State-of-the-art in Probabilistic Forecasting of Wind and Solar Power Generation

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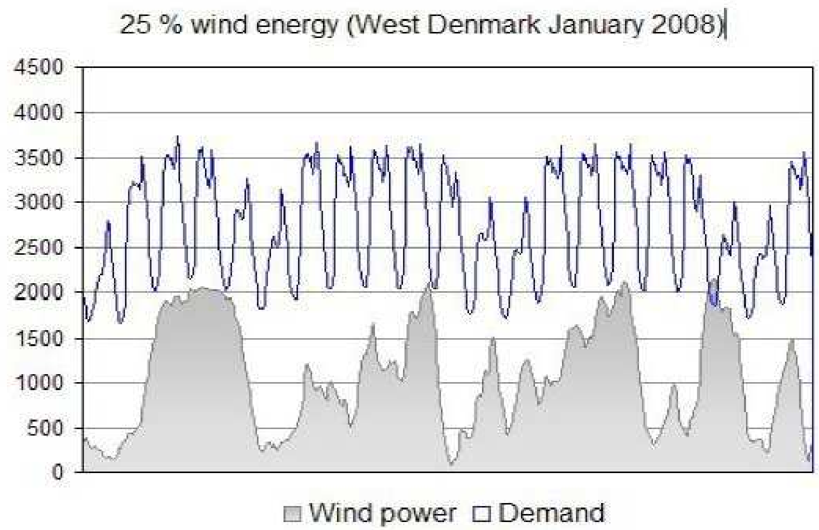
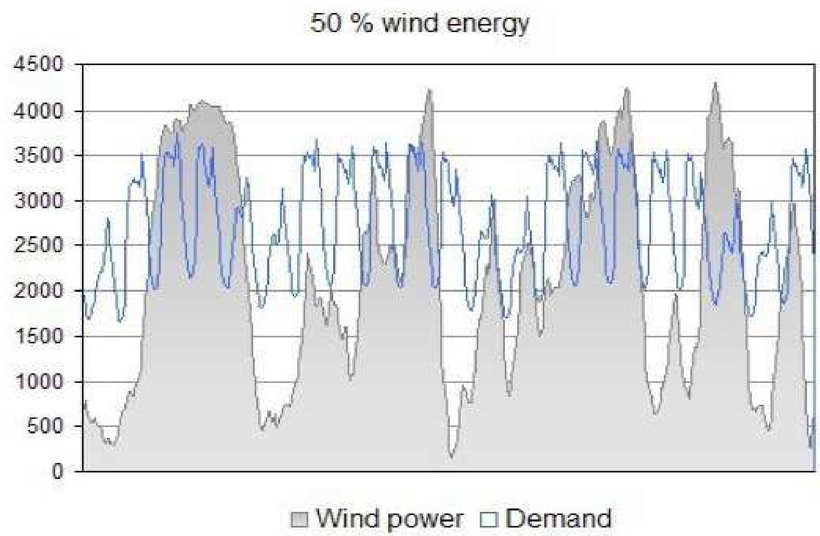
Outline

Focus on Wind Power - and briefly mention Solar Power:

- Status in Denmark
- Wind power point forecasting
- Use of several providers of MET forecasts
- Uncertainty and confidence intervals
- Scenario forecasting
- Space-time scenario forecasting
- Use of stochastic differential equations
- Examples on the use of probabilistic forecasts
- Optimal bidding for a wind farm owner
- Solar power forecasting
- Some advanced topics

Wind Power Statistics for Denmark





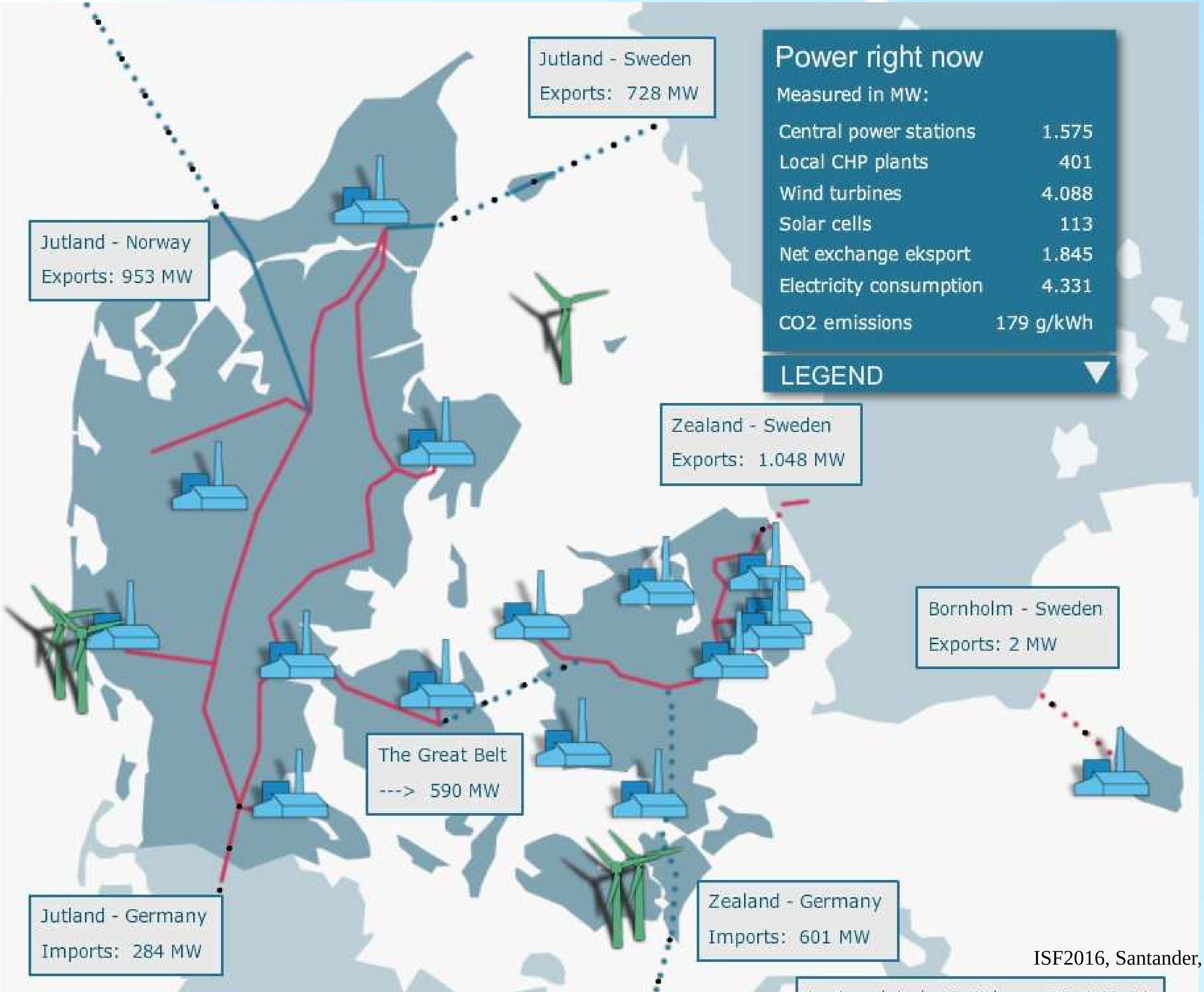
In 2008 wind power did cover the entire demand of electricity in 200 hours (West DK)

In 2015 more than 42 pct of electricity load was covered by wind power.

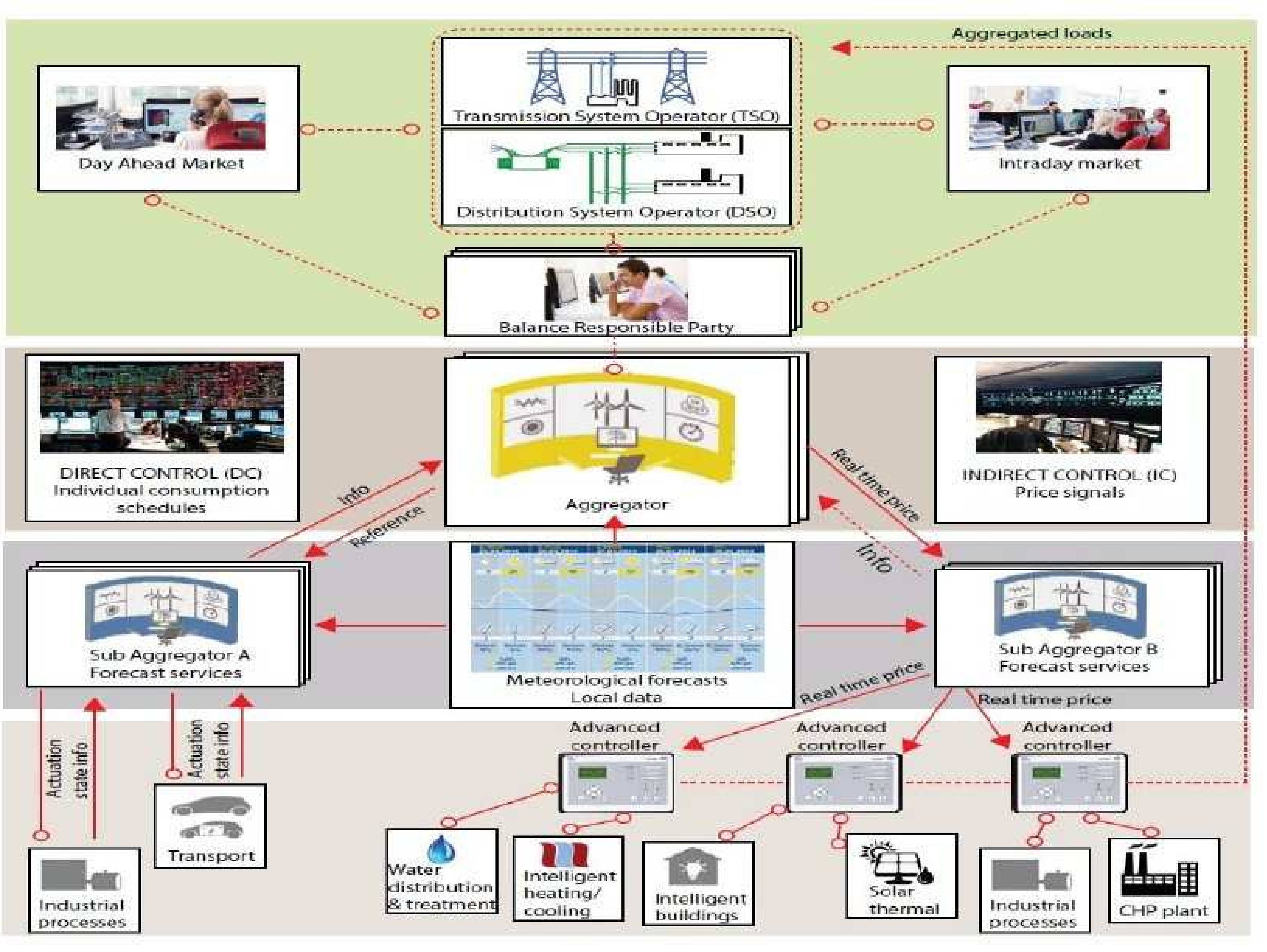
For several days in 2015 the wind power production was more than 120 pct of the power load.

July 10th, 2015 more than 140 pct of the power load was covered by wind power

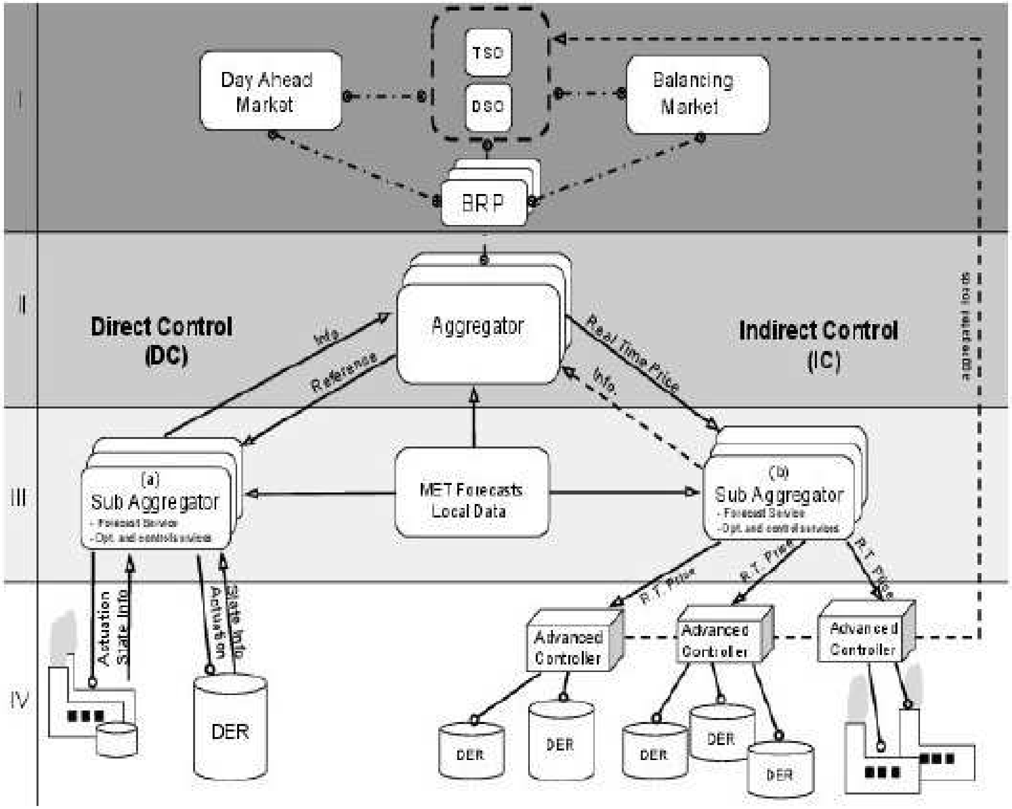
Power Grid. A snap-shot from 14. February



Smart-Energy OS



Stochastic Control and Optimization



In New Wiley Book: Control of Electric Loads in Future Electric Energy Systems, 2015

Day Ahead:

Stoch. Programming based on eg. Scenarios

Cost: Related to the market (one or two levels)

Direct Control:

Actuator: **Power**

Two-way communication

Models for DERs are needed

Constraints for the DERs (calls for state est.)

Contracts are complicated

Indirect Control:

Actuator: **Price**

Cost: E-MPC at **low (DER) level**, One-way communication

Models for DERs are not needed

Simple 'contracts'

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Direct vs Indirect Control

Level	Direct Control (DC)	Indirect Control (IC)
III	$\min_{x,u} \sum_{k=0}^{N} \sum_{j=1}^{J} \phi_j(x_{j,k}, u_{j,k})$	$\min_{\hat{z},p} \sum_{k=0}^{N} \phi(\hat{z}_k, p_k)$ s.t. $\hat{z}_{k+1} = f(p_k)$
	$\downarrow_{u_1} \cdots \downarrow_{u_J} \quad \uparrow_{x_1} \cdots \uparrow_{x_J}$	
IV	s.t. $x_{j,k+1} = f_j(x_{j,k}, u_{j,k}) \quad \forall j \in J$	$\min_{u} \sum_{k=0}^{N} \phi_j(p_k, u_k) \quad \forall j \in J$ s.t. $x_{k+1} = f_j(x_k, u_k)$

Table 1: Comparison between direct (DC) and indirect (IC) control methods. (DC) In direct control the optimization is globally solved at level III. Consequently the optimal control signals u_j are sent to all the J DER units at level IV. (IC) In indirect control the optimization at level III computes the optimal prices p which are sent to the J-units at level IV. Hence the J DERs optimize their own energy consumption taking into account p as the actual price of energy.

