Grey-Box Modeling; An approach to combined physical and statistical model building

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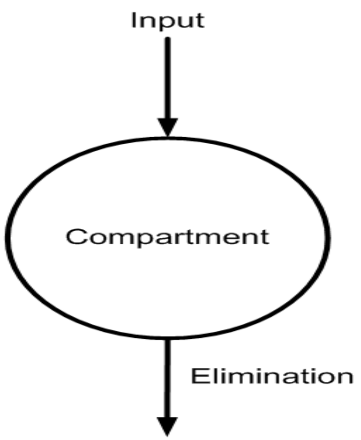
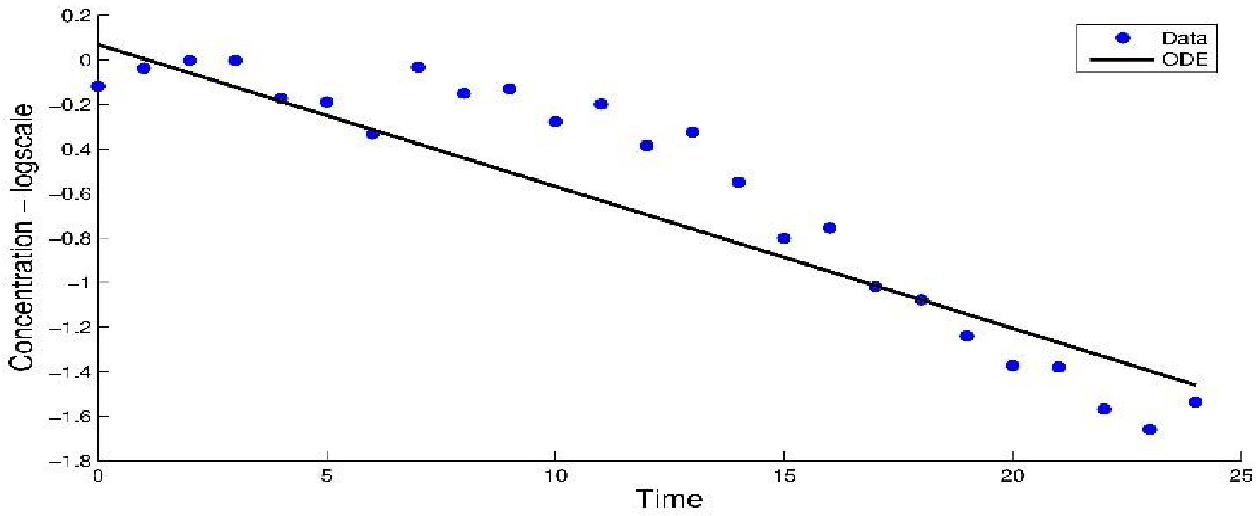
Traditional Dynamical Model

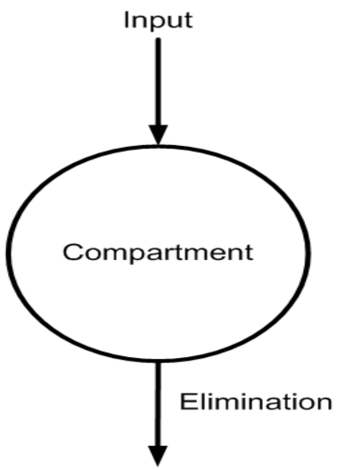


- Ordinary Differential Equation:

