Using stochastic differential equations for wind and solar power forecasting

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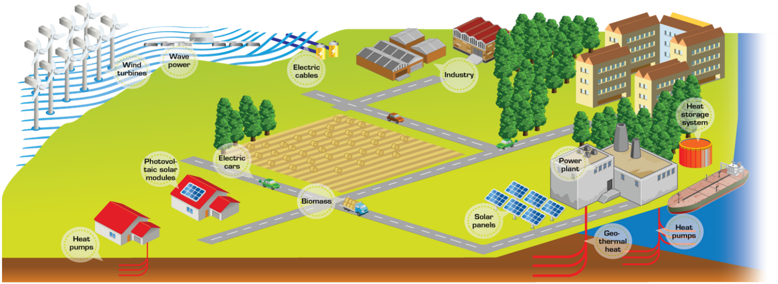
Outline

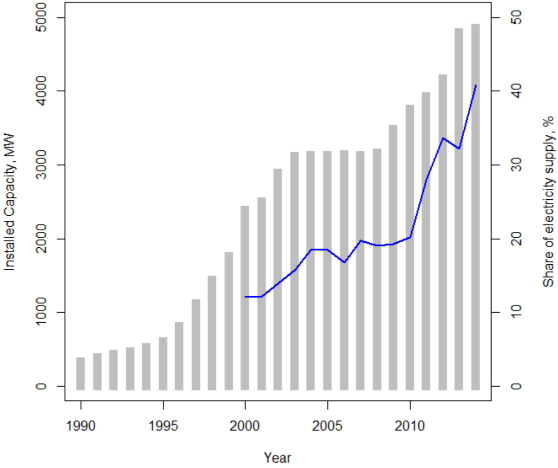
Motivation

An Energy System with a Large Renewable Component



source: http://www.imm.dtu.dk/ jbjo/smartenergy.html

Wind Power in Denmark



Renewable Energy and Uncertainty

Challenges with Renewable Energy

- Wind, solar and wave energy depend on the weather system.
- The weather is inherently uncertain, implying that
- Wind, wave and solar energy is intermittent and uncertain.
- This uncertainty affects the supply and demand for energy, the energy infrastructure and the economics of the energy system.

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Overcoming the Challenges

- Understanding the uncertainty associated with renewable energy becomes valuable.
- Input knowledge of uncertainty into decision problems.
- Solve decision problems for minimizing the issues related to this uncertainty.

Topics related to eSACP

Aims

- To produce probabilistic forecasts quantifying this uncertainty.
- To consider applications of such probabilistic forecasts.

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Approaches

- Modeling uncertainty using:
  - Stochastic differential equations
  - Stochastic partial differential equations
- Applying optimization tools incorporating uncertainty:
  - Stochastic Programming based on scenarios for future states)

Probabilistic Forecasting

What is Probabilistic Forecasting

Point Forecast

- Focus on describing typical or most likely outcome.
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Probabilistic Forecast

- Describes (features of) the predictive distribution.
- Probabilistic if it makes use of probabilities in the forecast.
- Examples of probabilistic forecasts include:
  - Quantile forecasts
  - Prediction intervals
  - Predictive densities
  - Scenarios
- These options are used in some state-of-the-art tools for wind power forecasting (WPPT) and solar power forecasting (SolarFor)
  For more information we refer to https://www.enfor.dk)

Stochastic Differential Equations in Forecasting

The Basic Setup

The basic stochastic differential equation formulation:

$$X_t \quad = \quad X_0 + \int_0^t f(X_s, s)ds + \int_0^t g(X_s, s,)dW_s,$$

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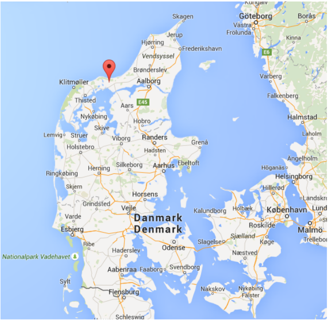
The predictive density, $j(x, t)$, can be found by solving (with $g(X_t, t) = \sqrt{2D(X_t, t)}$):

$$\frac{\partial}{\partial t} j(x, t) = -\frac{\partial}{\partial x}\left[f(x, t) j(x, t)\right] + \frac{\partial^2}{\partial x^2}\left[D(x, t) j(x, t)\right]. \qquad (1)$$

Example: A Probabilistic Model for Wind Power

The Data

- The Klim Fjordholme wind farm with a rated capacity of 21 MW.
- Hourly measurements for three years.
- Numerical weather predictions from Danish Meteorological Institute, updated every 6 hours.