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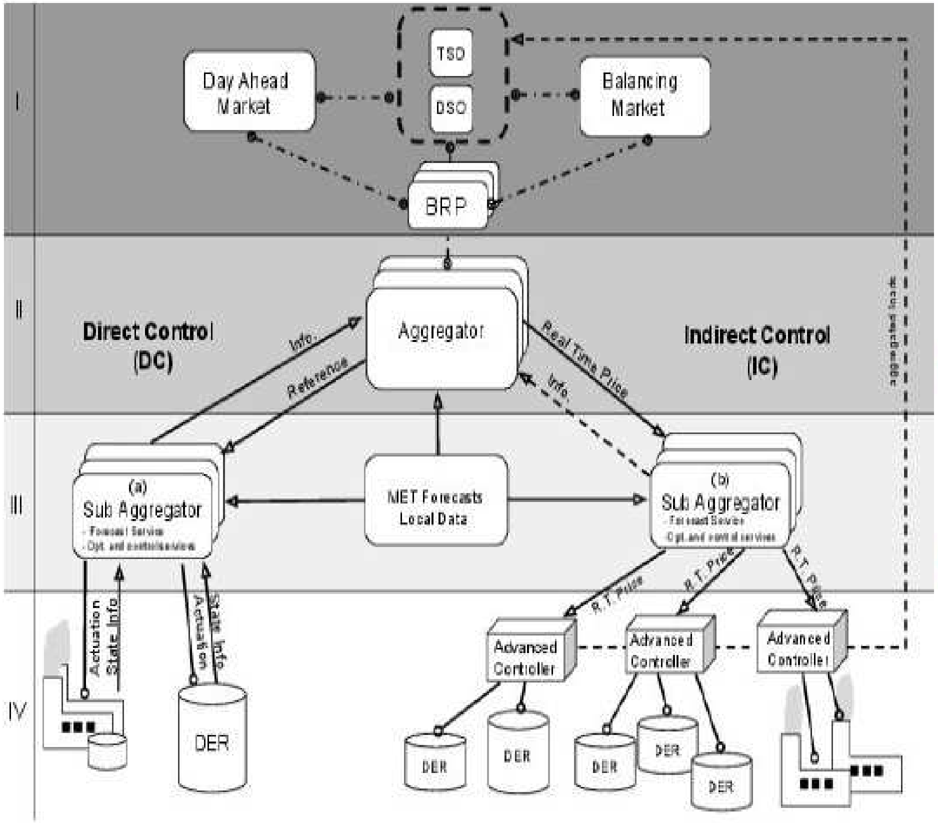
Forecast Requirements



Day Ahead:

- Forecasts of loads
- Forecast of Grid Capacity (using eg. DLR)
- Forecasts of production (eg. Wind and Solar)

Direct Control: .

- Forecasts of states of DERs
- Forecasts of load

Indirect Control:

- Forecasts of prices
- Forecasts of load

Wind Power Forecasting - History

Our methods for probabilistic wind power forecasting have been implemented in the **Anemos Wind Power Prediction System**, **Australian Wind Energy Forecasting Systems (AWEFS)** and **WPPT**

- The methods have been continuously developed since 1993 - in collaboration with
  - Energinet.dk,
  - Dong Energy,
  - Vattenfall,
  - Risø – DTU Wind,
  - The ANEMOS projects partners/consortium (since 2002),
  - Overspeed GmbH (Anemos: www.overspeed.de/gb/produkte/windpower.html)
  - ENFOR (WPPT: www.enfor.dk)
- Used operationally for predicting wind power in Denmark since 1996.
- Now used by all major players in Denmark (Energinet.dk, DONG, Vattenfall, ..)
- Anemos/WPPT is now used eg in Europe, Australia, and North America.
- Often used as forecast engine embedded in other systems.

Prediction of wind power

In areas with high penetration of wind power such as the Western part of Denmark and the Northern part of Germany and Spain, reliable wind power predictions are needed in order to ensure safe and economic operation of the power system.

Accurate wind power predictions are needed with different prediction horizons in order to ensure

- (**up to a few hours**) efficient and safe use of regulation power (spinning reserve) and the transmission system,
- (**12 to 36 hours**) efficient trading on the Nordic power exchange, NordPool,
- (**days**) optimal operation of eg. large CHP plants.

Predictions of wind power are needed both for the total supply area as well as on a regional scale and for single wind farms.

For some grids/in some situations the focus is on methods for **ramp forecasting**, in some other cases the focus in on reliable probabilistic forecasting.

Uncertainty and adaptivity

Errors in MET forecasts will end up in errors in wind power forecasts, but other factors lead to a need for adaptation which however leads to some uncertainties.

The total system consisting of wind farms measured online, wind turbines not measured online and meteorological forecasts will inevitably change over time as:

- the population of wind turbines changes,
- changes in unmodelled or insufficiently modelled characteristics (important examples: roughness and dirty blades),
- changes in the NWP models.

A wind power prediction system must be able to handle these time-variations in model and system. An adequate forecasting system may use **adaptive and recursive model estimation** to handle these issues.

We started (some 20 years ago) assuming Gaussianity; but this is a very serious (wrong) assumption !

Following the initial installation the software tool will automatically calibrate the models to the actual situation.

The power curve model

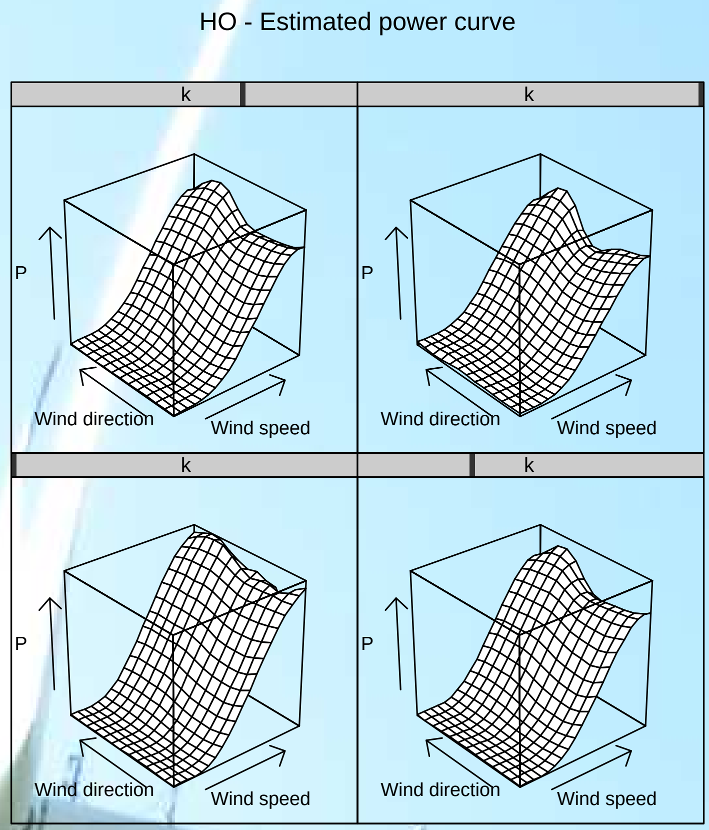
The wind turbine "power curve" model, $p^{tur} = f(w^{tur})$ is extended to a wind farm model, $p^{wf} = f(w^{wf}, \theta^{wf})$, by introducing wind direction dependency. By introducing a representative area wind speed and direction it can be further extended to cover all turbines in an entire region, $p^{ar} = f(\bar{w}^{ar}, \bar{\theta}^{ar})$.

The power curve model is defined as:

$$\hat{p}_{t+k|t} = f(\ \bar{w}_{t+k|t},\ \bar{\theta}_{t+k|t},\ k\ )$$

where
$\bar{w}_{t+k|t}$ is forecasted wind speed, and
$\bar{\theta}_{t+k|t}$ is forecasted wind direction.

The characteristics of the NWP change with the prediction horizon.



Plots of the estimated power curve for the Hollandsbjerg wind farm.

The dynamical prediction model

The power curve models are used as input for an adaptively estimated **dynamical model**, which (as a **simple example**) leads to the following k-stop ahead forecasts:

$$\begin{aligned}\hat{p}_{t+k|t} &= a_1 p_t + a_2 p_{t-1} + b\,\hat{p}^{pc}_{t+k|t} + \\ &\sum_{i=1}^{3} [c_i^c \cos \frac{2 i \pi h^{24}_{t+k}}{24} + c_i^s \sin \frac{2 i \pi h^{24}_{t+k}}{24}] + m + e_{t+k}\end{aligned}$$

where p_t is observed power production, $k \in [1;48]$ (hours) is prediction horizon, $\hat{p}^{pc}_{t+k|t}$ is power curve prediction and h^{24}_{t+k} is time of day.

Model features include

- the number of terms in the model depends on the prediction horizon,
- non-stationarity is handled by adaptive estimation of the model parameters,

A model for upscaling

The dynamic upscaling model for a region is defined as:

$$\hat{p}^{reg}_{t+k|t} = f(\ \bar{w}^{ar}_{t+k|t},\ \bar{\theta}^{ar}_{t+k|t},\ k\ )\ \hat{p}^{loc}_{t+k|t}$$

where

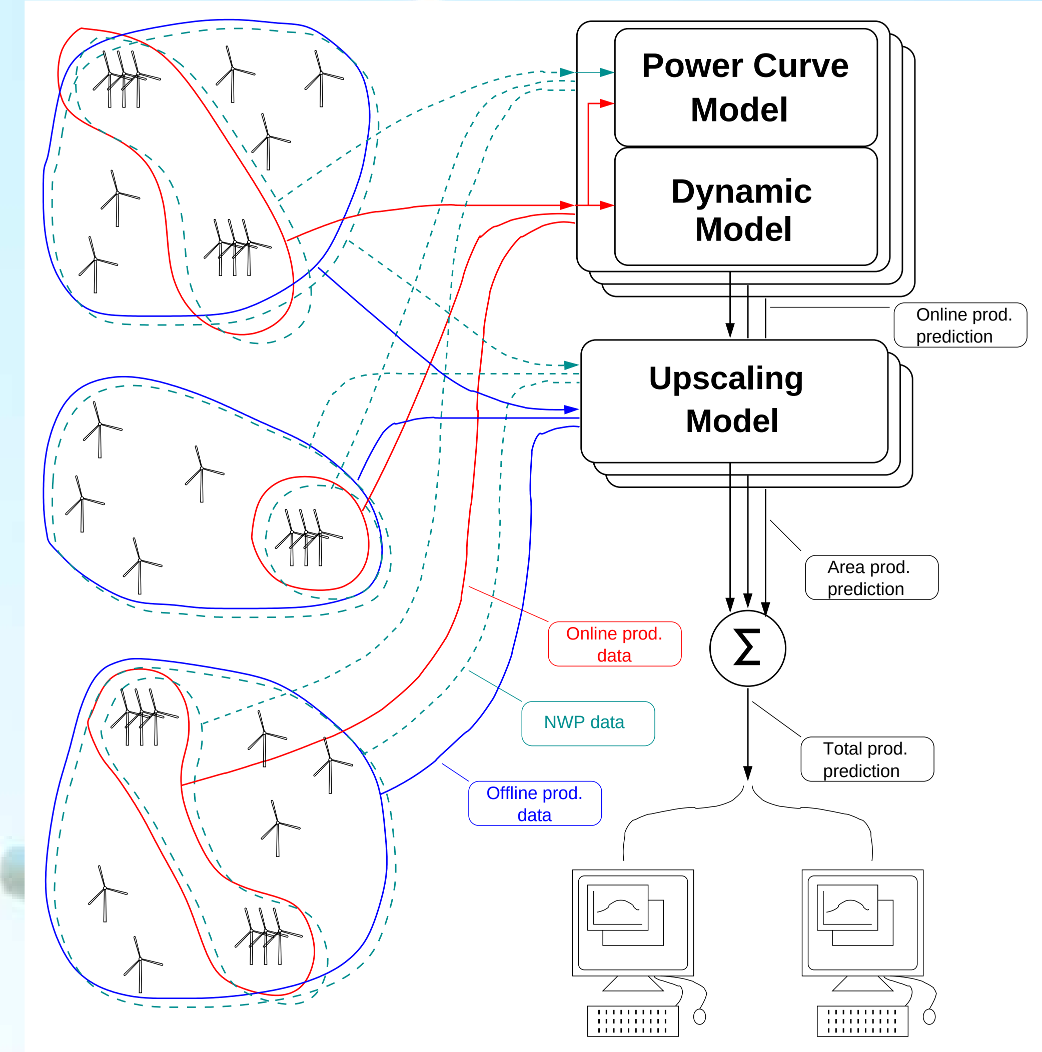
- $\hat{p}^{loc}_{t+k|t}$ is a local (dynamic) power prediction within the region,
- $\bar{w}^{ar}_{t+k|t}$ is forecasted regional wind speed, and
- $\bar{\theta}^{ar}_{t+k|t}$ is forecasted regional wind direction.

The characteristics of the NWP and $\hat{p}^{loc}$ change with the prediction horizon. Hence the dependendency of prediction horizon k in the model.

Configuration Example

This configuration of Anemos Prediction System/WPPT is used by a large TSO. Characteristics for the installation:

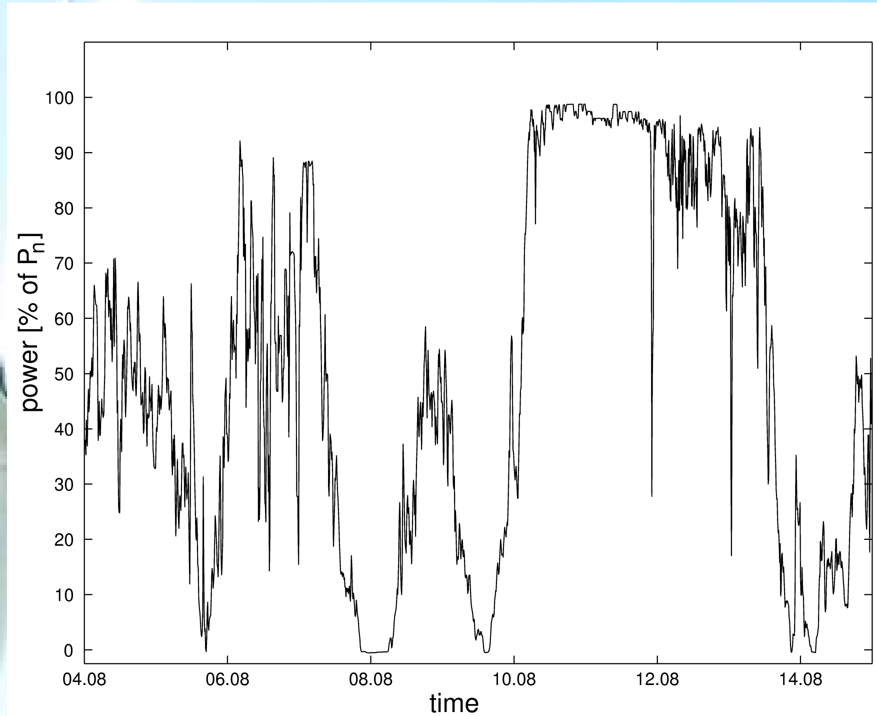
- A large number of wind farms and stand-alone wind turbines.
- Frequent changes in the wind turbine population.
- Offline production data with a resolution of 15 min. is available for more than 99% of the wind turbines in the area.
- Online data for a large number of wind farms are available. The number of online wind farms increases quite frequently.