$$\begin{aligned} dA &= -KAdt \\ Y &= A + \epsilon \end{aligned}$$

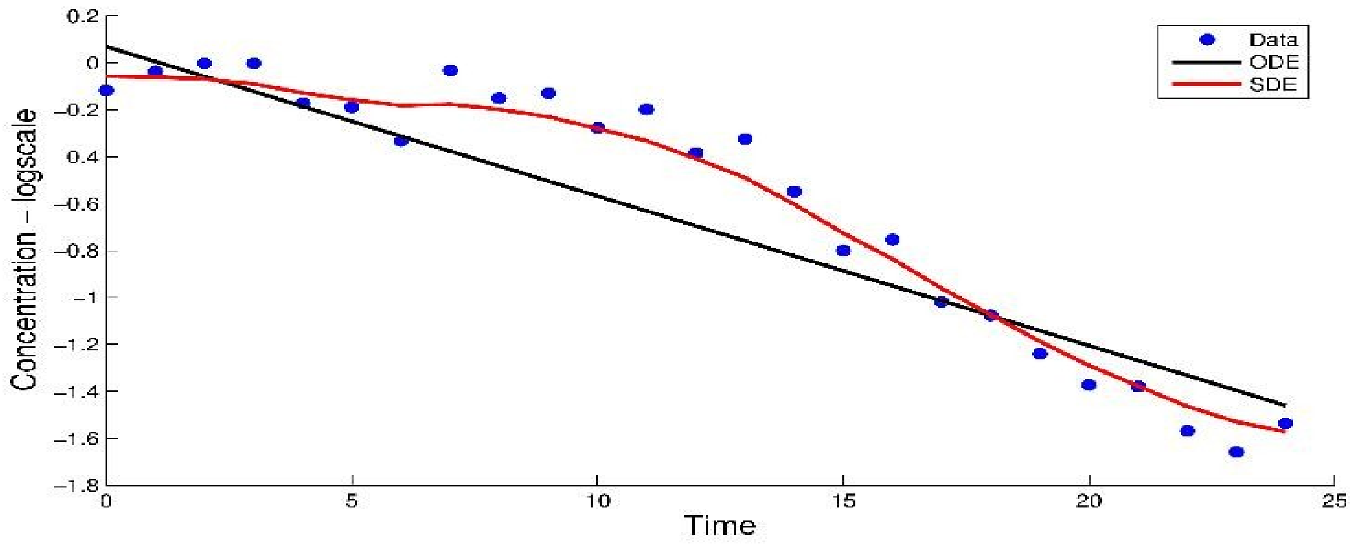


Stochastic Dynamical Model



- Stochastic Differential Equation:

$$\begin{aligned} dA &= -KAdt + \sigma dw \\ Y &= A + e \end{aligned}$$



The grey box model

Drift term

Diffusion term

$$dX_t = f(X_t, u_t, t, \theta)dt + \sigma(X_t, u_t, t, \theta)d\omega_t$$

System equation

$$Y_k = h(X_k, u_k, t_k, \theta) + e_k,$$

Observation equation

Observation noise

Notation:

X_t: State variables

u_t: Input variables

θ: Parameters

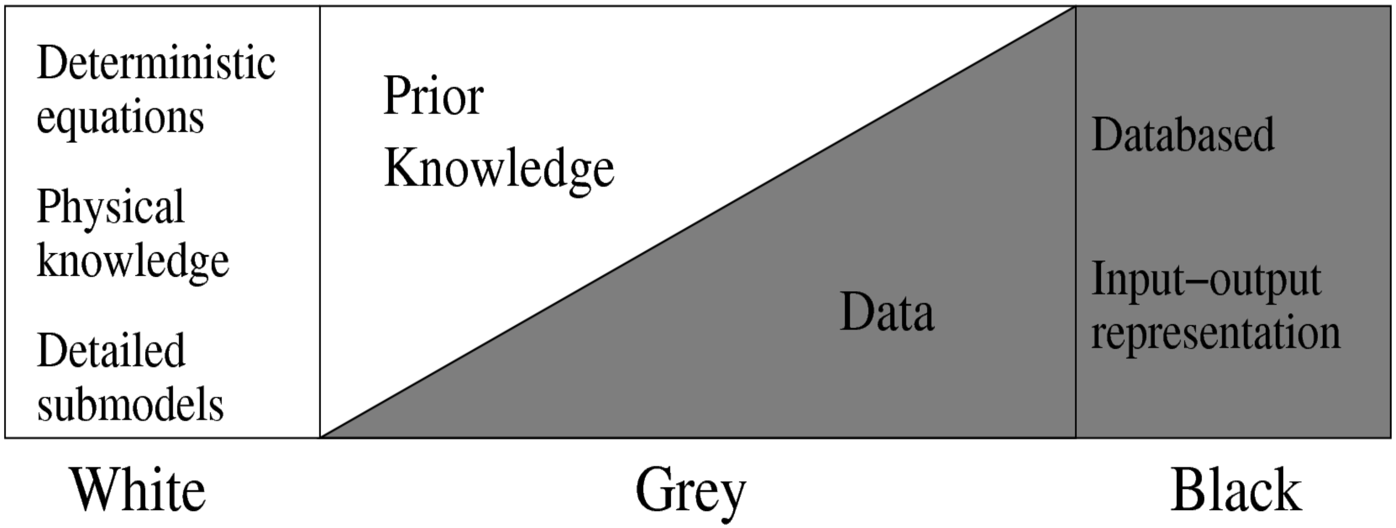
Y_k: Output variables

t: Time

ω_t: Standard Wiener process

e_k: White noise process with $N(0, S)$

Grey-box modelling concept



- Combines prior physical knowledge with information in data
- Equations and parameters are physically interpretable

