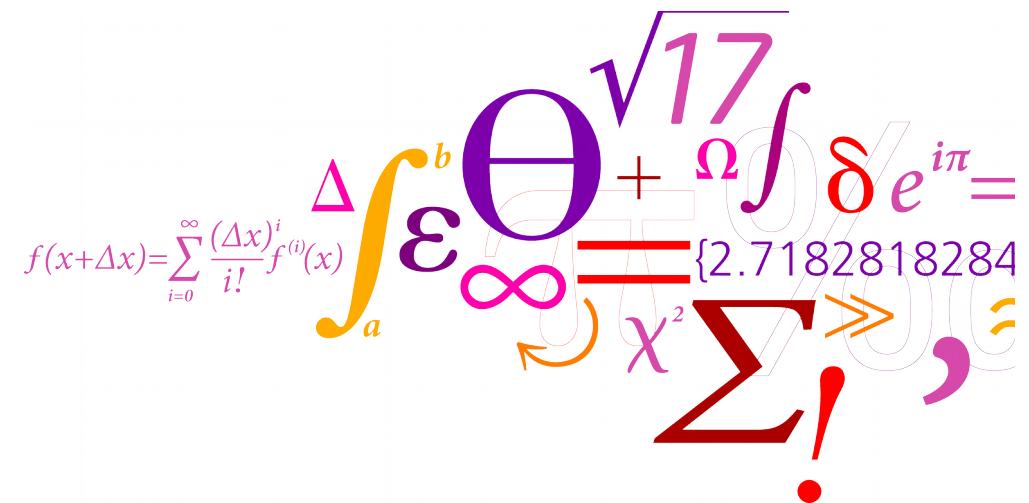


# **Grey-box modelling; Bridging the gap between physical and statistical modelling**

**ZEN Workshop, Energy Flexibility in Zero-Emission  
Neighborhoods, Oslo, November 2017**

**Henrik Madsen**

**[www.henrikmadsen.org](http://www.henrikmadsen.org)**

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$


# Thermal performance characterization using time series data - statistical guidelines

IEA EBC Annex 58

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BBRI, Belgian Building Research Institute  
Brussels, Belgium

November 28, 2016

# GUIDELINES FROM ANNEX 58

Static and dynamic conditions: estimate the Heat Loss Coefficient (HLC) and gA-value from 'simple' data:

- Constant indoor temperature
- *Model input:* ambient temperature and global radiation (wind not included in guideline models)
- *Model output:* heat load

Grey-box models for detailed building behavior characterization:

- Varying indoor temperature (turn the heating on/off)
- *Model input:* ambient temperature, global radiation, wind
- *Model output:* indoor air temperature