Fluctuations of offshore wind power

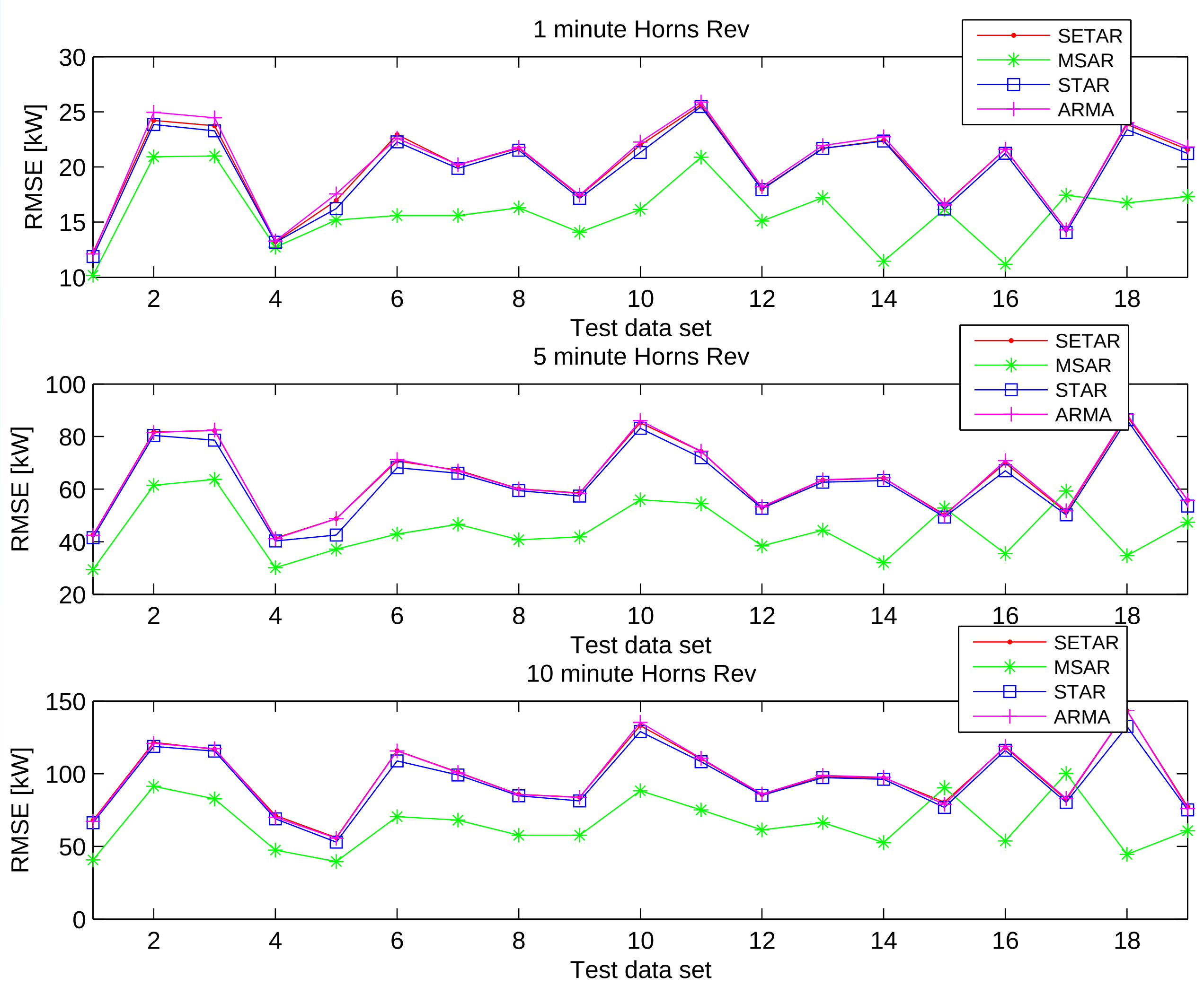
- Fluctuations at large offshore wind farms have a significant impact on the control and management strategies of their power output
- Focus is given to the minute scale. Thus, the effects related to the turbulent nature of the wind are smoothed out
- When looking at time-series of power production at Horns Rev (160MW/209MW) and Nysted (165 MW), one observes successive periods with fluctuations of larger and smaller magnitude

- We aim at building models
  - based on historical wind power measures only...
  - ... but able to reproduce this observed behavior
  - this calls for **regime-switching models**



Results - Horns Rev

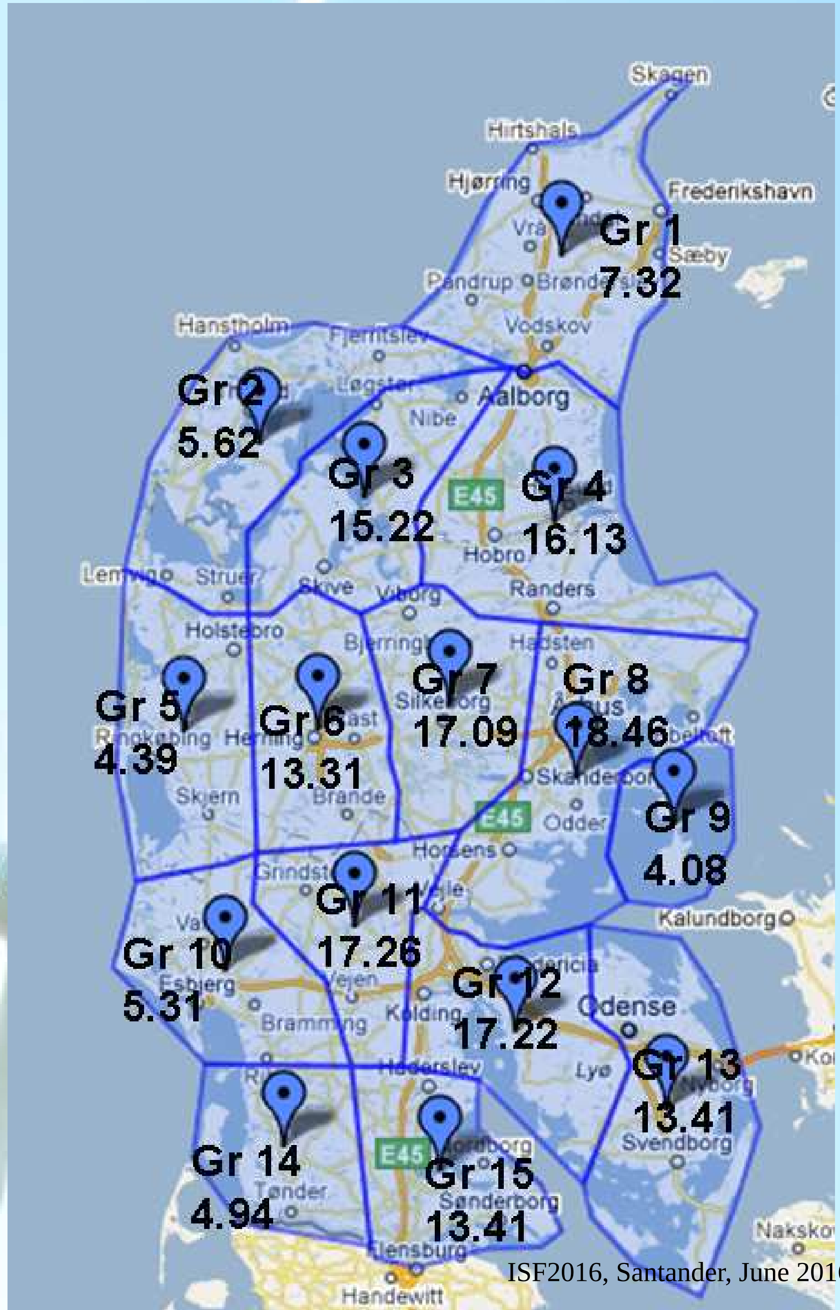
- The evaluation set is divided in 19 different periods of different lengths and characteristics
- MSAR models generally outperform the others
- In the RADAR@sea project the regime shift is linked to convective rain events – which are detected by a weather radar.



Spatio-temporal forecasting

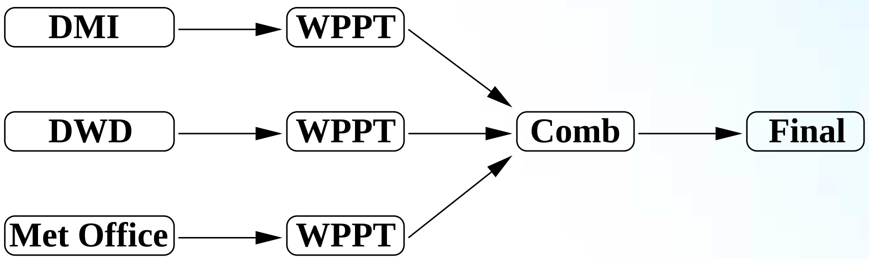
Predictive improvement (measured in RMSE) of forecasts errors by adding the spatio-temperal module in WPPT.

- 23 months (2006-2007)
- 15 onshore groups
- Focus here on 1-hour forecast only
- Larger improvements for eastern part of the region
- Needed for reliable ramp forecasting.
- The EU project NORSEWinD will extend the region

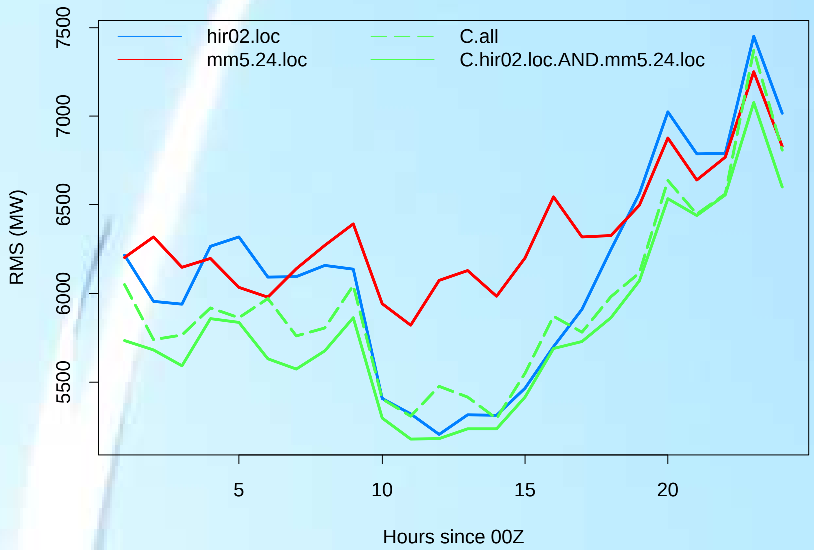


Combined forecasting

- A number of power forecasts are weighted together to form a new improved power forecast.
- These could come from parallel configurations of WPPT using NWP inputs from **different MET providers** or they could come from other power prediction providers.
- In addition to the improved performance also the robustness of the system is increased.



The example show results achieved for the Tunø Knob wind farms using combinations of up to 3 power forecasts.



Typically an improvement on 10-15 pct in accuracy of the point prediction is seen by including more than one MET provider. Two or more MET providers imply information about uncertainty

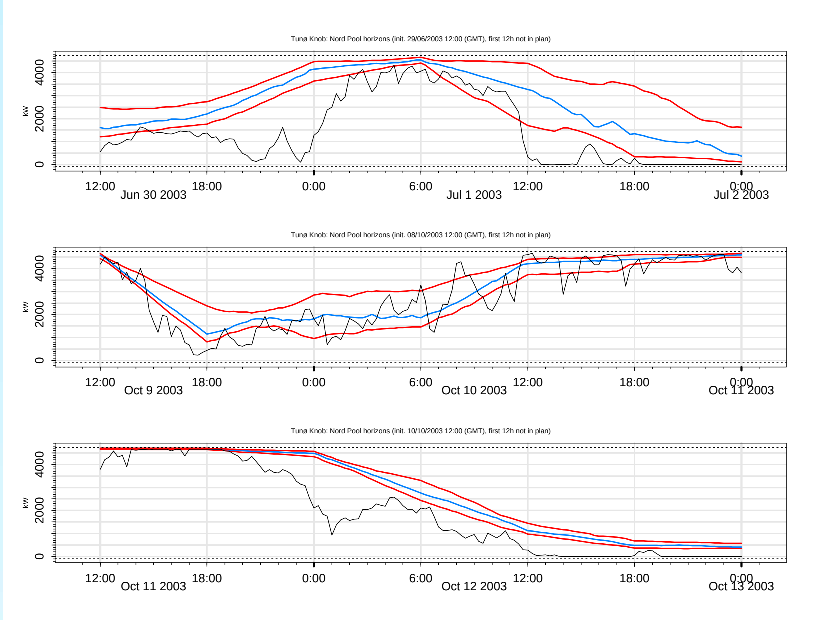
Uncertainty estimation

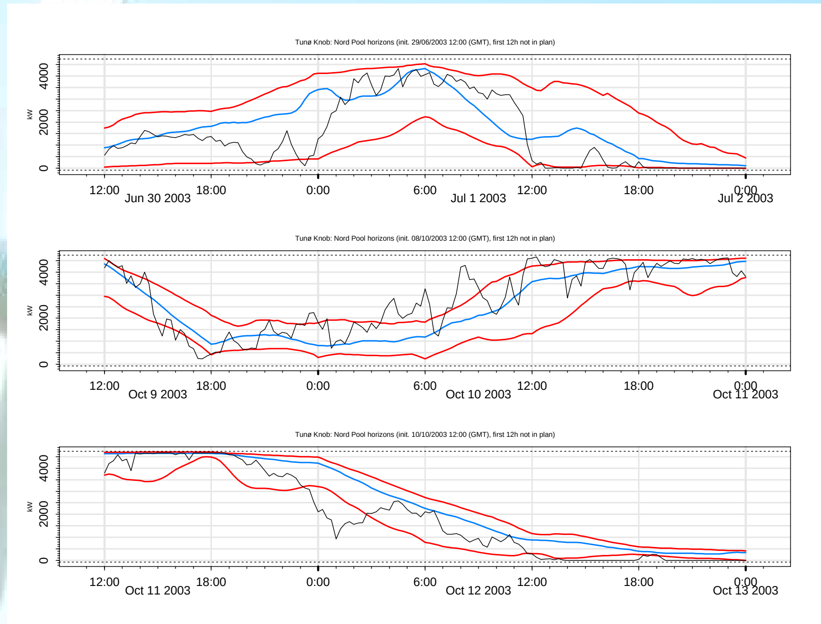
In many applications it is crucial that a prediction tool delivers reliable estimates (probabilistc forecasts) of the expected uncertainty of the wind power prediction.