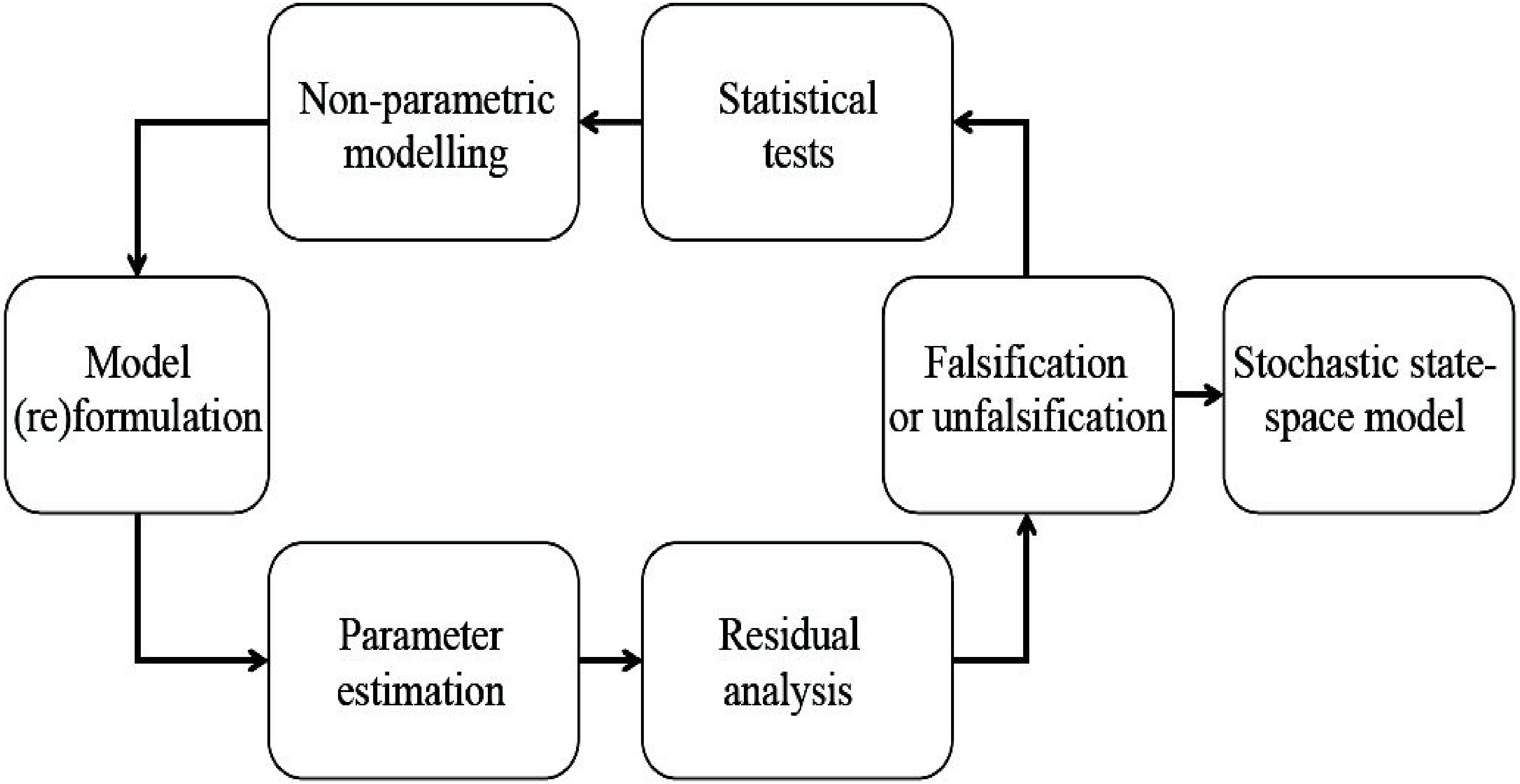
Forecasting and Simulation

Grey-Box models are well suited for ...

- One-step forecasts
- K-step forecasts
- Simulations
- Control
- … of both *observed* and *hidden* states.

- **It provides a framework for pinpointing model deficiencies – like:**
  - Time-tracking of unexplained variations in e.g. parameters
  - Missing (differential) equations
  - Missing functional relations
  - Lack of proper description of the uncertainty

Grey-box model building

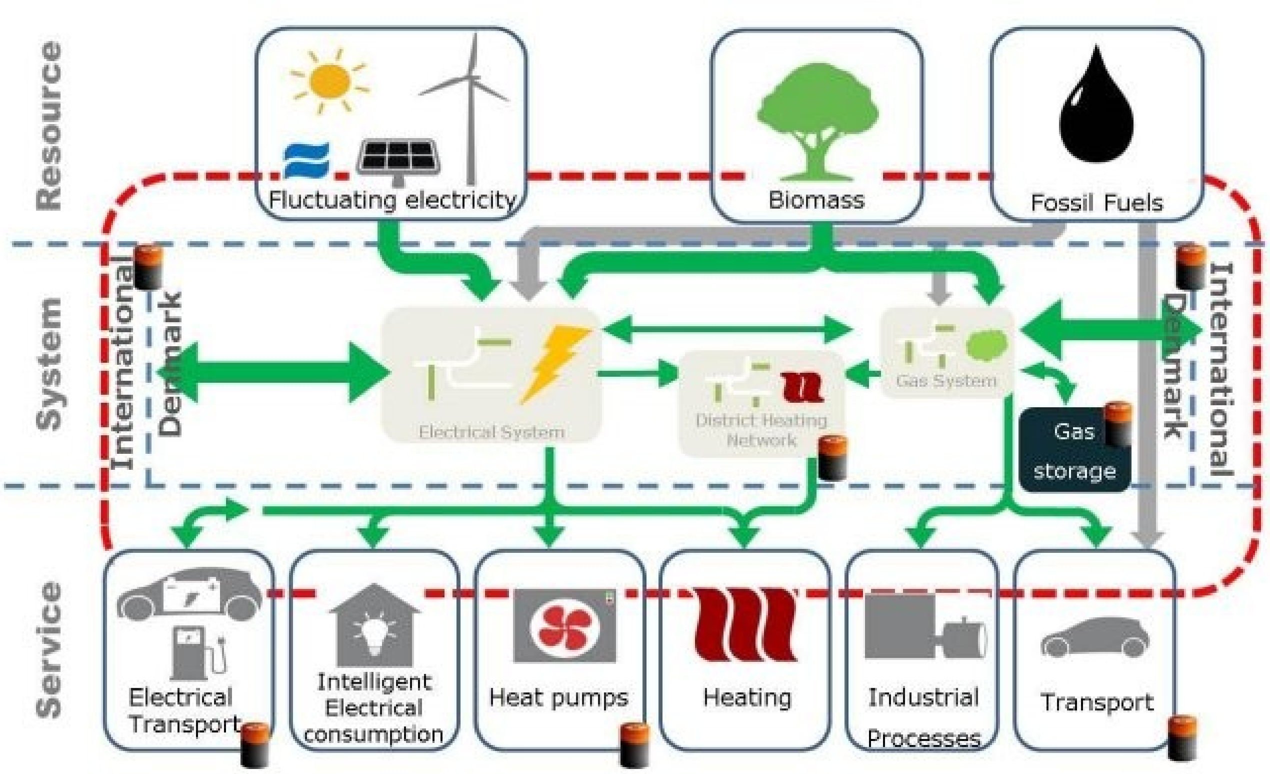


Grey-Box Modelling

- Bridge the gap between physical and statistical modelling
- Provides methods for model identification
- Provides methods for model validation
- Provides methods for pinpointing model deficiencies
- Enables methods for a reliable description of the uncertainties, which implies that the same model can be used for k-step forecasting, simulation and control

