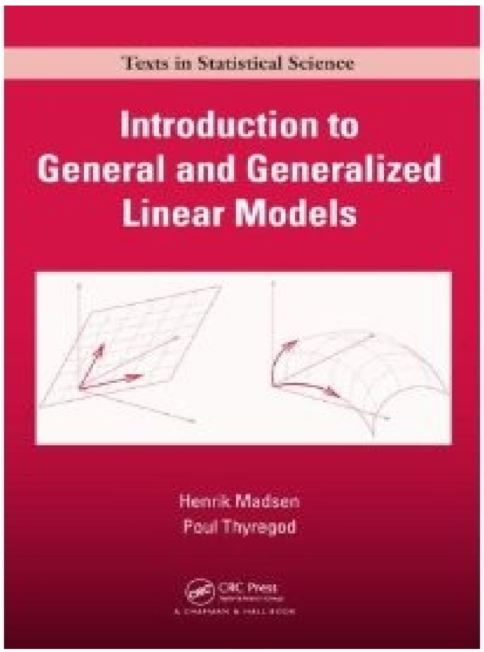
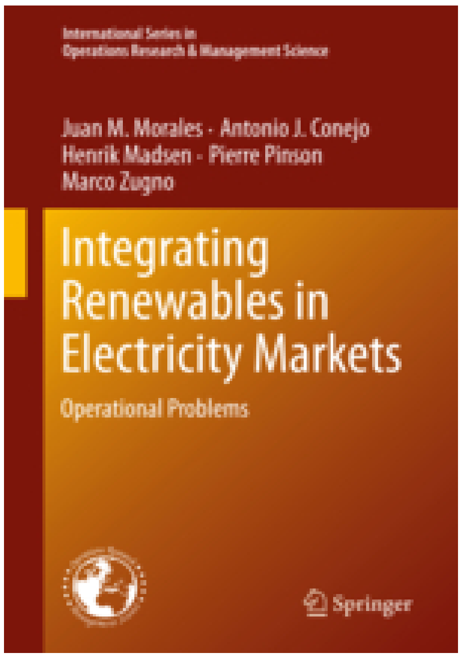
Some 'randomly picked' books on modeling ....







Thanks ...

- For more information

  www.ctsm.info

  www.henrikmadsen.org

  www.smart-cities-centre.org

- ...or contact
  - Henrik Madsen (DTU Compute)

    hmad@dtu.dk

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