We consider the following methods for estimating the uncertainty of the forecasted wind power production:

- Ensemble based - but corrected - quantiles.
- Quantile regression.
- Stochastic differential equations.

The plots show raw (top) and corrected (bottom) uncertainty intervales based on ECMEF ensembles for Tunø Knob (offshore park), 29/6, 8/10, 10/10 (2003). Shown are the 25%, 50%, 75%, quantiles.





Quantile regression

A (additive) model for each quantile:

$$Q(\tau) = \alpha(\tau) + f_1(x_1; \tau) + f_2(x_2; \tau) + \ldots + f_p(x_p; \tau)$$

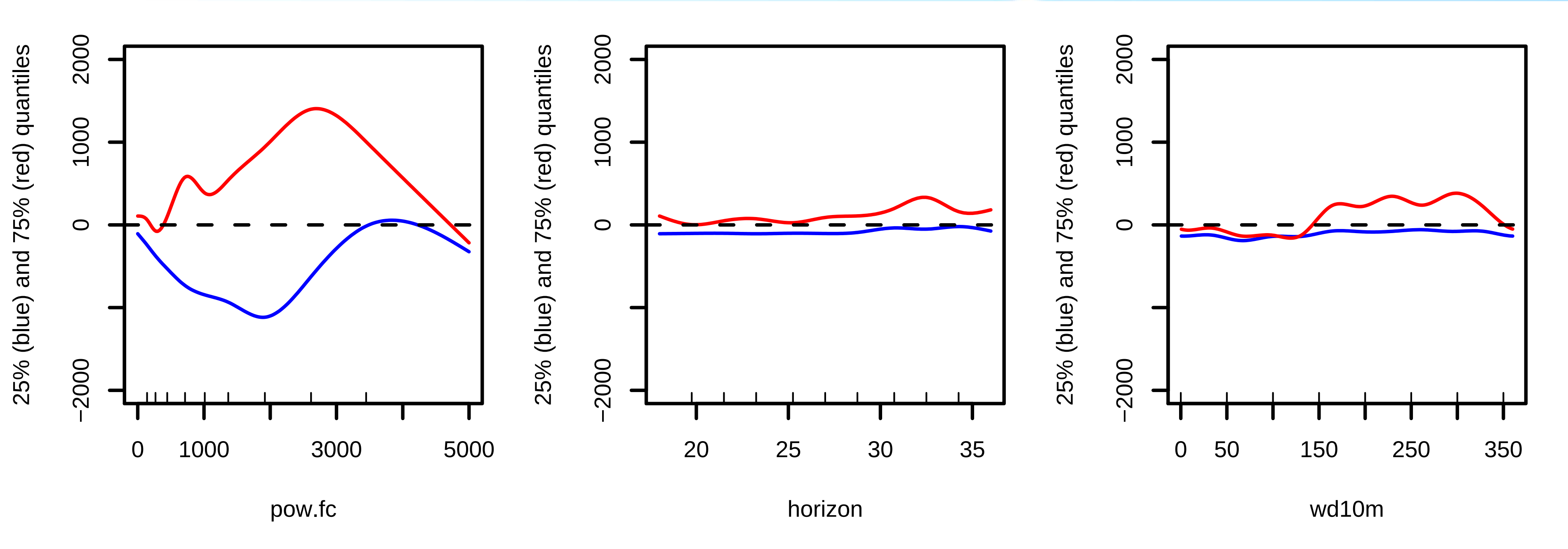
$Q(\tau)$	Quantile of **forecast error** from an **existing system**.
x_j	Variables which influence the quantiles, e.g. the wind direction.
$\alpha(\tau)$	Intercept to be estimated from data.
$f_j(\cdot; \tau)$	Functions to be estimated from data.

Notes on quantile regression:

- Parameter estimates found by minimizing a dedicated function of the prediction errors.
- The variation of the uncertainty is (partly) explained by the independent variables.

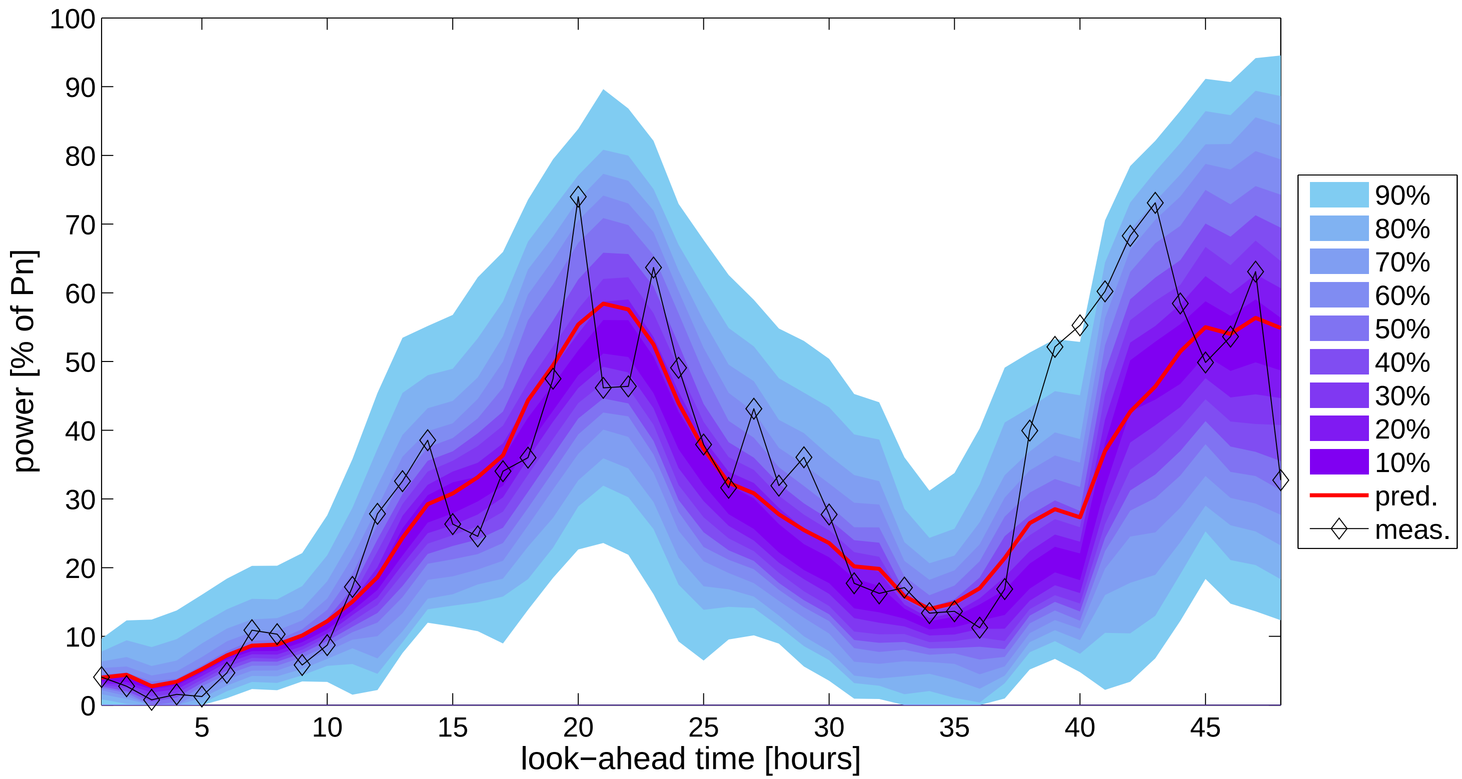
Quantile regression - An example

Effect of variables (- the functions are approximated by Spline basis functions):



- Forecasted power has a large influence.
- The effect of horizon is of less importance.
- Some increased uncertainty for Westerly winds.

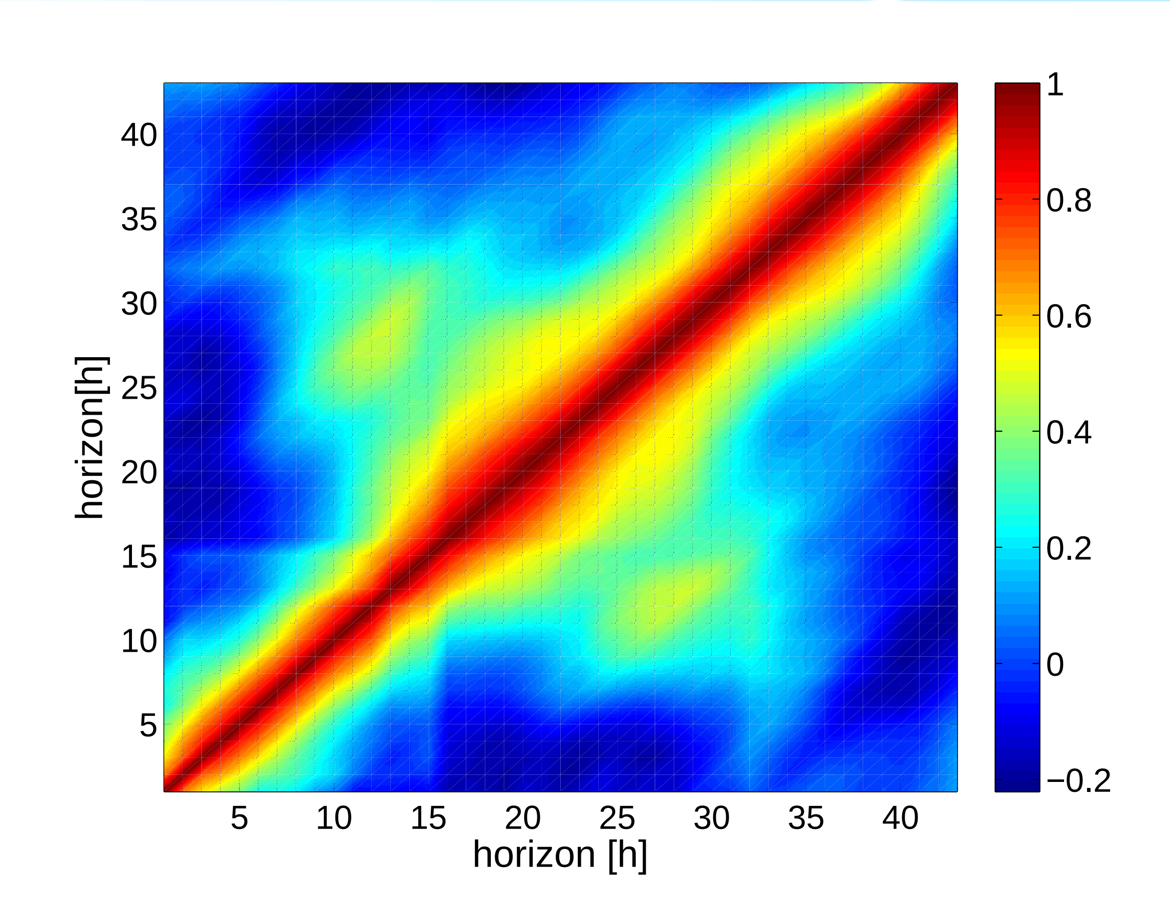
Example: Probabilistic forecasts



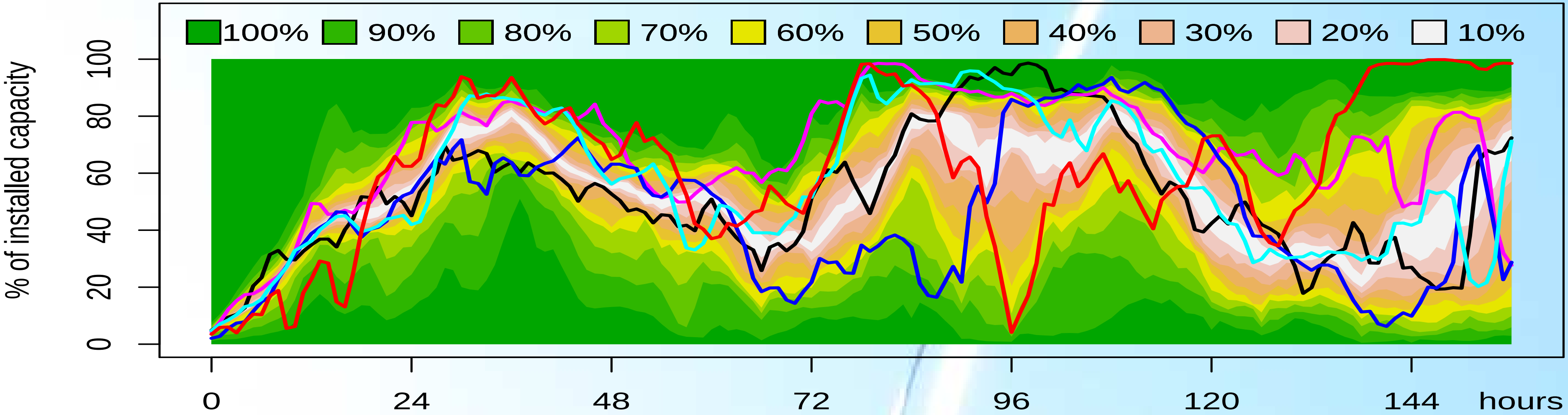
- Notice how the confidence intervals varies ...
- But the correlation in forecasts errors is not described so far.