Energy System Models for real time applications

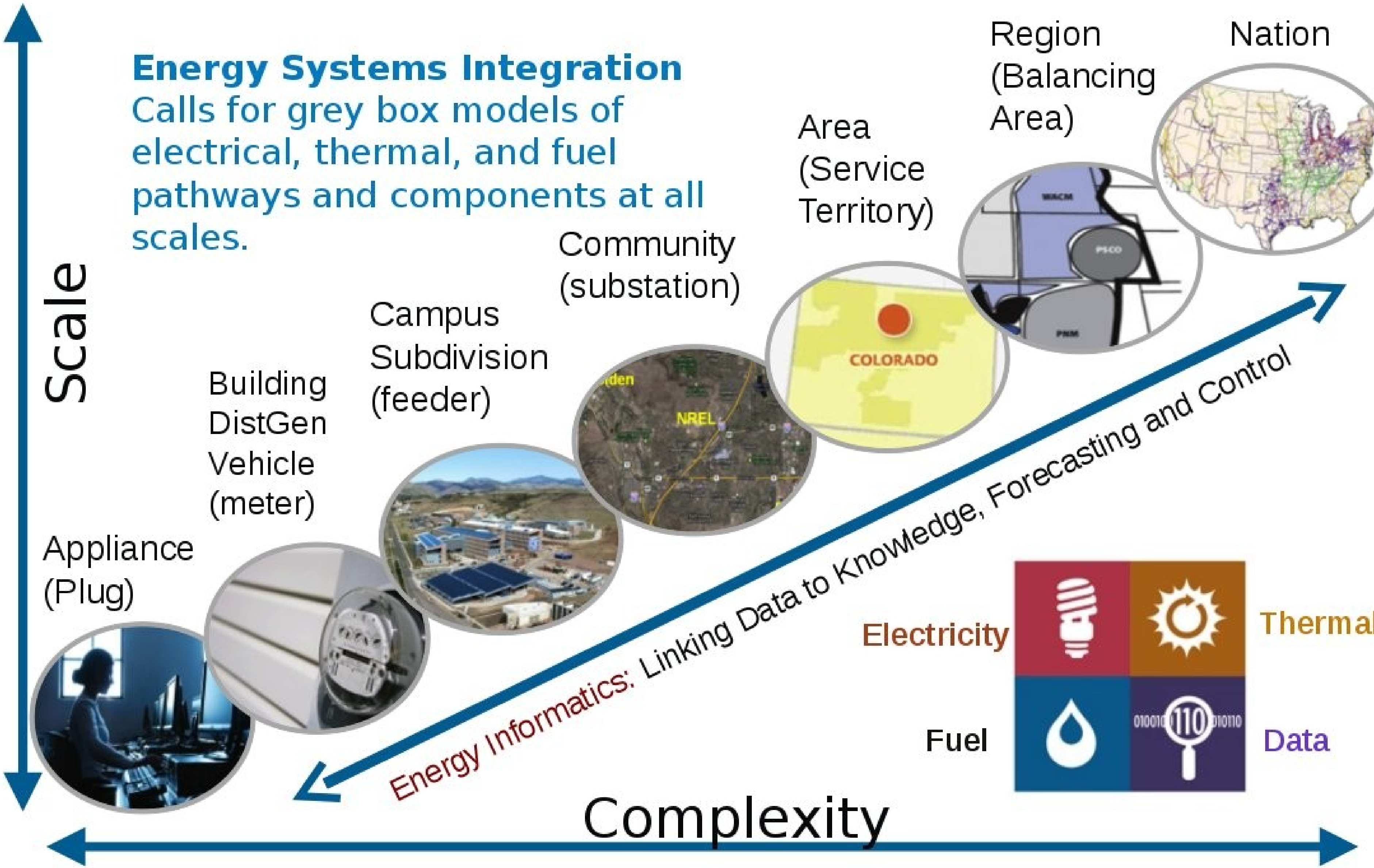
- **Grey-box models** are simplified models for the individual components facilitating system integration