A SDE Model

Wind dynamics given by:

$$
\begin{aligned}
dX_t &= \left(\left(1 - e^{-X_t}\right)(\rho_x \dot{p}_t + R_t) + \theta_x(p_t\mu_x - X_t)\right) dt + \sigma_x X_t^{0.5} dW_{x,t} \\
dR_t &= -\theta_r R_t dt + \sigma_r dW_{r,t} \\
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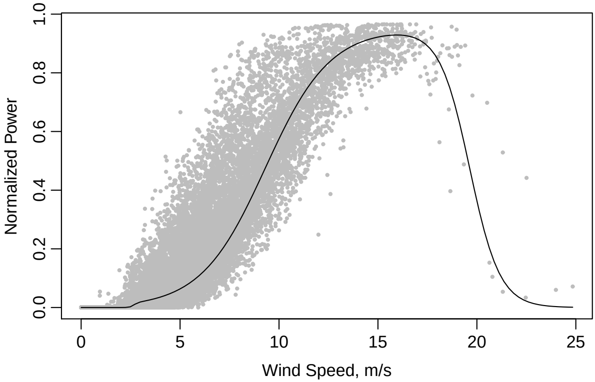
Wind to power dynamics given by:

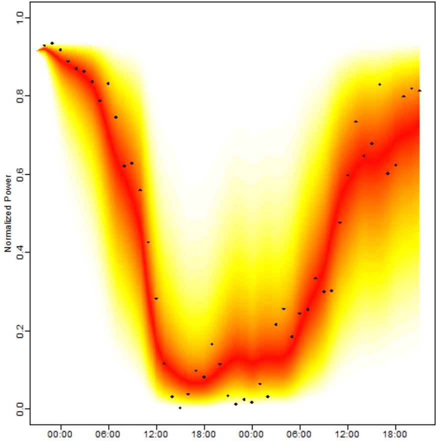
$$
\begin{aligned}
dQ_t &= (S_t - \theta_q Q_t) dt + \sigma_q dW_{q,t} \\
dS_t &= -\theta_s S_t dt + \sigma_s dW_{s,t} \\
Y_{2,k} &= (0.5 + 0.5 \tanh(5(X_{t_k} - \gamma_1)))(0.5 - 0.5 \tanh(\gamma_2 (X_{t_k} - \gamma_3))) \\
&\quad \frac{\zeta_3}{1 + e^{-\zeta_1 (X_{t_k} - \zeta_2 + Q_{t_k})}} + \epsilon_{2,k}
\end{aligned}
$$

Power Curve



Multi-Horizon Probabilistic Forecasts

Predictive density of production in percent out of rated power for the Klim wind farm:



Model Performance 1-step-ahead

Model performance on 1-hour ahead predictions on test set:

		Test Set		
Models	Parameters	MAE	RMSE	CRPS
Climatology	-	0.2208	0.2693	0.1417
Persistence	1	0.0509	0.0835	0.0428
AR	4	0.0527	0.0820	0.0417
ARX	5	0.0510	0.0795	0.0406
ARX - TN	7	0.0648	0.0848	0.0444
ARX - GARCH	9	0.0505	0.0797	0.0382
ARX - GARCH - TN	11	0.0575	0.0823	0.0401
Model	19	**0.0471**	**0.0773**	**0.0327**

Model Performance on Multiple Horizons

Model multi-horizons predictions performance on test set:

Models	CRPS for different horizons				Energy Score
	1 hour	4 hours	12 hours	24 hours	
ARX - GARCH	0.0382	0.0704	0.0787	**0.0789**	1.180
ARX - GARCH - TN - iterative	0.0401	0.0783	0.1043	0.1225	1.945
Model	**0.0327**	**0.0641**	**0.0779**	0.0836	**0.739**

Example: A Spatio-Temporal Forecast Model for Solar Power

The Data

- A solar power plant with a nominal output of 151 MW.
- Measurements of 91 inverters every second for one year.
- We consider a cutout of 5 by 14 inverters for modeling.

Motivation

Challenges

- A high dimensional problem, with 70 inverters and forecast horizon of two miutes.
- Classical methods are have a large high dimensional parameter space.
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Approach

- A model that incorporates the physics of the system.
- Good local models should lead to good global models.
- A physical understanding of the system leads to fewer parameters and lowered computational burden.