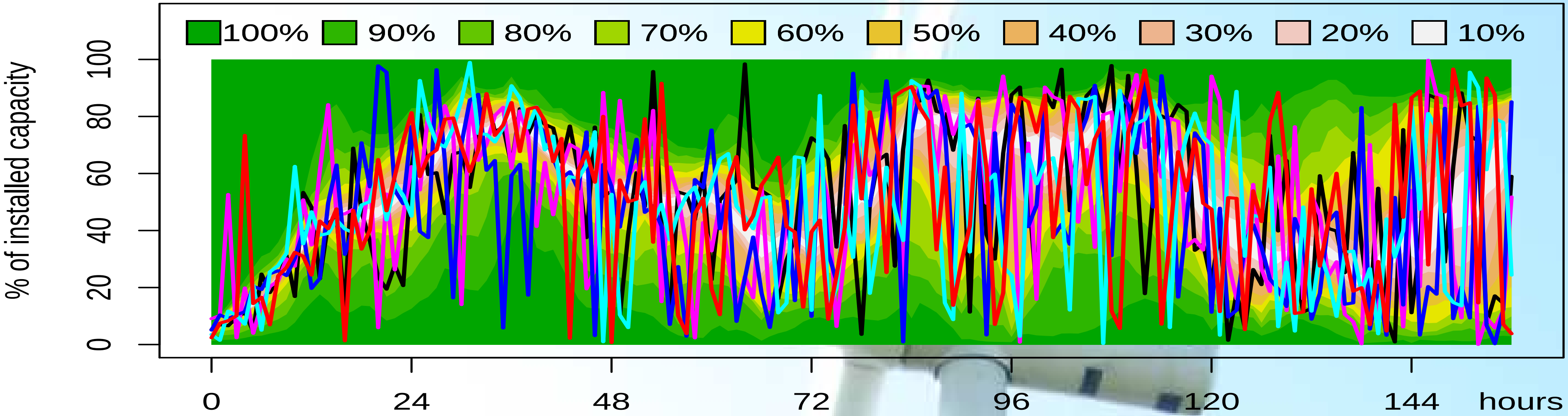
Correlation structure of forecast errors

- It is important to model the **interdependence structure** of the prediction errors.
- An example of interdependence covariance matrix:



Correct (top) and naive (bottom) scenarios





Use of Stoch. Diff. Equations

The state equation describes the future wind power production

$$dx_t = -\,\theta(\boldsymbol{u_t}) \cdot (x_t - \hat{p}_{t|0})dt +$$
$$2\sqrt{\theta(\boldsymbol{u_t})\alpha(\boldsymbol{u_t})\hat{p}_{t|0}(1-\hat{p}_{t|0})x_t \cdot (1-x_t)}dw_t,$$

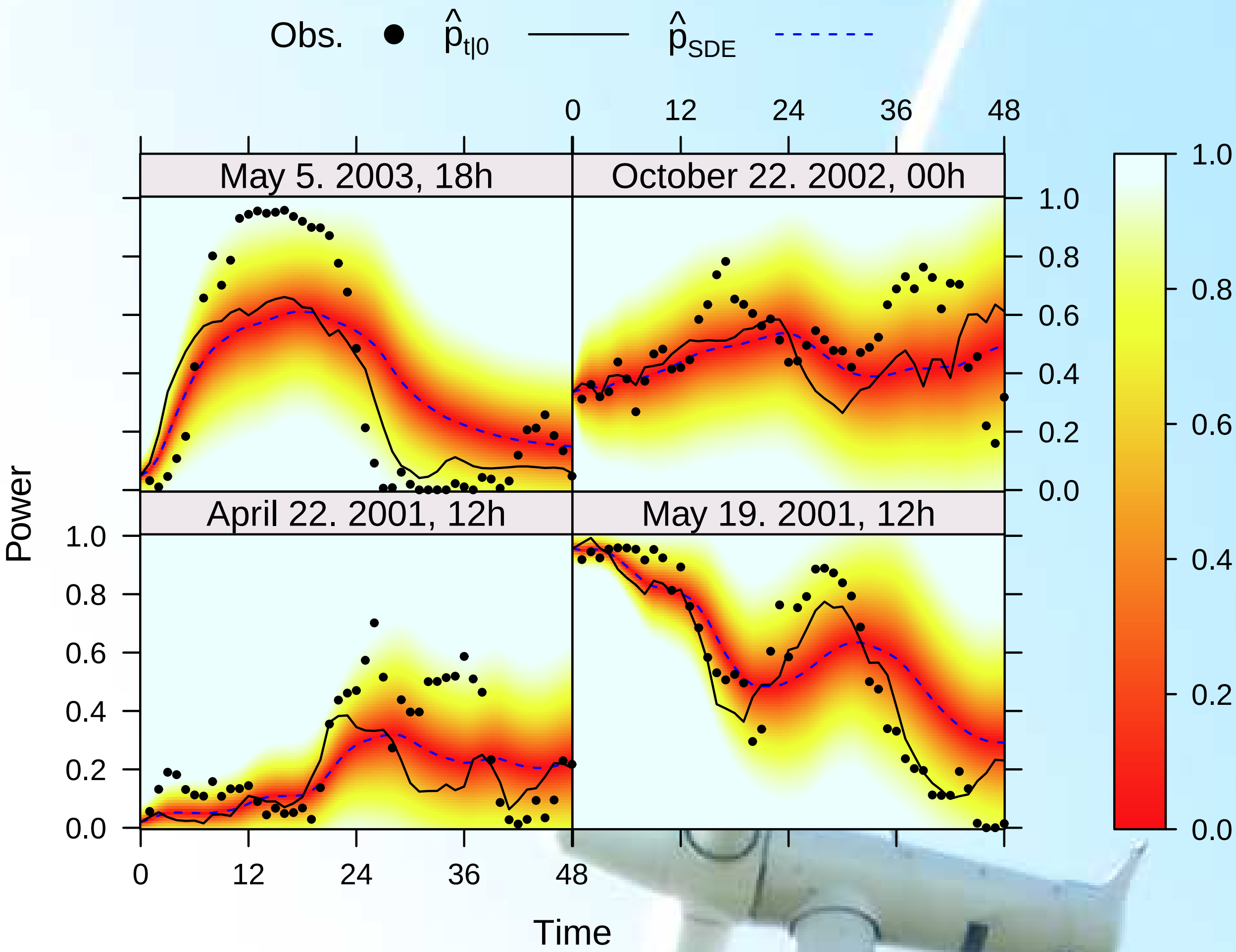
with $\alpha(\boldsymbol{u_t}) \in (0,1)$, and the observation equation

$$y_h = x_{t_h|0} + e_h,$$

where $h \in \{1, 2, ..., 48\}$, $t_h = k$, $e_h \sim N(0, s^2)$, $x_0 =$ *"observed power at t=0"*, and

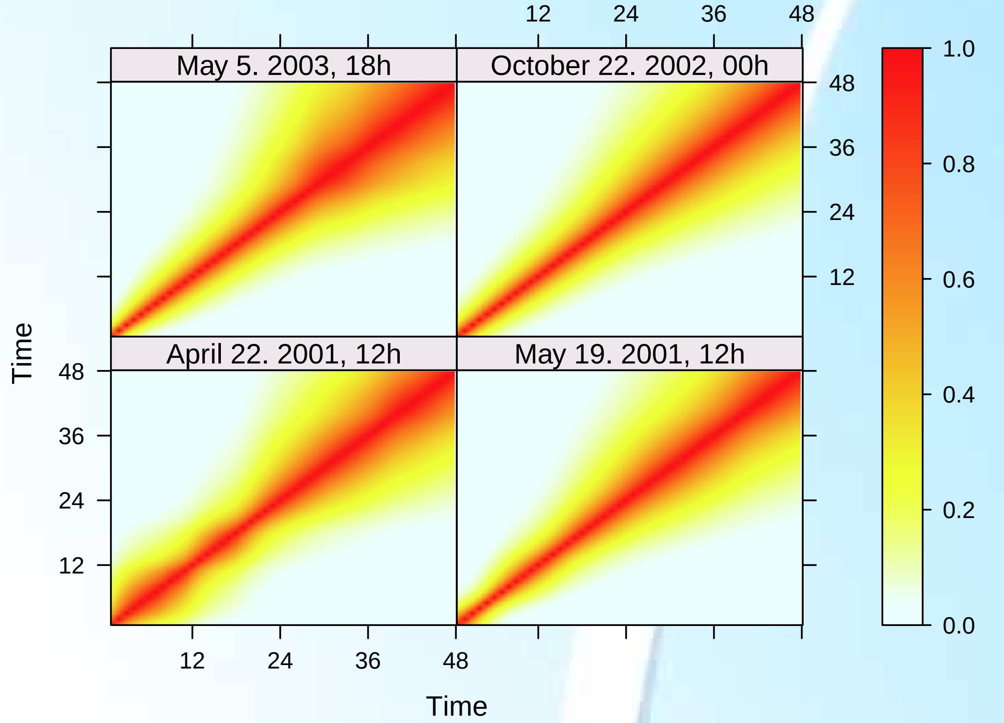
- $\hat{p}_{t|0}$ point forecast by **WPPT** (Wind Power Prediction Tool)
- $\boldsymbol{u_t}$ input vector (here t and $\hat{p}_{t|0}$)

Examples of using SDEs



Use of SDEs provides a possibility for a joint description of both non-symmetrical conditional densities as well as the interdependence of the forecasts.

SDE approach – Correlation structures

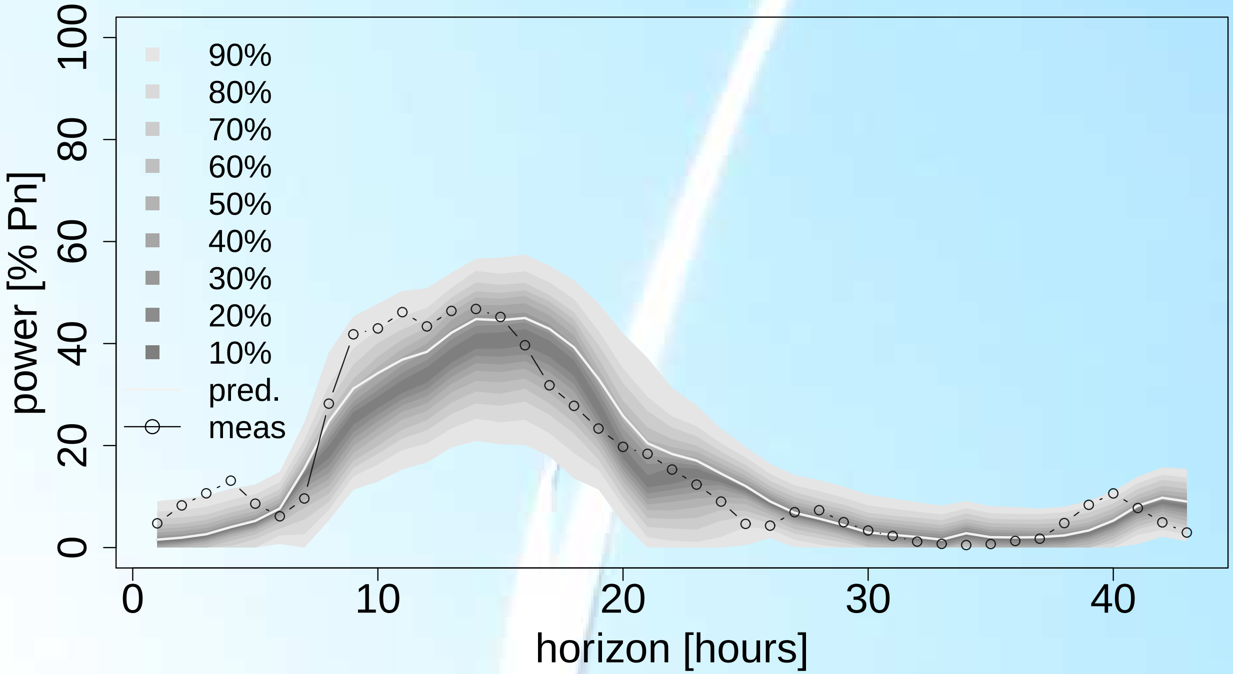


Use of SDEs provides a possibility to model eg. time varying and wind power dependent correlation structures.

SDEs provide a perfect framework for **combined wind and solar power forecasting**.

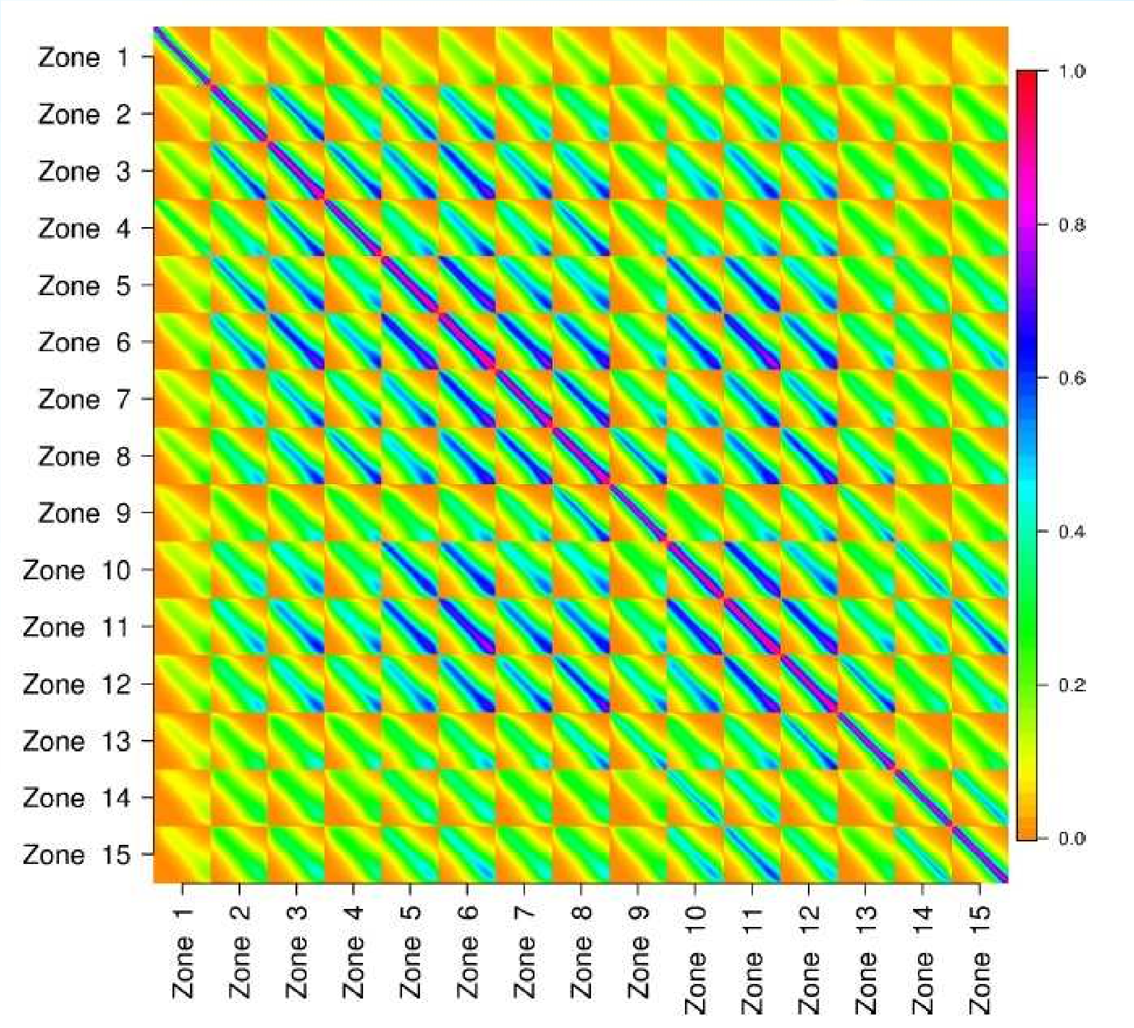
Today both **Anemos Wind Power Prediction System** and **WPPT** provide operations forecasts of both wind and solar power production (used eg. all over Australia)

Motivation - Space-Time Dependencies

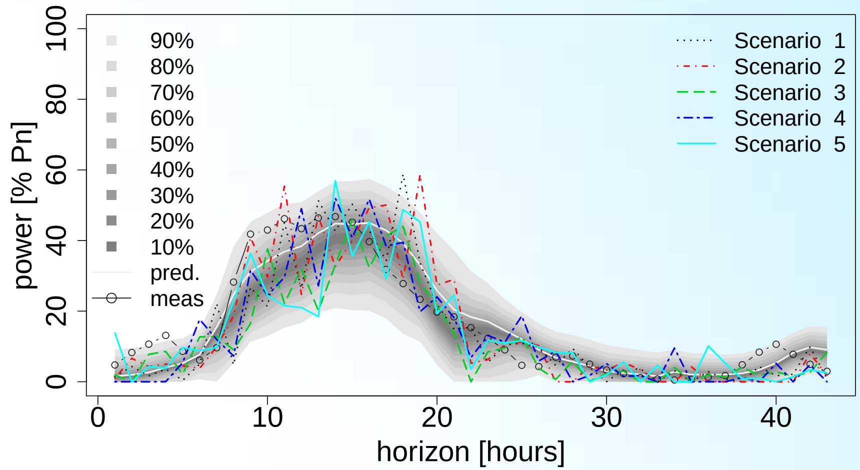


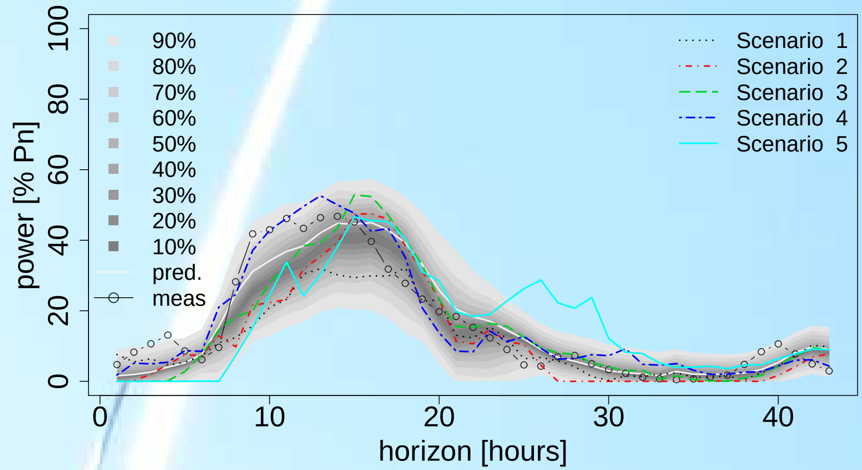
This is not enough...