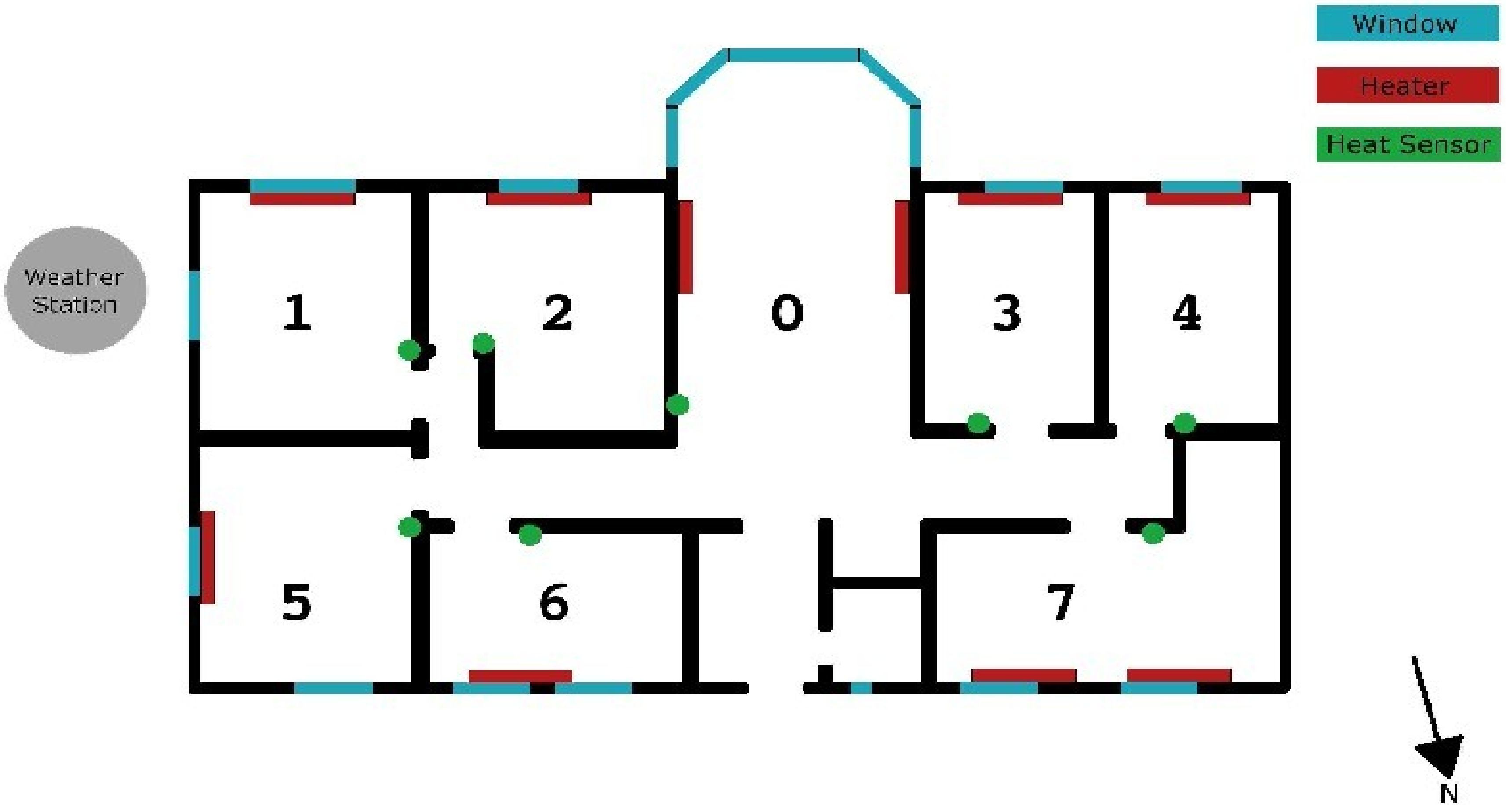
Energy Systems Integration Continuum

(From Ben Kroposki, NREL)