**Procedures (recipes) for model selection and validation, with examples in R**

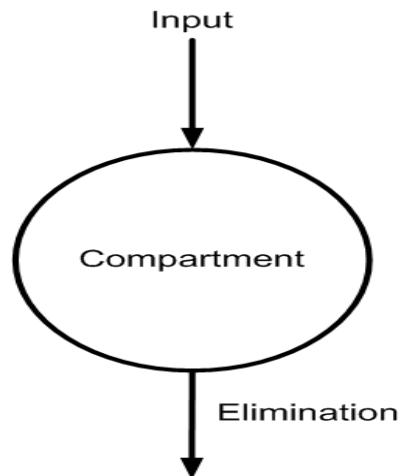


# Introduction to Grey-Box modelling



**ZEN Workshop,  
SINTEF Byggforsk, Oslo, November 2017**

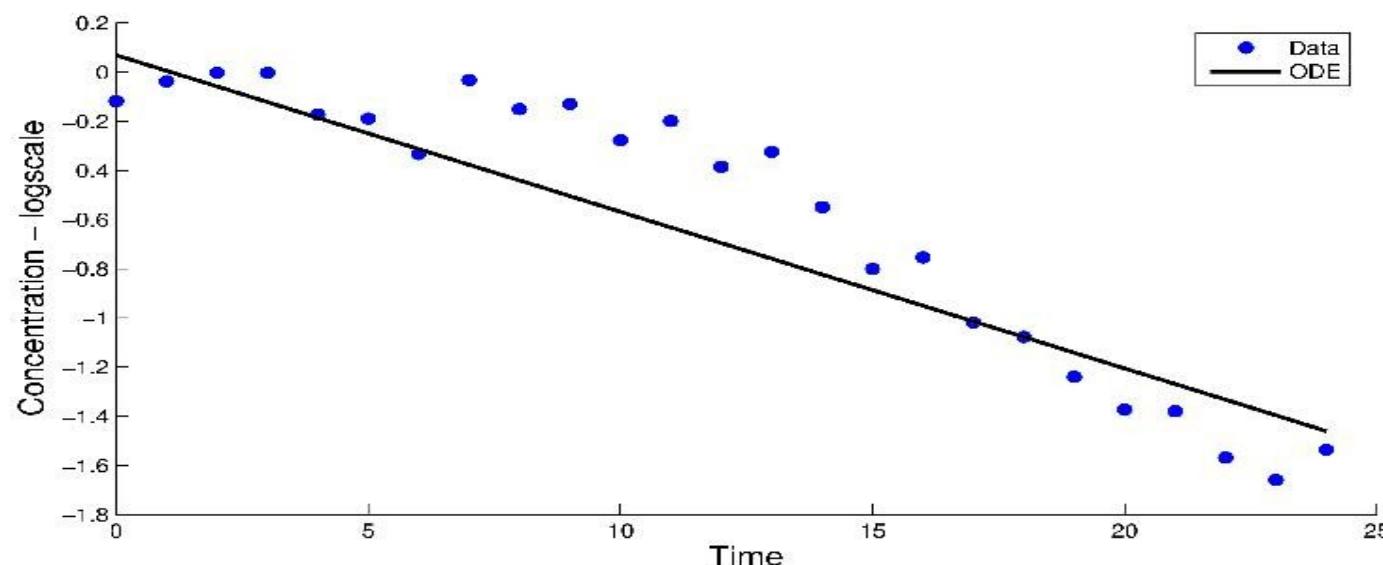
# Traditional Dynamical Model



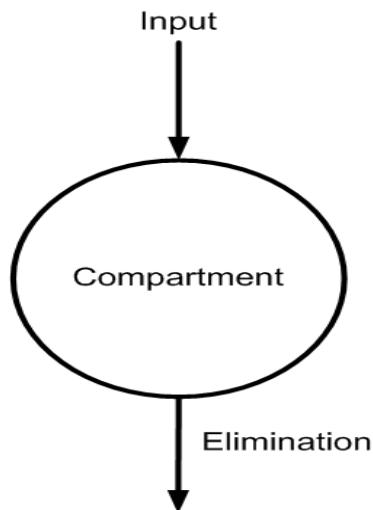
- Ordinary Differential Equation:

$$dA = -K A dt$$

$$Y = A + \epsilon$$

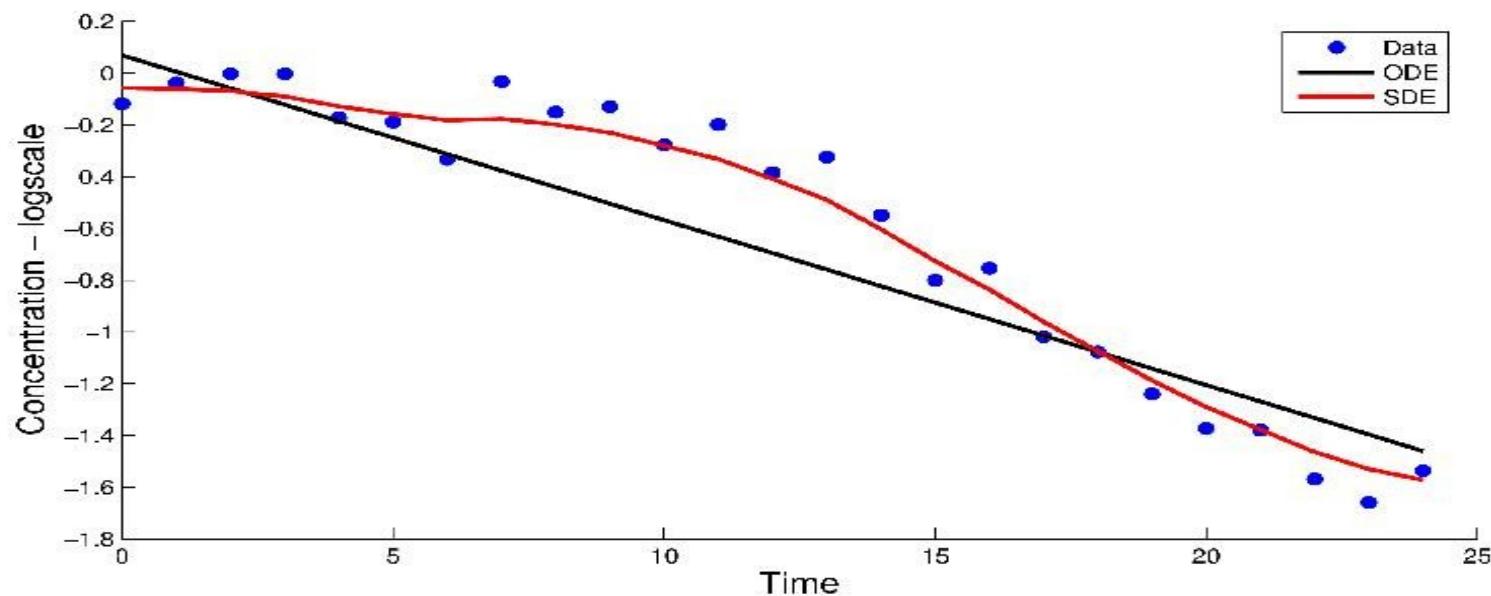


# Stochastic Dynamical Model



- Stochastic Differential Equation:

$$\begin{aligned} dA &= -KA dt + \sigma dw \\ Y &= A + e \end{aligned}$$



# The grey box model

Drift term

$$dX_t = f(X_t, u_t, t, \theta) dt + \sigma(X_t, u_t, t, \theta) d\omega_t$$

$$Y_k = h(X_k, u_k, t_k, \theta) + e_k,$$

Diffusion term

System equation

Observation equation

Observation noise

Notation:

$X_t$ : State variables

$u_t$ : Input variables

$\theta$ : Parameters

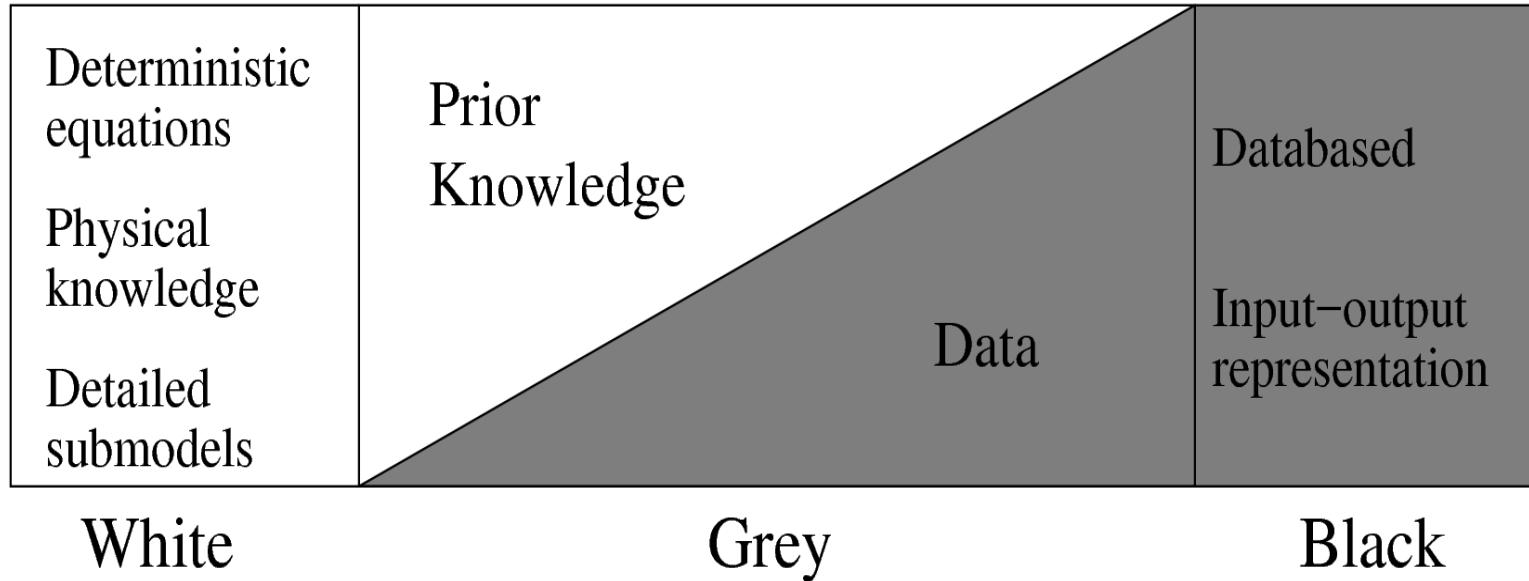
$Y_k$ : Output variables

$t$ : Time

$\omega_t$ : Standard Wiener process

$e_k$ : White noise process with  $N(\mathbf{0}, \mathbf{S})$

# Grey-box modelling concept



- Combines prior physical knowledge with information in data
- Equations and parameters are physically interpretable

# Grey-Box Modelling

- Bridges the gap between physical and statistical modelling
- Provides methods for model identification
- Provides methods for model validation
- Provides methods for pinpointing model deficiencies
- Enables methods for a reliable description of the uncertainties, which implies that the same model can be used for **k-step forecasting, simulation and control**

## Case study

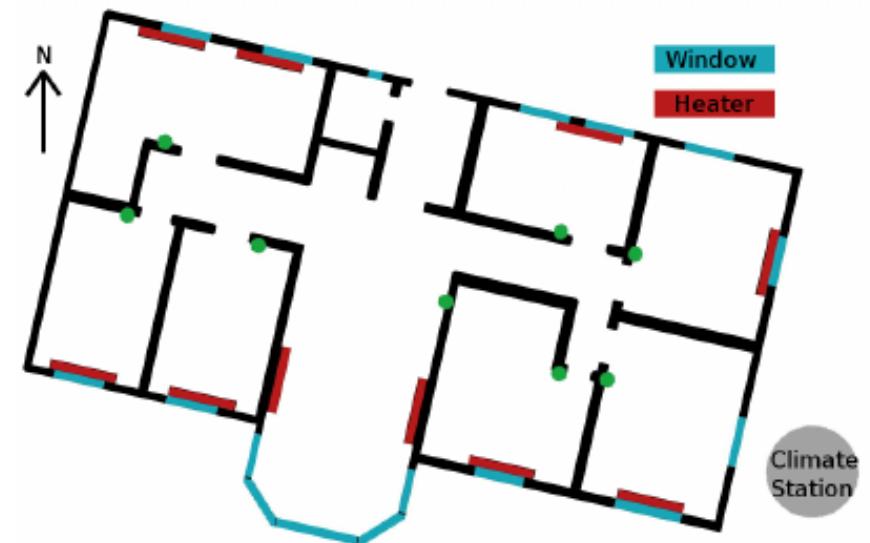
# Model for the thermal characteristics of a small office building



# TEST CASE: ONE FLOORED 120 M<sup>2</sup> BUILDING

## Objective

Find the best model describing the heat dynamics of this building  
([1], [4])