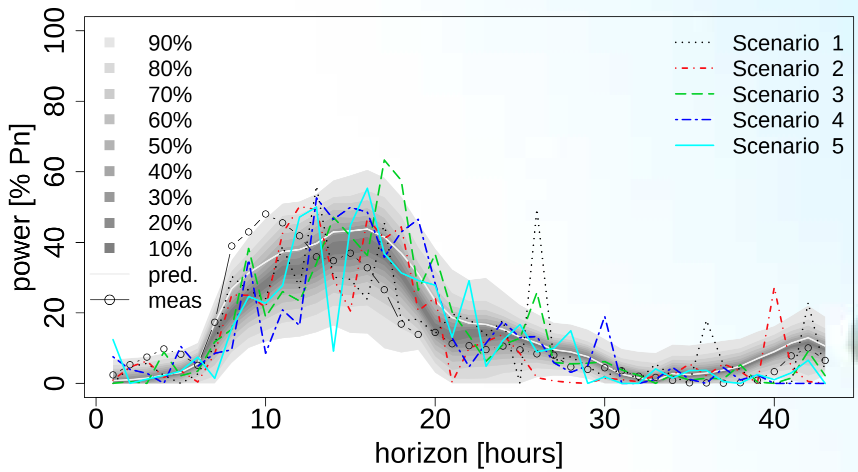
Space-Time Correlations

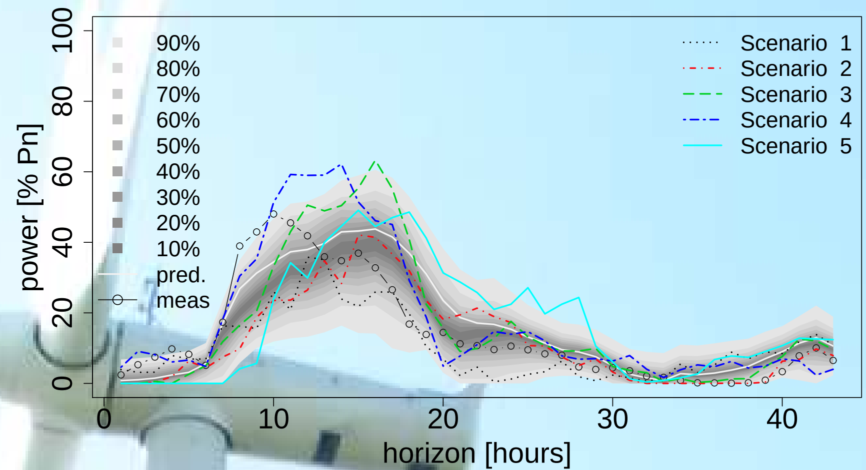


Space-time trajectories





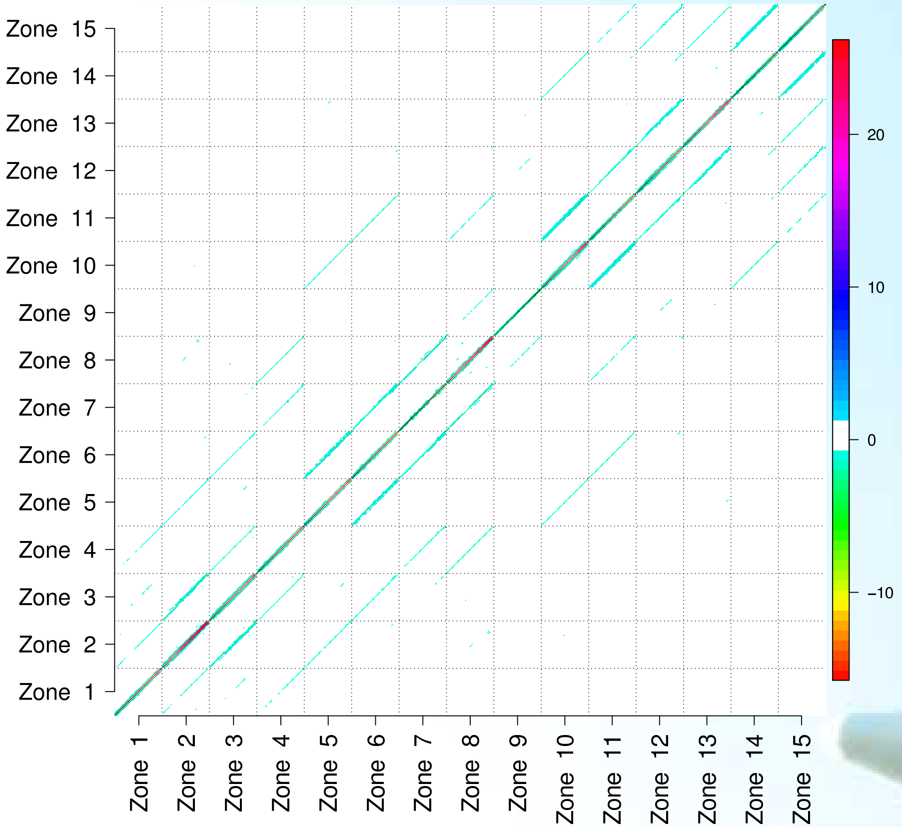




no space-time correlation

appropriate space-time correlati

Precision matrix

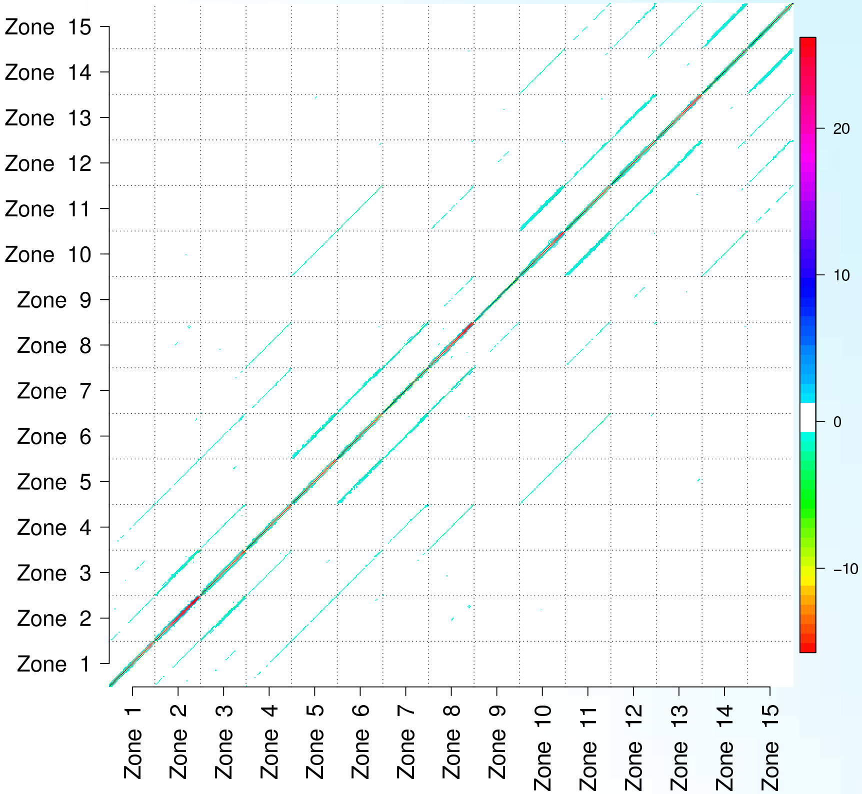


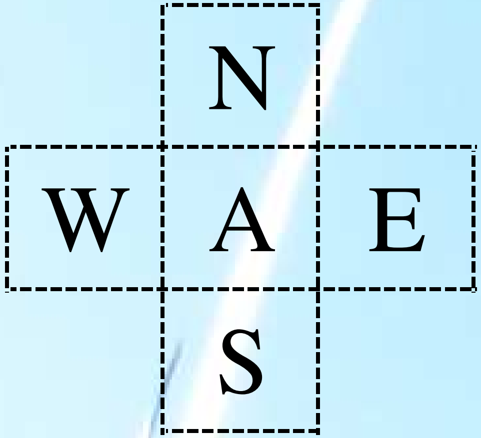
Sample precision matrix, $Q = \Sigma^{-1}$.

- <1->Inference complexity is reduced from $\mathcal{O}(n^3)$ to the range $\mathcal{O}(n)$ - $\mathcal{O}(n^{3/2})$.

Parametrization of the precision matrix

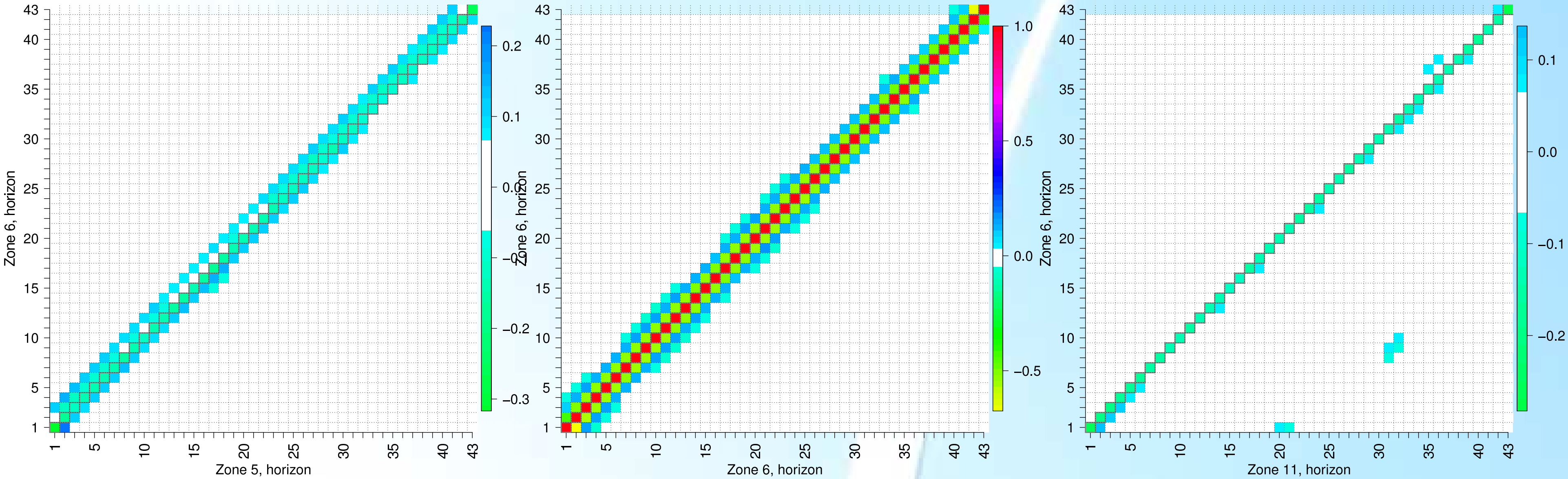
Spatial neighbourhood





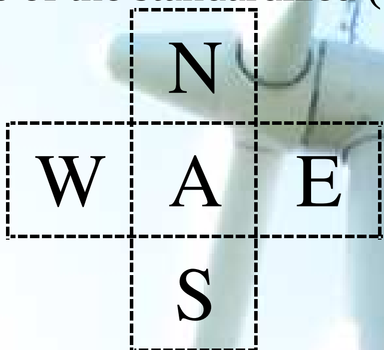
Parametrization of the precision matrix

Temporal neighbourhood



W **A** **S**

Zoomed in blocks of the standardized (by its diagonal) sample precision matrix



Type of forecasts required

- **Point forecasts (normal forecasts)**; a single value for each time point in the future. Sometimes with simple error bands.

- **Probabilistic or quantile forecasts**; the full conditional distribution for each time point in the future.

- **Scenarios**; probabilistic correct scenarios of the future wind power production.

Value of knowing the uncertainties

- Case study: A 15 MW wind farm in the Dutch electricity market, prices and measurements from the entire year 2002.
- From a phd thesis by Pierre Pinson (2006).
- The costs are due to the imbalance penalties on the regulation market.
- Value of an advanced method for point forecasting: **The regulation costs are diminished by nearly 38 pct.** compared to the costs of using the persistance forecasts.
- Added value of reliable uncertainties: **A further decrease of regulation costs – up to 39 pct.**

Wind power – asymmetrical penalties

- The revenue from trading a specific hour on NordPool can be expressed as

$$P_S \times \text{Bid} + \begin{cases} P_D \times (\text{Actual} - \text{Bid}) & \text{if} \quad \text{Actual} > \text{Bid} \\ P_U \times (\text{Actual} - \text{Bid}) & \text{if} \quad \text{Actual} < \text{Bid} \end{cases}$$

  P_S is the spot price and P_D/P_U is the down/up reg. price.

