Identification of stochastic models for describing the thermal performance of buildings

DTU-Tsinghua Workshop on Energy, Technology and Behaviour in Buildings

Beijing, June 2017

Henrik Madsen

www.henrikmadsen.org
George Box:

All models are wrong – but some are useful
Modeling made simple

Suppose we have a time series of data:

\[ \{X_t\} = X_1, X_2, ..., X_t, ... \]

The purpose of any modeling is to find a nonlinear function \( h(\{X_t\}) \) such that

\[ h(\{X_t\}) = \varepsilon_t \]

Where \( \{\varepsilon_t\} \) is white noise – ie. no autocorrelation.
Some 'randomly picked' books on modeling ....
Contents

1. A single sensor (a smart meter)
2. Several sensors (and grey-box modelling)
3. Special sensors (model for occupant behavior)
Part 1
A single sensor (smart meter)

- Smart Meters and data splitting
- Smart Meters and Thermal Characteristics
  - Problem setting
  - Simple tool
Case Study No. 1

Split of total readings into space heating and domestic hot water using data from smart meters
Data separation principle

- House Characteristic
  - e.g. size, insulating power, solar absorption

- Occupants Characteristic
  - e.g. open/close windows, turn up/down the heating, night-time drop

- Raw Data
- Heating Consumption
- Hot Water Consumption
  - e.g. shower, dishwashing

DTU-Tsinghua Workshop,
Beijing, June 2017
Data

• 10 min averages from a number of houses

<table>
<thead>
<tr>
<th>House 1</th>
<th>House 2</th>
<th>House 3</th>
<th>House 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year build</td>
<td>Year build</td>
<td>Year build</td>
<td>Year build</td>
</tr>
<tr>
<td>1963</td>
<td>1937</td>
<td>1963</td>
<td>1967</td>
</tr>
<tr>
<td>House size</td>
<td>House size</td>
<td>House size</td>
<td>House size</td>
</tr>
<tr>
<td>119 m²</td>
<td>86 m²</td>
<td>140 m²</td>
<td>137 m²</td>
</tr>
<tr>
<td>Occupants</td>
<td>Occupants</td>
<td>Occupants</td>
<td>Occupants</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>
Holiday period

House: 2, Occupants: 2

DTU-Tsinghua Workshop,
Beijing, June 2017
Robust Polynomial Kernel

To improve the kernel method

Rewrite the kernel smoother to a Least Square Problem

\[
\arg \min_{\theta} \frac{1}{N} \sum_{s=1}^{N} w_s(x) (Y_s - \theta)^2 \quad w_s(x) = \frac{k\{x - X_s\}}{\frac{1}{N} \sum_{s=1}^{N} k\{x - X_s\}}
\]

Make the method robust by replacing \((Y_s - \theta)^2\) with

\[
\rho_{\text{Huber}}(\varepsilon) = \begin{cases} 
\frac{1}{2\gamma} \varepsilon^2 & \text{if } |\varepsilon| \leq \gamma \\
|\varepsilon| - \frac{1}{2}\gamma & \text{if } |\varepsilon| > \gamma
\end{cases}
\varepsilon_s = Y_s - \theta
\]

Make the method polynomial by replacing \(\theta\) with

\[
P_s = \theta_0 + \theta_1(X_t - x) + \theta_2(X_t - x)^2
\]
Case Study No. 2

Identification of Thermal Performance using Smart Meter Data
Characterization Smart Meter Data

- Energy labelling
- Estimation of UA and gA values
- Estimation of energy signature
- Estimation of dynamic characteristics
- Estimation of time constants
Simple estimation of UA-values

Consider the following model (t=day No.) estimated by kernel-smoothing:

\[ Q_t = Q_0(t) + c_0(t)(T_{i,t} - T_{a,t}) + c_1(t)(T_{i,t-1} - T_{a,t-1}) \]  \hspace{1cm} (1)

The estimated UA-value is

\[ \hat{U}A(t) = \hat{c}_0(t) + \hat{c}_1(t) \]  \hspace{1cm} (2)