# DATA

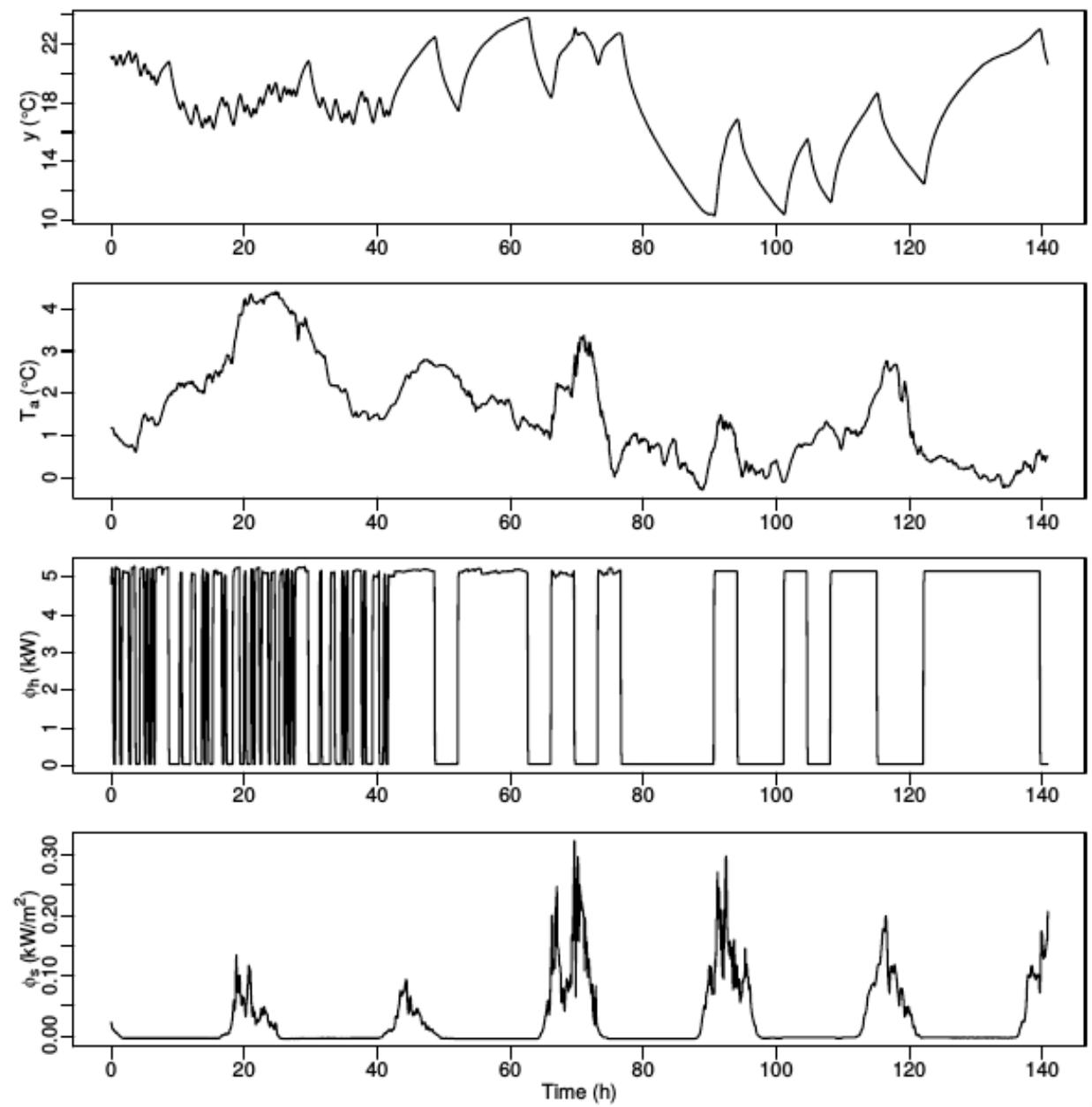
Measurements of:

$y_t$  Indoor air temperature

$T_a$  Ambient temperature

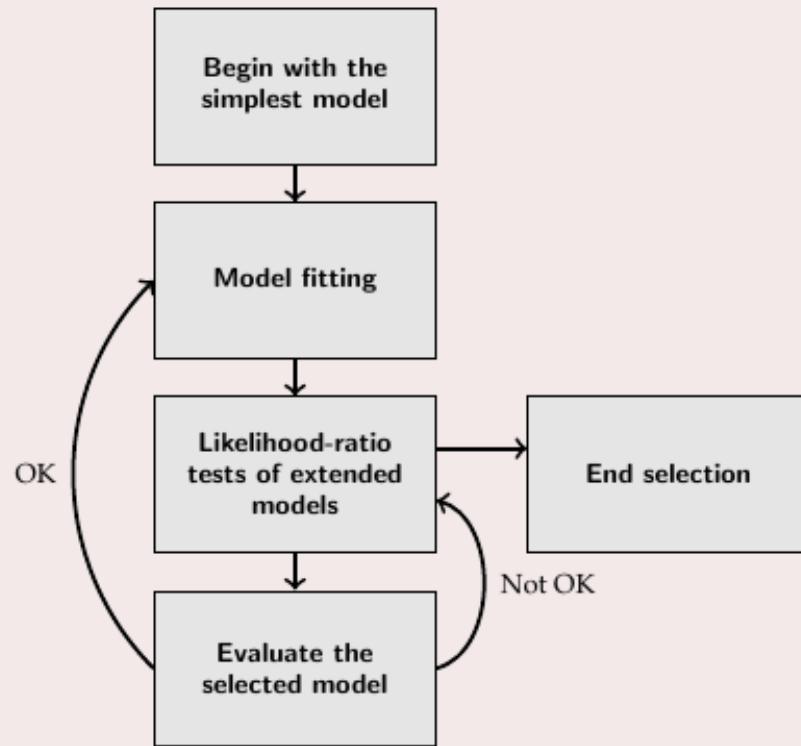
$\Phi_h$  Heat input

$\Phi_s$  Global irradiance

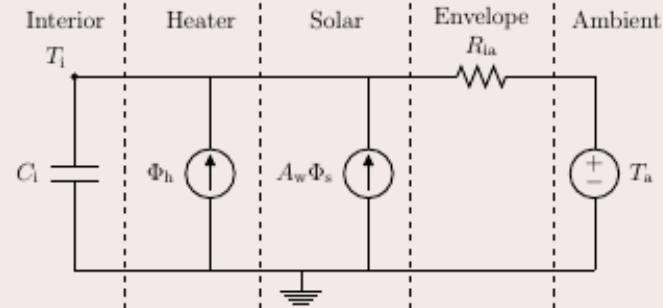


# SELECTION PROCEDURE

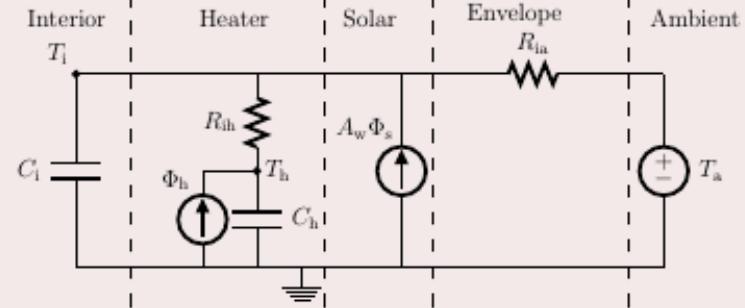
Iterative procedure using statistical tests