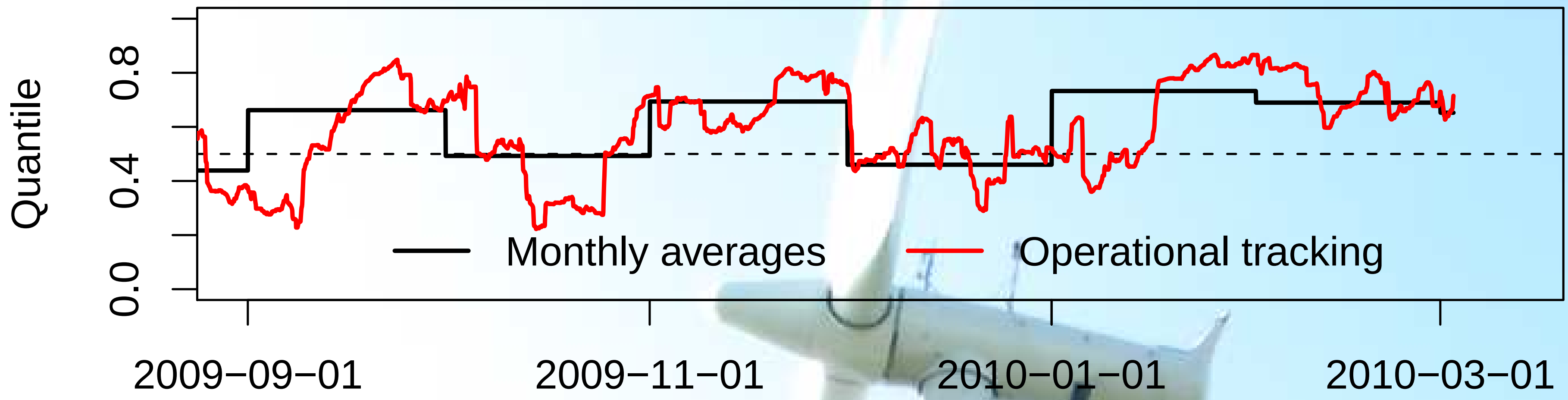
- The bid maximising the expected revenue is the following **quantile**

$$\frac{E[P_S] - E[P_D]}{E[P_U] - E[P_D]}$$

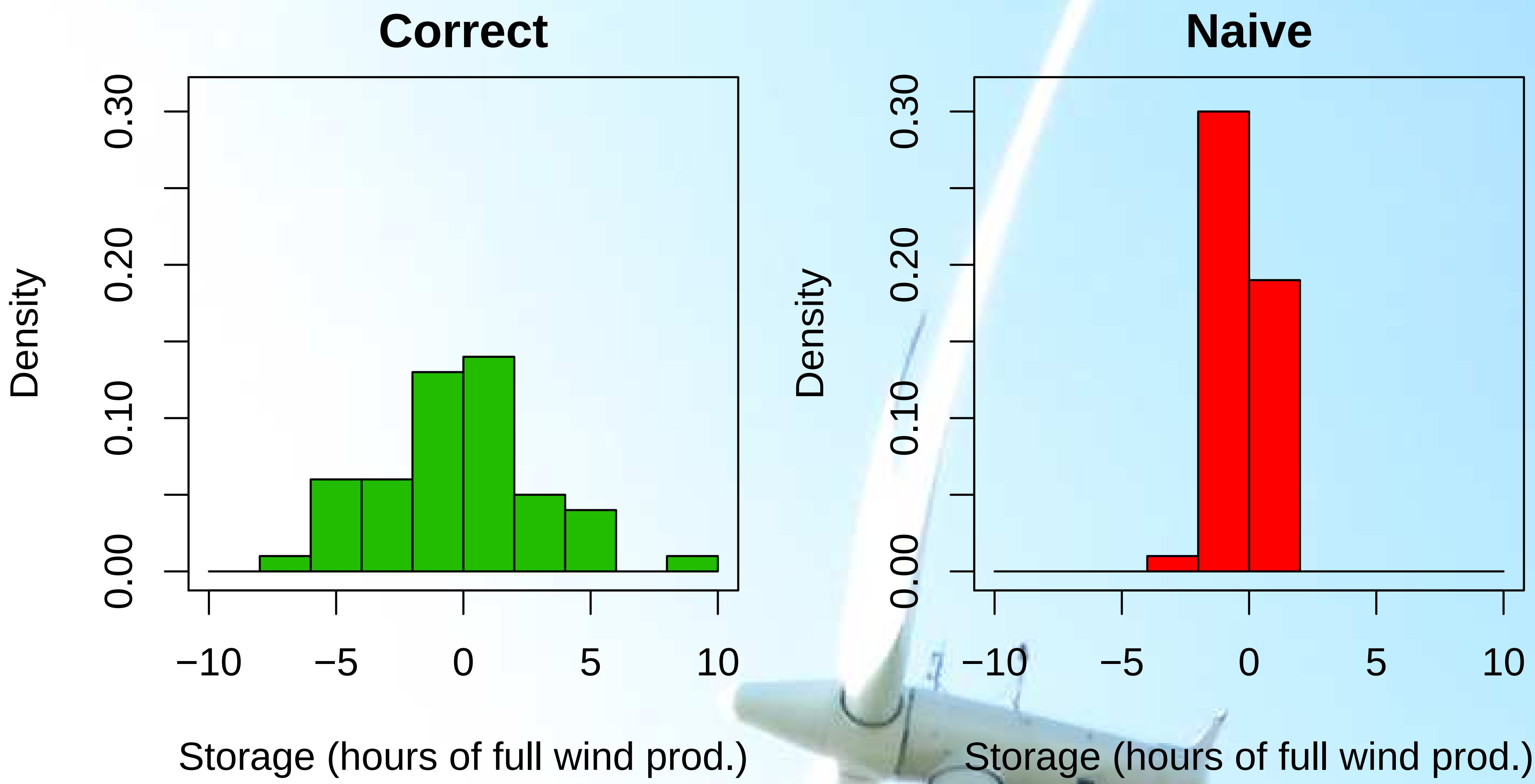
  in the conditional distribution of the future wind power production.

Wind power – asymmetrical penalties

- It is difficult to know the regulation prices at the day ahead level – research into forecasting is ongoing.
- The expression for the quantile is concerned with expected values of the prices – just getting these somewhat right will increase the revenue.
- A simple tracking of C_D and C_U is a starting point.
- **The bids maximizing the revenue during the period September 2009 to March 2010:**

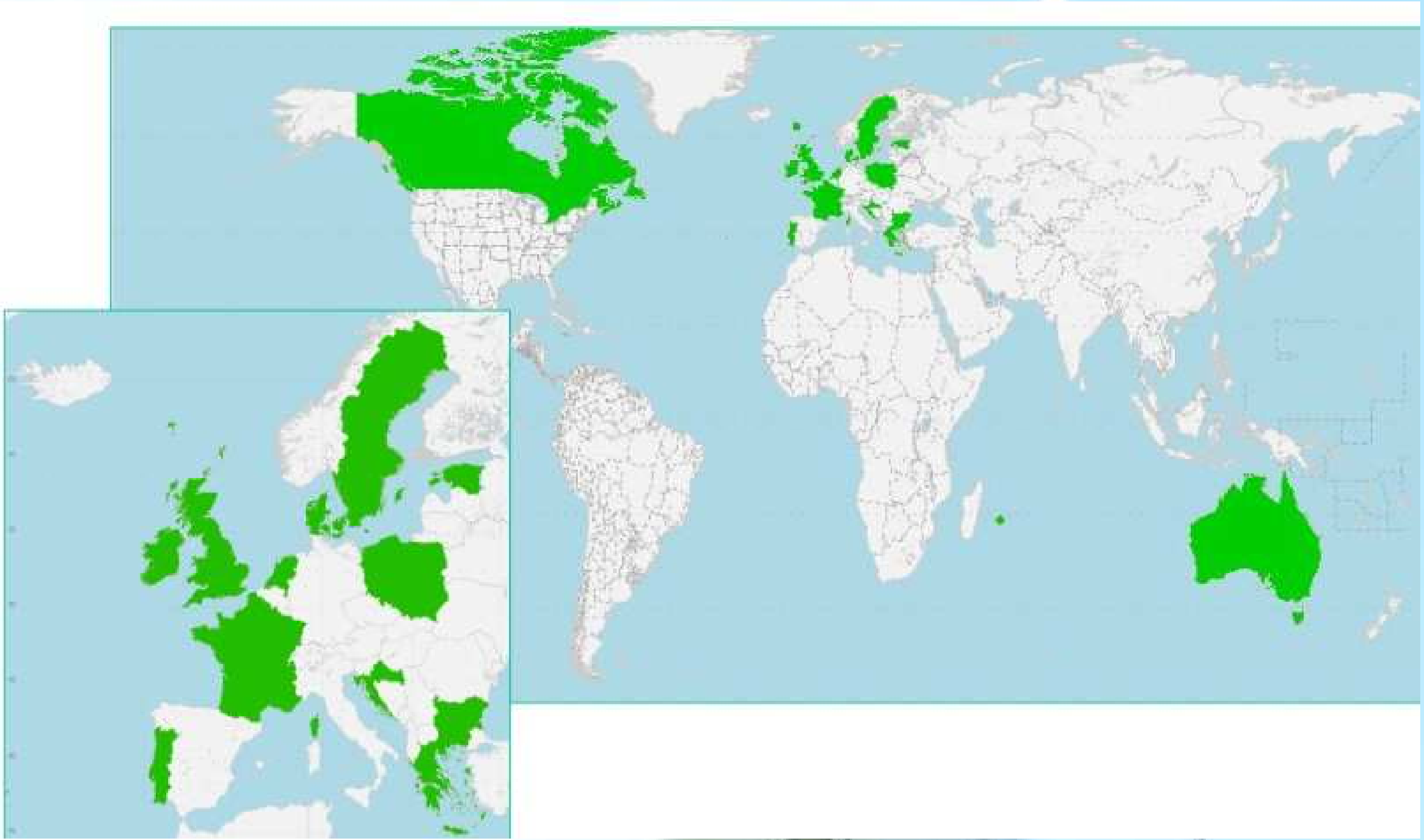


Sizing of Energy Storage



(Illustrative example based on 50 day ahead scenarios. Used for calculating the risk for a storage to be too small)