With more involved (but similar models) also gA and wA values can be estimated.
Results

<table>
<thead>
<tr>
<th></th>
<th>UA</th>
<th>$\sigma_{UA}$</th>
<th>$gA^\text{max}$</th>
<th>$wA_E^\text{max}$</th>
<th>$wA_S^\text{max}$</th>
<th>$wA_W^\text{max}$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W/°C</td>
<td></td>
<td>W</td>
<td>W/°C</td>
<td>W/°C</td>
<td>W/°C</td>
<td>°C</td>
</tr>
<tr>
<td>4218598</td>
<td>211.8</td>
<td>10.4</td>
<td>597.0</td>
<td>11.0</td>
<td>3.3</td>
<td>8.9</td>
<td>23.6</td>
</tr>
<tr>
<td>4218600</td>
<td>98.7</td>
<td>10.8</td>
<td>-96.2</td>
<td>23.6</td>
<td>10.1</td>
<td>13.0</td>
<td>22.3</td>
</tr>
<tr>
<td>4381449</td>
<td>228.2</td>
<td>12.6</td>
<td>1012.3</td>
<td>29.8</td>
<td>42.8</td>
<td>39.7</td>
<td>19.4</td>
</tr>
<tr>
<td>4711160</td>
<td>155.4</td>
<td>6.3</td>
<td>518.8</td>
<td>14.5</td>
<td>4.4</td>
<td>9.1</td>
<td>22.5</td>
</tr>
<tr>
<td>4711176</td>
<td>178.5</td>
<td>7.3</td>
<td>800.0</td>
<td>1.9</td>
<td>-7.6</td>
<td>8.5</td>
<td>26.4</td>
</tr>
<tr>
<td>4836681</td>
<td>155.3</td>
<td>8.1</td>
<td>591.0</td>
<td>39.5</td>
<td>28.0</td>
<td>21.4</td>
<td>23.5</td>
</tr>
<tr>
<td>4836722</td>
<td>236.0</td>
<td>17.7</td>
<td>1578.3</td>
<td>4.3</td>
<td>3.3</td>
<td>18.9</td>
<td>23.5</td>
</tr>
<tr>
<td>4986050</td>
<td>159.6</td>
<td>10.7</td>
<td>715.7</td>
<td>10.2</td>
<td>7.5</td>
<td>7.2</td>
<td>20.8</td>
</tr>
<tr>
<td>5069878</td>
<td>144.8</td>
<td>10.4</td>
<td>87.6</td>
<td>3.7</td>
<td>1.6</td>
<td>17.3</td>
<td>21.8</td>
</tr>
<tr>
<td>5069913</td>
<td>207.8</td>
<td>9.0</td>
<td>962.5</td>
<td>3.7</td>
<td>8.6</td>
<td>10.6</td>
<td>22.6</td>
</tr>
<tr>
<td>5107720</td>
<td>189.4</td>
<td>15.4</td>
<td>657.7</td>
<td>41.4</td>
<td>29.4</td>
<td>16.5</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Notice: Still some issues with negative values but often they are not significant.
Perspectives for using Smart Meters

- Reliable Energy Signature.
- Energy Labelling
- Time Constants (eg for night setback)

Proposals for Energy Savings:
- Replace the windows?
- Put more insulation on the roof?
- Is the house too untight?
- ......

- Optimized Control

- Integration of Solar and Wind Power using DSM
Part 2
Several sensors

- Introduction to Grey-Box Modelling (a continuous-discrete state space models)
- A model for the thermal characteristics of a small office building
- Models for control
Introduction to Grey-Box modelling
The grey box model

Notation:

\( X_t \): State variables
\( u_t \): Input variables
\( \theta \): Parameters
\( Y_k \): Output variables
\( t \): Time
\( \omega_t \): Standard Wiener process
\( e_k \): White noise process with \( N(0, S) \)
Grey-box modelling concept

- Combines prior physical knowledge with information in data
- Equations and parameters are physically interpretable
Forecasting and Simulation

Grey-Box models are well suited for ...