## Simplest model

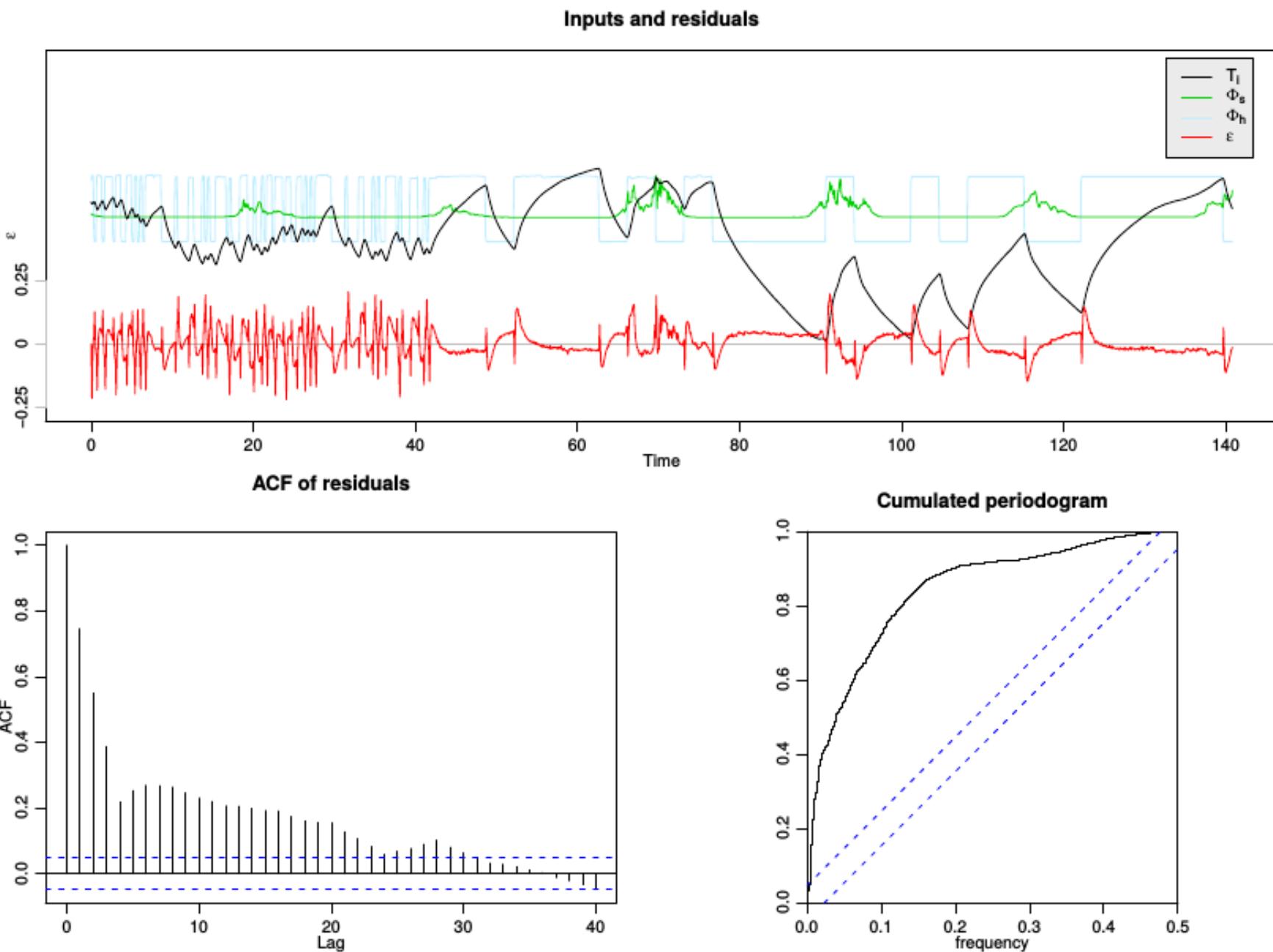


## First extension: heater part

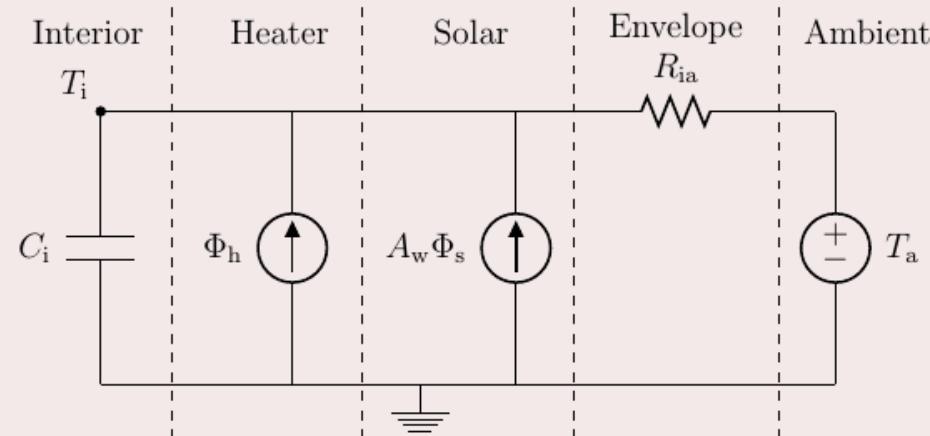


Start	$Model_{Ti}$	$l(\theta; \mathcal{Y}_N)$	$m$	$Model_{TiTe}$	$Model_{TiTm}$	$Model_{TiTs}$	$Model_{TiTh}$
1	2482.6		6				
$l(\theta; \mathcal{Y}_N)$	3628.0			3639.4		3884.4	3911.1
$m$	10			10		10	10
2 ...							