Wind Power Forecasting Installations

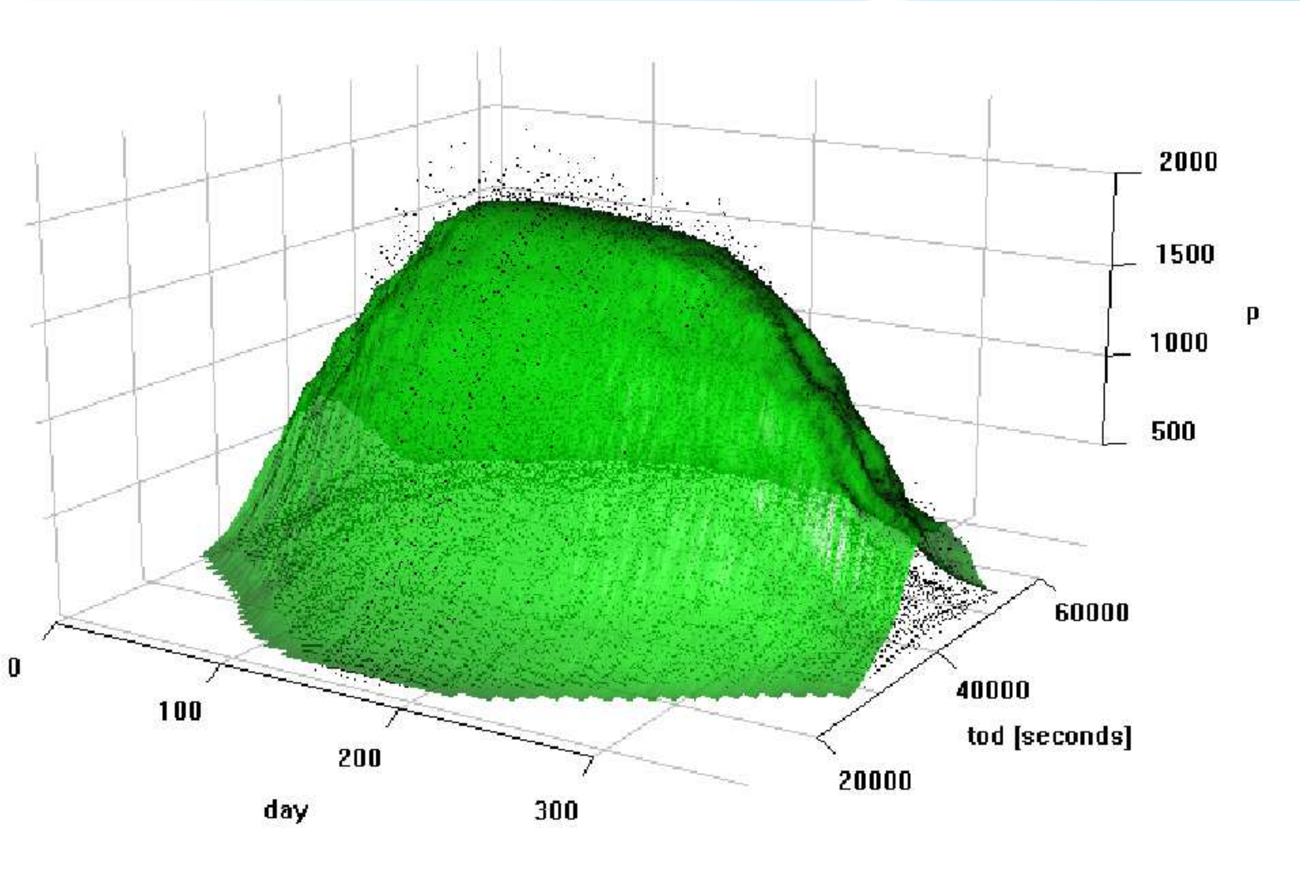


Solar Power Forecasting

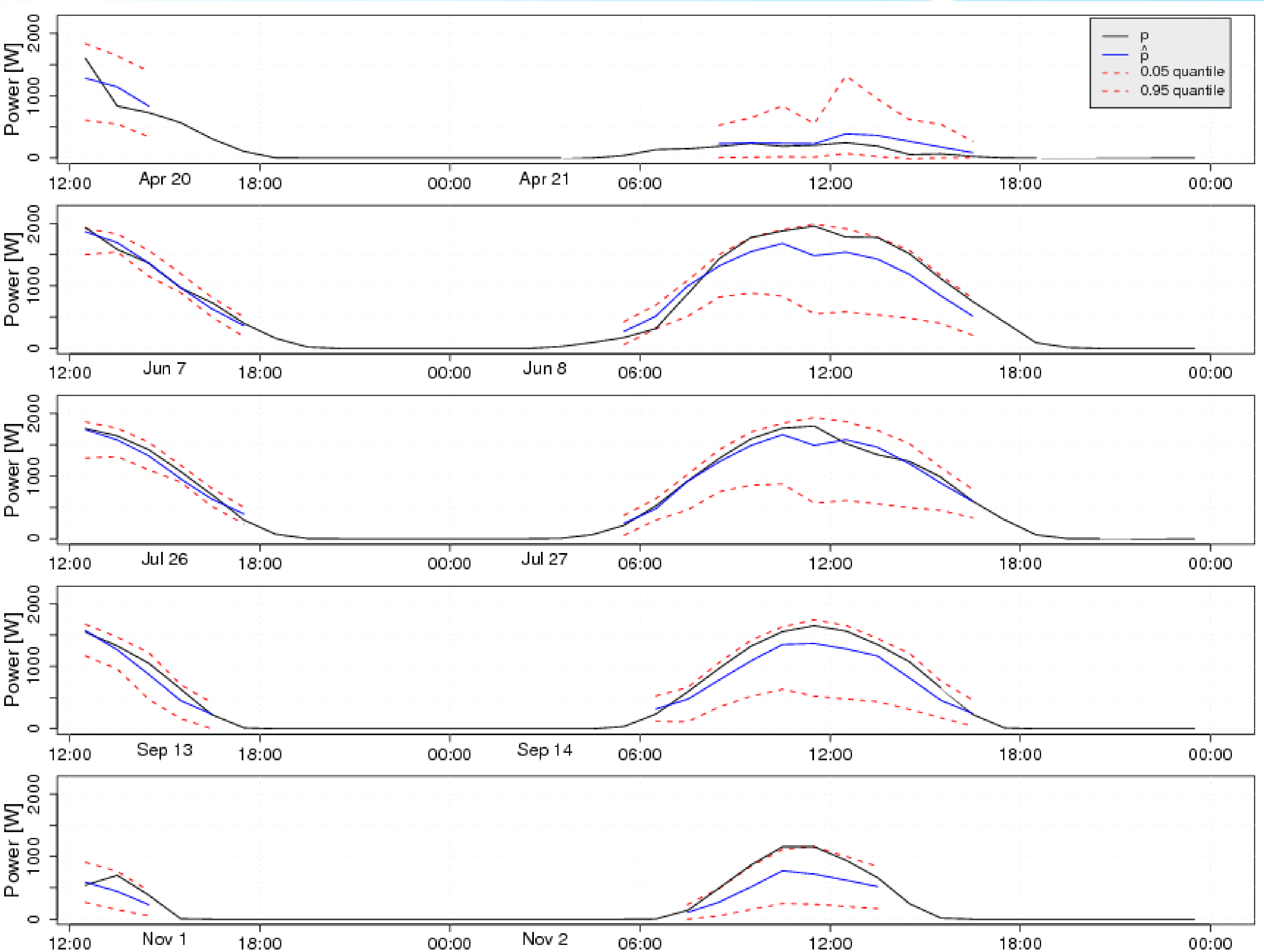
- Same principles as for wind power ....
- Developed for grid connected PV-systems mainly installed on rooftops
- Average of output from 21 PV systems in small village (Brædstrup) in DK

Method

- Based on readings from the systems and weather forecasts
- Two-step method
- Step One: Transformation to atmospheric transmittance τ with statistical clear sky model (see below). Step Two: A dynamic model (see paper).



Example of hourly forecasts



Some Advanced Topics

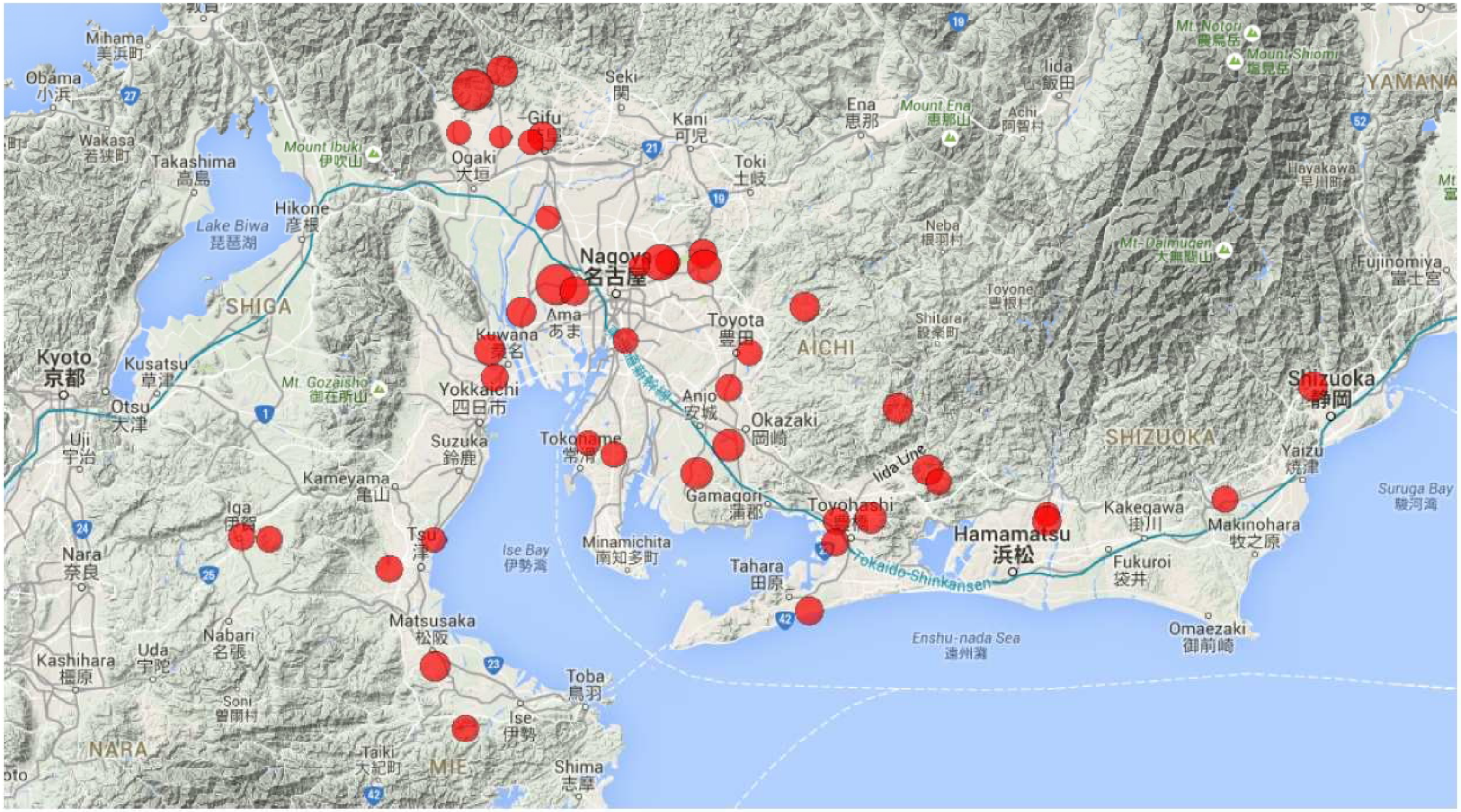


Figure 1: Location of the 42 PV installations. The size of the mark is scaled by installed capacity.

Spatio-Temp. solar power forecasting in Japan

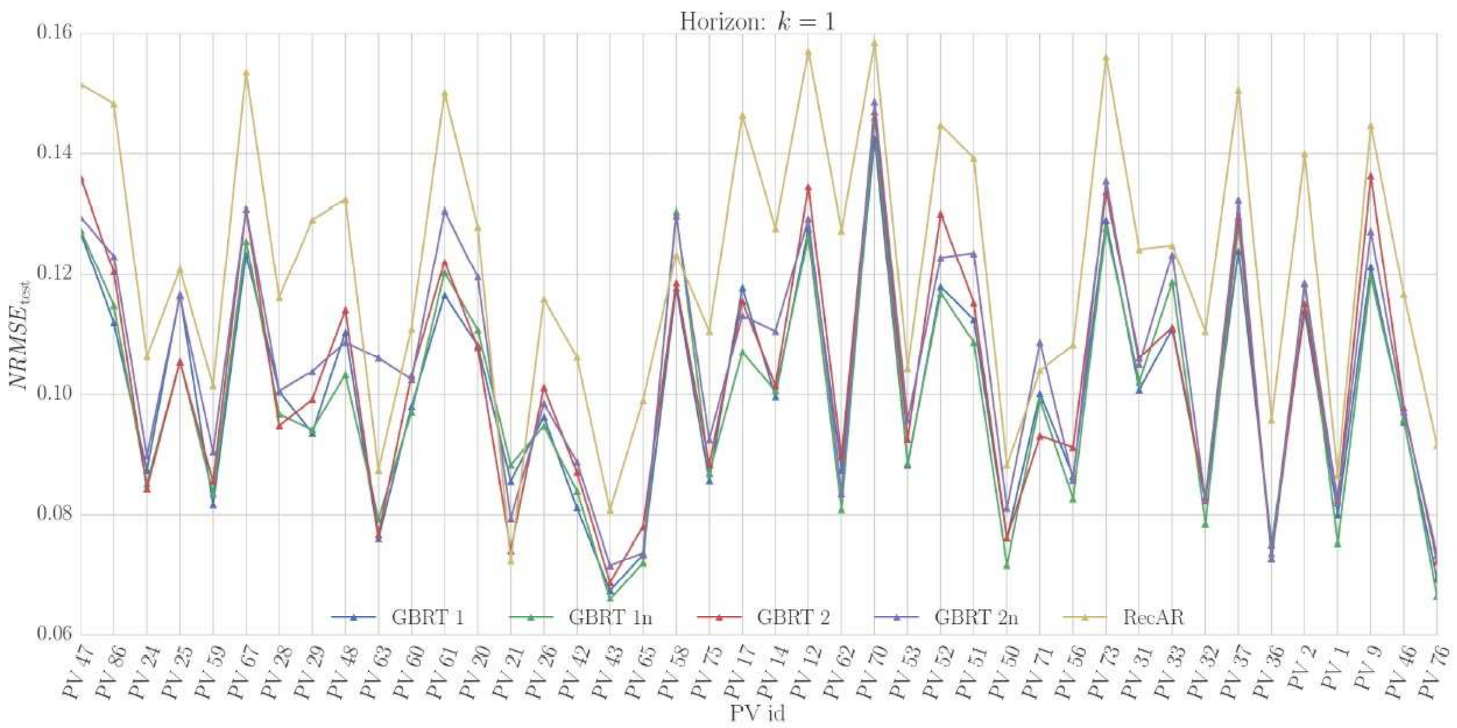


Figure 9: $NRMSE_{test}$ for the AR model and the GBRT models on a horizon of one hour for each PV installation.

Forecasting for a large Solar Power Plant

Figure 1: *An aerial photo of the Copper Mountain Solar Facility. The small red rectangle indicates the size of a single inverter, which is 125m by 125m. The large red rectangle indicates the 5 by 14 inverters for which we provide forecasts of the power production.*

Forecasting for a large Solar Power Plant

$$
\begin{aligned}
dU_{i,j,t} &= \theta|v_t|\Big(\mathrm{n}_t(U_{i,j+1,t}-U_{i,j,t})\mathbb{1}_{\{j+1\le J\}}+\mathrm{e}_t(U_{i+1,j,t}-U_{i,j,t})\mathbb{1}_{\{i+1\le I\}} \\
&\quad +\mathrm{s}_t(U_{i,j-1,t}-U_{i,j,t})\mathbb{1}_{\{j-1\ge 1\}}+\mathrm{w}_t(U_{i-1,j,t}-U_{i,j,t})\mathbb{1}_{\{i-1\ge 1\}} \qquad (11)\\
&\quad -\mu U_{i,j,t}\left(\mathrm{n}_t\mathbb{1}_{\{j=J\}}+\mathrm{s}_t\mathbb{1}_{\{j=1\}}+\mathrm{e}_t\mathbb{1}_{\{i=I\}}+\mathrm{w}_t\mathbb{1}_{\{i=1\}}\right)\Big)dt \\
&\quad +\sigma dW_{i,j,t} \\
dQ_{i,j,t} &= U_{i,j,t}dt \qquad (12)\\
Y_{i,j,k} &= Q_{i,j,t_k}+\epsilon_{i,j,k}. \qquad (13)
\end{aligned}
$$

$$
dU(x,t) = \bar{v}\theta\nabla U(x,t)dt+\sigma dW(x,t) \qquad (20)
$$

Forecasting for a large Solar Power Plant

Score	Cloud Speed Persistence	Ramp Speed Persistence	Auto-Regressive	Model (12)-(13)
$RMSE_5$	0.334	0.612	0.464	**0.636**
$RMSE_{20}$	0.289	0.284	0.319	**0.523**
$RMSE_{60}$	0.168	-0.203	0.113	**0.254**
$RMSE_{120}$	0.062	-0.434	0.039	**0.097**
MAE_5	0.258	0.597	0.431	**0.612**
MAE_{20}	0.213	0.301	0.280	**0.497**
MAE_{60}	0.136	-0.145	0.045	**0.246**
MAE_{120}	0.048	-0.396	-0.064	**0.096**
$CRPS_5$	–	–	0.00262	**0.00131**
$CRPS_{20}$	–	–	0.00982	**0.00666**
$CRPS_{60}$	–	–	0.02886	**0.02455**
$CRPS_{120}$	–	–	0.04883	**0.04675**

Table 2: *The MAE skill score, RMSE skill score and CRPS for benchmarks as well as for the proposed model for horizons of 5, 20, 60 and 120 seconds.*

Software Modules for Wind Power Forecasting

- Point prediction module
- Probabilistic (quantile) forecasting module
- Scenario generation module
- Spatio-temporal forecasting module
- Space-time scenario generation module
- Even-based prediction module (eg. cut-off prob.)
- Ramp prediction module

Same modules are available for solar Power Forecasting

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VG-Integration: Lessons Learned in Denmark

- (> 5 pct wind): Tools for Wind/Solar Power forecasting are important
- (> 10 pct wind): Tools for reliable probabilistic forecasting are needed
- (> 15 pct wind): Consider Energy Systems Integration (not Power alone)
- (> 20 pct wind): Consider Methods for Demand Side Management
- (> 25 pct wind): New methods for finding the optimal spinning reserve are needed (based on prob. forecasting of wind/solar power production)
- Joint forecasts of wind, solar, load and prices are essential
- Limited need - or no need - for classical storage solutions
- Huge need for virtual storage solutions
- Intelligent interaction between power, gas, DH and biomass very important
- ICT and use of data, adaptivity, intelligence, and stochastic modelling is very important

The largest national strategic research project: **Centre for IT-Intelligent Energy Systems in Cities - CITIES** have been launched 1. January 2014.

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Wind Power Forecasting - Lessons Learned

- The forecasting models must be **adaptive** (in order to taken changes of dust on blades, changes roughness, etc., into account).
- Reliable estimates of the **forecast accuracy** is very important (check the reliability by eg. reliability diagrams).
- Reliable probabilistic forecasts are important to gain the **full economical value**.
- Use **more than a single MET provider** for delivering the input to the prediction tool – this improves the accuracy of wind power forecasts with 10-15 pct.
- Estimates of the **correlation in forecasts errors** important.
- Forecasts of ’**cross dependencies**’ between load, prices, wind and solar power are important.
- **Probabilistic forecasts are very important for asymmetric cost functions**.
- Probabilistic forecasts can provide **answers** for questions like
  - What is the probability that a given storage is large enough for the next 5 hours?
  - What is the probability of an increase in wind power production of more that 50 pct of installed power over the next two hours?
  - What is the probability of a down-regulation due to wind power on more than x GW within the next 4 hours.

The same conclusions hold for our tools for **eg. solar power forecasting**.

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