The Framework

We propose a model of the form:

$$dU_{i,j,t} = f\left(\mathbf{U}_{i,j,t}, t\right) dt + g(U_{i,j,t}, t) dW_{i,j,t} \tag{2}$$

$$Y_{l,k} = h(\mathbf{U}_{t_k}, t_k) + \epsilon_{l,k}, \tag{3}$$

where $\mathbf{U}_{i,j,t} = \{U_{i,j,t}, U_{i-1,j,t}, U_{i+1,j,t}, U_{i,j-1,t}, U_{i,j+1,t}\}$.

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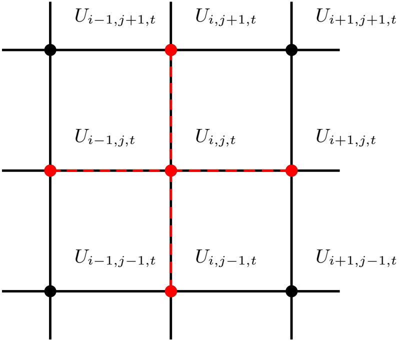

SPDE Model

Stochastic Partial Differential Equation

- Normalize the parameters with the spatial distance in appropriate way.
- Parameters become grid-invariant.
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The dynamical model interpretation:

$$dU(x,t) = \bar{v}\theta\nabla U(x,t)dt + \sigma dW(x,t),$$

with the deterministic part $dU(x,t) = \bar{v}\theta\nabla U(x,t)dt$ being a uni-directional wave equation.

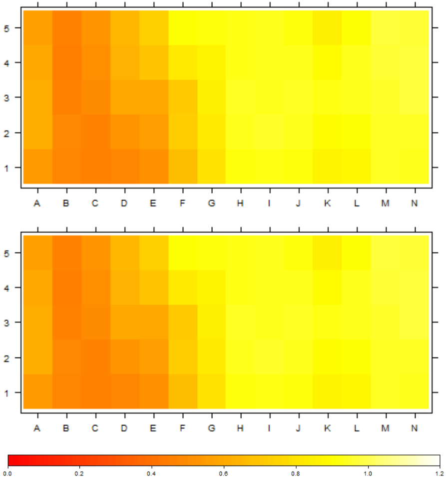
Parameters

End up with a model with 4 parameters and the accompanying estimates:

$\hat{\theta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\sigma}_\epsilon$
0.0631	0.703	0.00865	10^{-10}

Predicting Spatio-Temporal Power Output

Power production in percent out of rated power.



Model Performance:

Model compared to benchmarks.

Score	Cloud Speed Persistence	Ramp Speed Persistence	Auto-Regressive	Model
RMSE_5	0.334	0.612	0.464	**0.636**
RMSE_{20}	0.289	0.284	0.319	**0.523**
RMSE_{60}	0.168	-0.203	0.113	**0.254**
RMSE_{120}	0.062	-0.434	0.039	**0.097**
MAE_5	0.258	0.597	0.431	**0.612**
MAE_{20}	0.213	0.301	0.280	**0.497**
MAE_{60}	0.136	-0.145	0.045	**0.246**
MAE_{120}	0.048	-0.396	-0.064	**0.096**
CRPS_5	–	–	0.00262	**0.00131**
CRPS_{20}	–	–	0.00982	**0.00666**
CRPS_{60}	–	–	0.02886	**0.02455**
CRPS_{120}	–	–	0.04883	**0.04675**

Using Probabilistic Forecasts

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Challenges and advantages of using Probabilistic Forecasts

- Probabilistic forecasts potentially contain large amounts of information.
- Probabilistic forecasts are potentially difficult to interpret for non-specialists.
- Gives a possibility to choose the appropriate probabilistic forecast for a specific application.
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Applications of Probabilistic Forecasts

- Trading energy from renewable generation with asymmetric cost structures.
- Setting reserve capacity in the electrical grid.
- Modeling consumer demand for electricity, heating, water etc.

Conclusions

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- We believe that we have obtained state-of-the-art methodologies for wind and solar power forecasting for operational purposes.
- The SDE formulation allows for an introducing a physical understanding, which again may improve probabilistic forecasting.
- A proper understanding of the system error allows for generating multi-horizon probabilistic forecasts.
- Using probabilistic forecasts in connection with decision making tools may alleviate issues related to introducing renewable energy generation.
- Choosing the right probabilistic forecast product is important for solving operational problems.