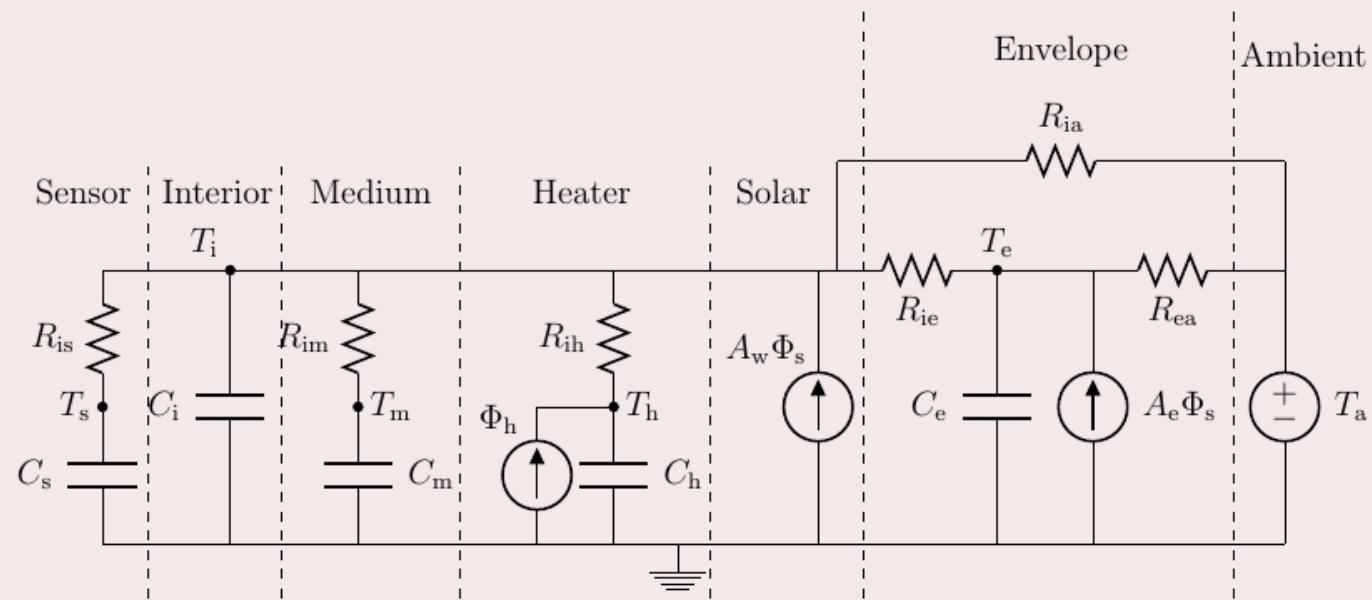
# EVALUATE THE SIMPLEST MODEL



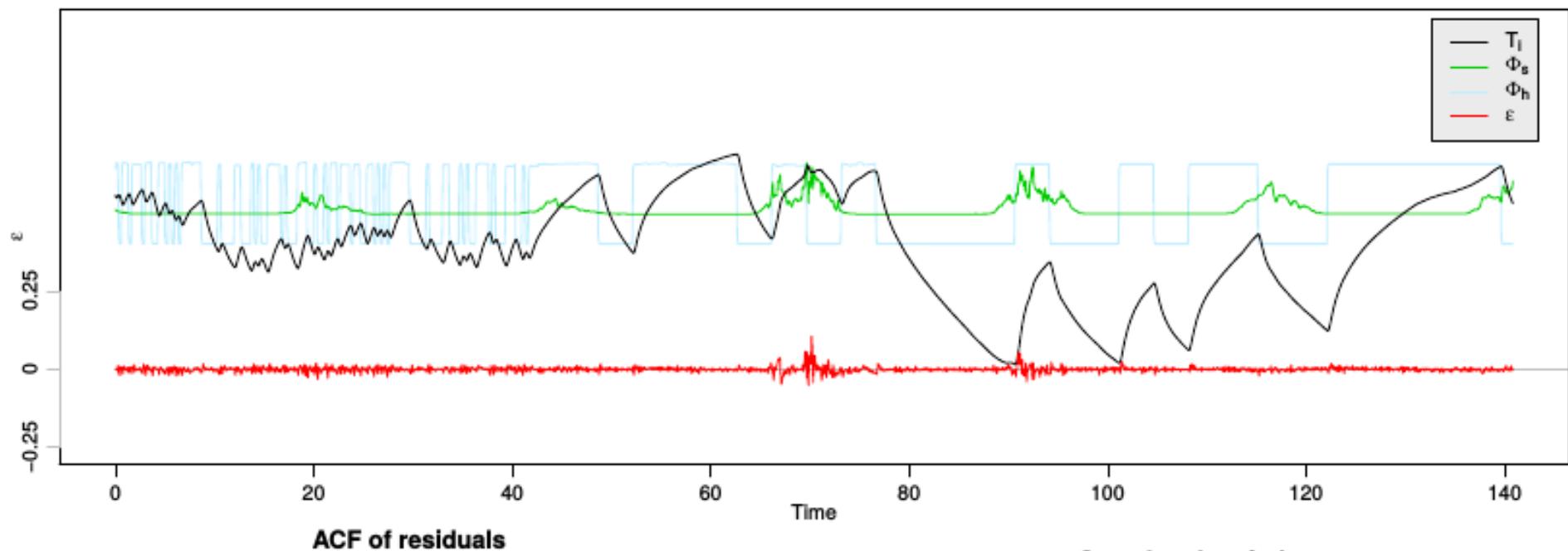
## Simplest model



## Most complex model applied

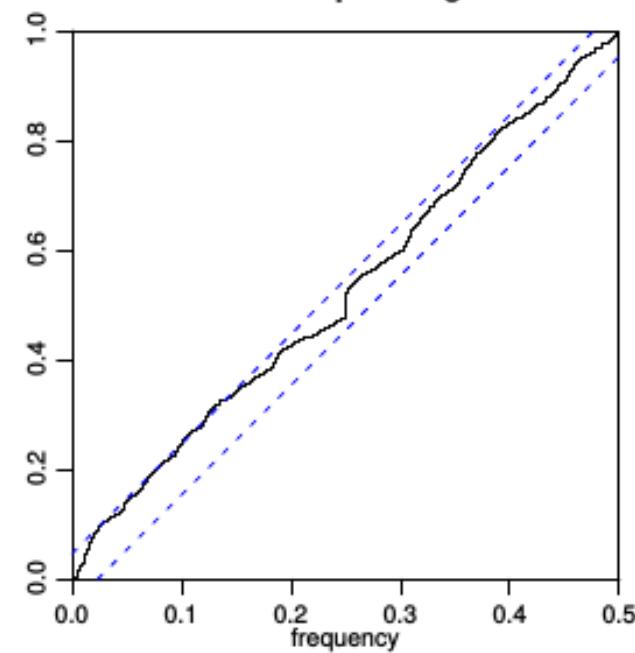
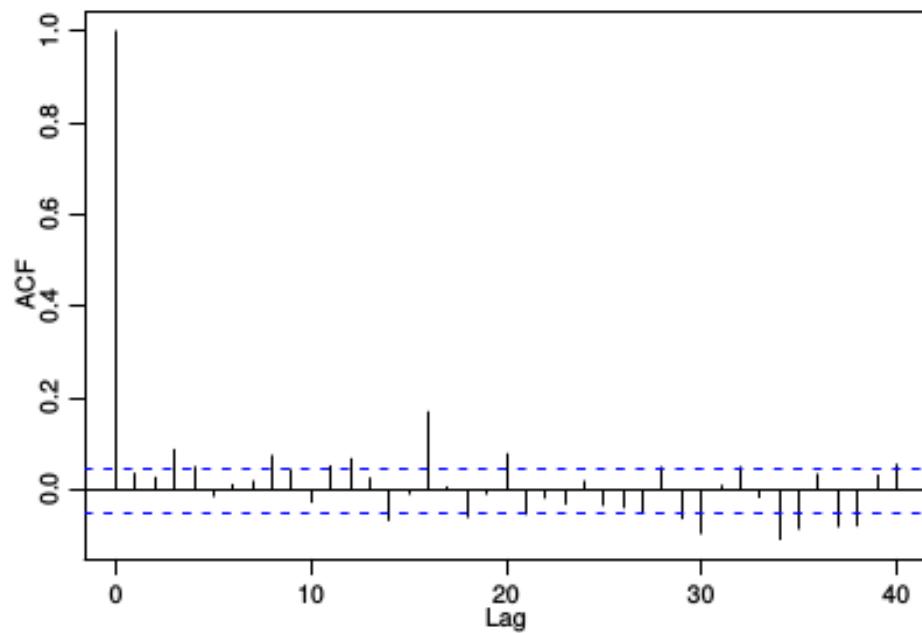


