



# Model Predictive Control in Urban Systems

## Optimal and Stochastic Utilization of the infrastructure of sewer systems

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## Introduction

The utility of sewer systems to both transport sewage and rainwater results in issues with capacity limits during larger rain events. When the volume capacity of the sewer is reached, then the water exits the sewers wherever it is possible; streets, parks, harbour basins, or basements. This polluting outflow of sewage water leads to damages of both environmental and proprietary nature, as well as a health hazard for the population. The issue can be prevented/reduced by introducing SuDS structures to direct the rainwater to the natural environment, or by expanding/updating the sewer infrastructure to increase its capacity and/or becoming a separate sewer system. Both of these solutions require time-expensive investments in changing the infrastructure, as well as requiring the availability of the needed space for the new infrastructure.

Another approach is to achieve better performance with the existing infrastructure. For this, we must control the flows within the sewers, so an optimal control method is useful. Given the physical limits of sewers, the optimal method: Model Predictive Control (MPC) is useful, as it allows for the inclusion of constraints in its formulation of the optimal control, as well as to account for the future evolution of the system, relying on weather forecasts.

## The MPC Methods

The MPC methods consist of a cost function describing the definition of the best performance over a prediction horizon. The MPC also includes a process model of the sewer system with constraints describing the physical and operational limits of the system.

The optimal control is computed using a receding approach; recomputing the optimal control in the entire prediction horizon at each time step.

The sewer model used, are simplified versions of the sewer, only considering the flow and stored volumes.

While the best performance was defined in prioritized order: the minimum amount of overflow, the maximum amount of flow to treatment plants, and lastly reduced amount of equipment wear in form of control change.

### Overview of considerations for MPC in Sewers

.1st the simplified sewer models are constructed with the virtual tank approach. In which large sections of the systems are lumped together like a giant storage tank, creating an interconnected network of tanks with flows in between.

.2nd the overflows to the environment are then given by the volume and flow capacity of the tanks and interconnections respectively. The inclusion of overflows in the models results in logical model structures, which affect computation time.

.3rd the usage of weather forecasts to predict the future behaviour of the sewers introduces uncertainty into the optimization program (unless you have perfect forecasts). Which has to be accounted for.

.4th accounting for the uncertainty in the control design leads to considerations on how to handle the computation of the uncertainty used in the control.

## Case studies

The methods for handling the open considerations were tested on several case study models. The case study of the Barcelona sewer systems was used for evaluating alternatives to the natural logical description of overflows. The Astlingen benchmark model was used for the evaluations of how to include uncertainty in MPC for sewers, while the Aarhus case study was used for evaluating an alternative computation of the uncertainty for the control.

## Results

Methods handling the open considerations for applying MPC to sewers were proposed and evaluated. The benefits of approximating the logical formulation of overflow with minimized variables in the optimization program were explored. Resulting in practical rules for weighting the usage of the overflow variables, in order to conserve the physical true behaviour of the system.

The presence of overflow structures in the models turned out to be a hindrance for using the method Chance-Constrained MPC (CC-MPC) to handle the uncertainty directly. An alternative approach to CC-MPC was formulated; in order to include the overflows and preserve the feasibility of the controller.

The CC-MPC for sewers would then consider the probability of the overflow not happening, rather than considering the probability of the volume/flow staying below the capacity, which would naturally follow from the models.

With the CC-MPC method fit for the control of the system, its reliance on probability distributions was considered. Given the changes to distributions, accumulated as they propagate through a model; an estimated approach to CC-MPC was proposed, where an ensemble of scenarios of the uncertainties was propagated instead of distributions, with the final constraint distributions being estimated as part of the control computations.

## Discussion

In the evaluation of overflow formulations, it was found that the approximating approach could describe the proper logical formulation equivalent with respect to the physical behavior of the system, while also providing faster and more robust computation times. Where the maximum computation times were up to 5000 times faster than the logical approach, and less than a 1500ths of the sampling time. Though, this improvement in the computation was found to come with a trade-off in design freedom regarding the cost functions describing the best performance; enforcing strict minimization of overflows through the introduction of weighting rules.

The proposal of the reformulated CC-MPC method was found to be less sensitive to more aspect of how uncertainty can vary than a standard MPC method trusting its forecast; e.g. While the CC-MPC and MPC both perform worse with overestimated forecasts, only the performance of MPC deteriorated with underestimated forecasts by more than 1%, while CC-MPC improve by 0.1% in the simulations. Due to the nature of using finite-horizons in MPCs, the CC-MPC were found to occasionally perform better than the MPC with perfect forecasts, giving 0.27% less overflow volumes.

The Ensemble-based CC-MPC were found to perform similarly to the CC-MPC in a linear evaluation using Gaussian uncertainty. The performs were similar enough to indicate it as a possible practical alternative when propagations of distributions would be analytically difficult to do. In a sewer system, it would in general be a suitable approach; given that it is not uncommon for the uncertainty in weather forecasts to initially be given in ensembles.

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