- One-step forecasts
- K-step forecasts
- Simulations
- Control
- … of both observed and hidden states.

It provides a framework for pinpointing model deficiencies – like:

- Time-tracking of unexplained variations in e.g. parameters
- Missing (differential) equations
- Missing functional relations
- Lack of proper description of the uncertainty
Case study

Model for the thermal characteristics of a small office building
**Test case: One floored 120 m² building**

**Objective**

Find the best model describing the heat dynamics of this building ([1], [4])

DTU-Tsinghua Workshop, 
Beijing, June 2017
**Data**

**Measurements of:**

- $y_t$ Indoor air temperature
- $T_a$ Ambient temperature
- $\Phi_h$ Heat input
- $\Phi_s$ Global irradiance
**Selection Procedure**

**Iterative procedure using statistical tests**

1. Begin with the simplest model
2. Model fitting
3. Likelihood-ratio tests of extended models
4. Evaluate the selected model
5. End selection

**Simplest model**

- **Diagram**:
  - Interior
  - Heater
  - Solar
  - Envelope
  - Ambient
  - $T_i$, $T_h$, $T_a$
  - $C_i$, $R_h$

**First extension: heater part**

- **Diagram**:
  - Interior
  - Heater
  - Solar
  - Envelope
  - Ambient
  - $T_i$, $T_h$, $T_a$
  - $C_i$, $R_h$

**Start**

<table>
<thead>
<tr>
<th>Model</th>
<th>$l(\theta; \mathcal{Y}_N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Model}_{T_1}$</td>
<td>2482.6</td>
</tr>
<tr>
<td>$\text{Model}_{TiTe}$</td>
<td>3628.0</td>
</tr>
<tr>
<td>$\text{Model}_{TiTm}$</td>
<td>3639.4</td>
</tr>
<tr>
<td>$\text{Model}_{TiTs}$</td>
<td>3884.4</td>
</tr>
<tr>
<td>$\text{Model}_{TiTh}$</td>
<td>3911.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
Evaluate the simplest model

Inputs and residuals

ACF of residuals

Cumulated periodogram

Beijing, June 2017
**Grey-box modelling**

Continuous time models (*grey-box: stochastic state-space model*)

\[
\begin{align*}
\text{States} &= \text{Fun}_1(\text{States, Inputs}) + \text{Fun}_2(\text{Inputs}) \cdot \text{SystemError} \\
\text{Measurements} &= \text{Fun}_3(\text{States, Inputs}) + \text{Fun}_4(\text{Inputs}) \cdot \text{MeasurementError}
\end{align*}
\]

- **Used for buildings** (single- and multi-zone), **walls, systems** (hot water tank, integrated PV, heat pumps, heat exchanger, solar collectors, ...)

- **Formulate the model based on physical knowledge**

- **Maximum likelihood estimation**
  (we have the entire statistical framework available)

- **Description of the system noise is part of the model provides some very useful possibilities**
  (e.g. control the weight of data in the estimation depending on input signals)

- **Software, for example our R package CTSM-R**

  \(^1\)http://ctsm.info
Part 3
Special Data (eg Non-Gaussian)

Identification of Occupant Behavior

- Use of CO2 measurements to model occupant behavior in summer houses
Summer houses represent a special challenge

- Large variation in the number of people present in the house
- Power Grids in summer house areas represent a special problem for some DSOs
- Time series of CO2 measurements are the key to the classification
The Model Space

\[ \theta \sim f(\beta_{\text{fixed}}, t, \cdots) + g(U_{\text{random}}, t, \cdots) \]  
\[ dX_t \sim \text{Dynamical model} (\theta) \]  
\[ Y_t^{(1)} = \text{Electrical consumption} \]  
\[ Y_t^{(2)} = \text{Noise (indoor)} \]  
\[ Y_t^{(3)} = \text{CO}_2 \ (\text{indoor}) \]  
\[ \vdots \]  

