Grey-Box Modeling;  
An approach to combined physical and statistical model building

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Traditional Dynamical Model

- Ordinary Differential Equation:

\[ dA = -KA dt \]
\[ Y = A + \epsilon \]
Stochastic Dynamical Model

Stochastic Differential Equation:

\[ dA = -KA dt + \sigma dw \]

\[ Y = A + \epsilon \]
The grey box model

Notation:

- $X_t$: State variables
- $u_t$: Input variables
- $\theta$: Parameters
- $Y_k$: Output variables
- $t$: Time
- $\omega_t$: Standard Wiener process
- $e_k$: White noise process with $N(0, S)$
Grey-box modelling concept

- Combines prior physical knowledge with information in data
- Equations and parameters are physically interpretable
Forecasting and Simulation

Grey-Box models are well suited for ...

- One-step forecasts
- K-step forecasts
- Simulations
- Control
- ... of both observed and hidden states.

- It provides a framework for pinpointing model deficiencies – like:
  - Time-tracking of unexplained variations in e.g. parameters
  - Missing (differential) equations
  - Missing functional relations
  - Lack of proper description of the uncertainty
Grey-box model building

- Non-parametric modelling
- Statistical tests
- Falsification or unfalsification
- Stochastic state-space model
- Model (re)formulation
- Parameter estimation
- Residual analysis
Grey-Box Modelling

- Bridge the gap between physical and statistical modelling
- Provides methods for model identification
- Provides methods for model validation
- Provides methods for pinpointing model deficiencies
- Enables methods for a reliable description of the uncertainties, which implies that the same model can be used for k-step forecasting, simulation and control
Energy System Models for real time applications

Grey-box models are simplified models for the individual components facilitating system integration
Energy Systems Integration

Calls for grey box models of electrical, thermal, and fuel pathways and components at all scales.

Energy Systems Integration Continuum

Scale

Appliance (Plug)

Building DistGen Vehicle (meter)

Campus Subdivision (feeder)

Community (substation)

Area (Service Territory)

Region (Balancing Area)

Nation

Complexity

Energy Informatics: Linking Data to Knowledge, Forecasting and Control

Electricity

Fuel

Data

Thermal
Flexhouse at SYSLAB (DTU Risø)
Data and the first principal component
A first order model often used for simulation.
Model evaluation of the first order model

- Model is not adequate since residuals are not white noise
Final model found using Grey-box techniques
Model evaluation - Extended model

- This model is OK, since residuals are uncorrelated (white noise)
Grundfos Case Study

Schematic of the heating system
Modeling Heat Pump and Solar Collector

Simplified System
Advanced Controller
Economic Model Predictive Control

Formulation

The Economic MPC problem, with the constraints and the model, can be summarized into the following formal formulation:

\[
\begin{align*}
\min_{\{u_k\}_{k=0}^{N-1}} & \quad \phi = \sum_{k=0}^{N-1} c' u_k \\
\text{Subject to} & \quad x_{k+1} = Ax_k + Bu_k + Ed_k \quad k = 0, 1, \ldots, N - 1 \\
& \quad y_k = Cx_k \quad k = 1, 2, \ldots, N \\
& \quad u_{\min} \leq u_k \leq u_{\max} \quad k = 0, 1, \ldots, N - 1 \\
& \quad \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \quad k = 0, 1, \ldots, N - 1 \\
& \quad y_{\min} \leq y_k \leq y_{\max} \quad k = 0, 1, \ldots, N 
\end{align*}
\]
EMPC for heat pump with solar collector (savings 35 pct)
The software ...

- CTSM-R – *Continuous Time Stochastic Modelling in R*
- Download from http://ctsm.info
- User Guide and Math Guide available
- For more information. Email to info@ctsm.info

- Our intention is to provide relevant model examples and guidelines for at the homepage
Thanks for your time ...
Some 'randomly picked' books on modeling ....

2008

2011