### Inputs and residuals



ACF of residuals

Cumulated periodogram



# GREY-BOX MODELLING

Continuous time models (*grey-box: stochastic state-space model*)

$$\text{States} = \text{Fun}_1(\text{States}, \text{Inputs}) + \text{Fun}_2(\text{Inputs}) \cdot \text{SystemError}$$

$$\text{Measurements} = \text{Fun}_3(\text{States}, \text{Inputs}) + \text{Fun}_4(\text{Inputs}) \cdot \text{MeasurementError}$$

- Used for buildings (single- and multi-zone), walls, systems (hot water tank, integrated PV, heat pumps, heat exchanger, solar collectors, ...)
- Formulate the model based on physical knowledge
- Maximum likelihood estimation  
(we have the entire statistical framework available)
- Description of the system noise is part of the model provides some very useful possibilities  
(e.g. control the weight of data in the estimation depending on input signals)
- Software, for example our R package CTSM-R<sup>1</sup>

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<sup>1</sup><http://ctsm.info>

# Thanks ...

- For more information

[www.ctsm.info](http://www.ctsm.info)

[www.henrikmadsen.org](http://www.henrikmadsen.org)

[www.smart-cities-centre.org](http://www.smart-cities-centre.org)

- ...or contact

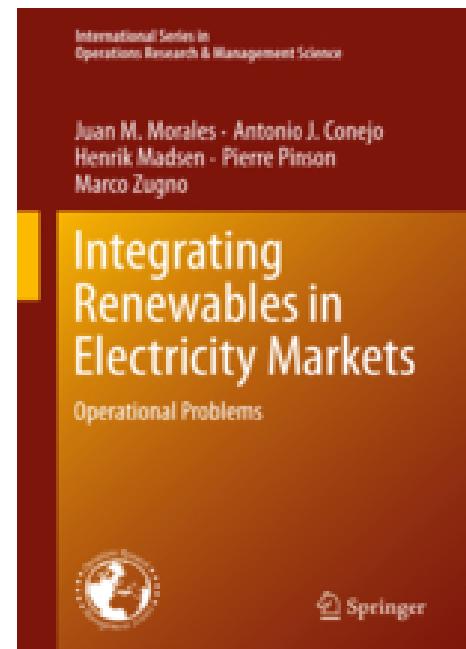
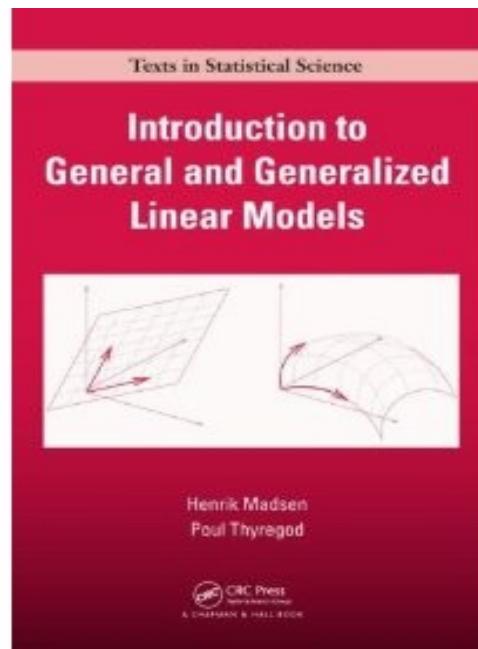
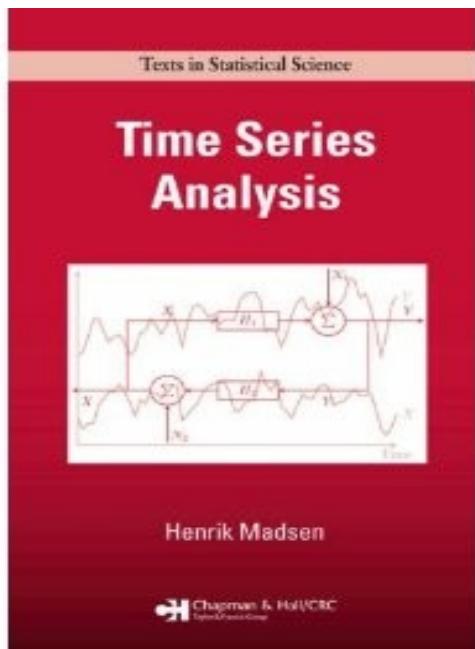
– Henrik Madsen (DTU Compute)

[hmad@dtu.dk](mailto:hmad@dtu.dk)

- Acknowledgement CITIES (DSF 1305-00027B)



# Some 'randomly picked' books on modeling ....



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