- \( \theta \) parameter vector for population/hierarchical model
  - Time, weather, demographics
- \( dX_t \) state vector described by some dynamical model depending on \( \theta \)
  - People, consumption, windows
- \( Y \)’s: Observed measurements related to occupancy behavior, including measurements inside and outside the building and smart metering data
Hidden Markov Model
First Order Markov Property

\[ p(X_t|X_{t-1}) = p(X_t|X^{(t-1)}), \quad t \in \mathbb{N} \]  \hspace{1cm} (2)

\[ p(Y_t|X_t) = p(Y_t|X^{(t)}, Y^{(t-1)}), \quad t \in \mathbb{N} \]  \hspace{1cm} (3)

Figure: Directed graph of basic HMM. The index denotes time.
Markov Chains

Discrete state vector at time $t$, $X_t$, with $m$ states.

Transition probability

$$p(X_t = j | X_{t-s} = i)$$  \hspace{1cm} (4)

One-step transition probability

$$\gamma_{ij,t} = p(X_t = j | X_{t-1} = i)$$  \hspace{1cm} (5)

One-step transition probability matrix from time $t-1$ to $t$

$$\Pi_t = \begin{pmatrix} \gamma_{11,t} & \cdots & \gamma_{1m,t} \\ \vdots & \ddots & \vdots \\ \gamma_{m1,t} & \cdots & \gamma_{mm,t} \end{pmatrix}$$  \hspace{1cm} (6)

where the row must sum to 1.
Homogeneous Hidden Markov Model

Setting

\[ y_t = h(CO_2, t) \]
\[ p(x_t | x_{t-1}) \sim \Gamma \]
\[ p(y_t | x_t) \sim \mathcal{N}(\mu_i, \sigma_i^2) \text{ for } i = 1, 2, \ldots, m \]

Note that there is no time dependence in the transition probabilities in the homogen case.
Figure 8.7: Global Decoding of the HMM (log $CO_2$) with 5 states.
Inhomogeneous Hidden Markov Model

Setting

\[ y_t = h(CO_2, t) \]
\[ p(x_t | x_{t-1}) \sim \Gamma_t \]
\[ p(y_t | x_t) \sim N(\mu_i, \sigma^2_i) \text{ for } i = 1, 2, \ldots, m \]

Note that there is time dependence in the transition probabilities in the inhomogenous case.
Inhomogeneous Markov-switching with auto-dependent observations

![Directed graph of Markov switching AR(1)](image)

Figure 8.10: Directed graph of Markov switching AR(1).
Inhomogen Markov-switching AR(1)

Setting

\[ y_t = h(CO_{2,t}) \]

\[ p(x_t|x_{t-1}) \sim \Gamma_t \]

\[ p(y_t|x_t,y_{t-1}) \sim \mathcal{N}(c_i + \phi_i y_{t-1}, \sigma_i^2) \] for \( i = 1, 2, \cdots, m \)

Note that there is time dependence in the transition probabilities in the inhomogen case.
Interpretation of the states

- State 1: Absence or sleeping
- State 2: Long term absence
- State 3: Outdoor interaction
- State 4: Presence (high activity)
- State 5: Presence (long term, low activity)
Figure 8.11: Model diagnostics of the final model.
Figure 8.16: Transition probabilities over the day of the final model. The lower right plot is the stationary distribution.
Some conclusions:

That the low activity state 5 is not very likely from 10 am to 11 pm. The highest activity is seen in the late afternoon.

Figure 8.17: Profile of the states over the course of the day. I.e. Stacked stationary probabilities over the course of the day of the final model.
Some references


Some references (cont.)


Thanks ...

- For more information
  - www.ctsm.info
  - www.henrikmadsen.org
  - www.smart-cities-centre.org

- ...or contact
  - Henrik Madsen (DTU Compute)
    hmad@dtu.dk

Acknowledgement CITIES (DSF 1305-00027B)