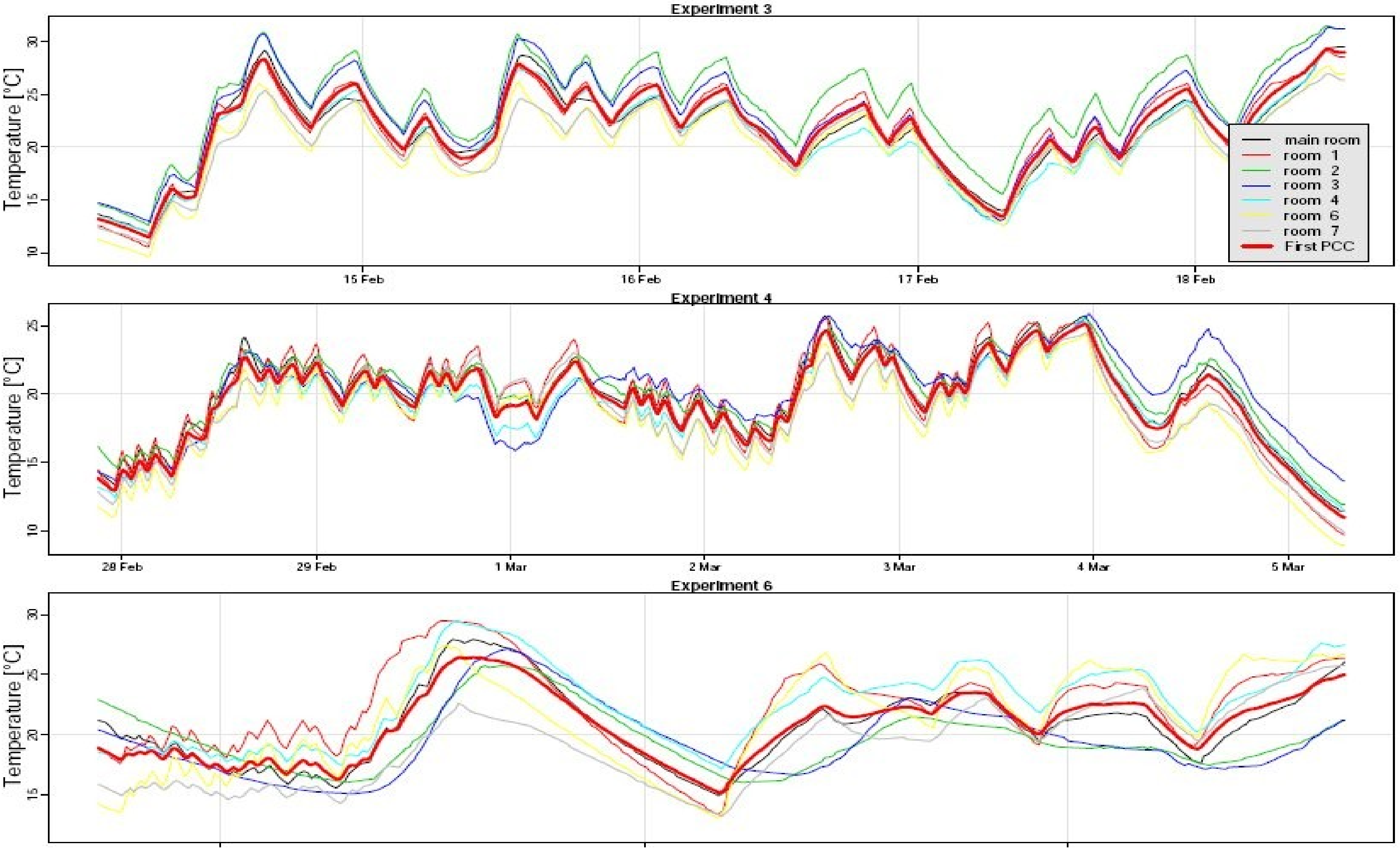
Energy Systems Integration
Calls for grey box models of electrical, thermal, and fuel pathways and components at all scales.



Flexhouse at SYSLAB (DTU Risø)



Data and the first principal component



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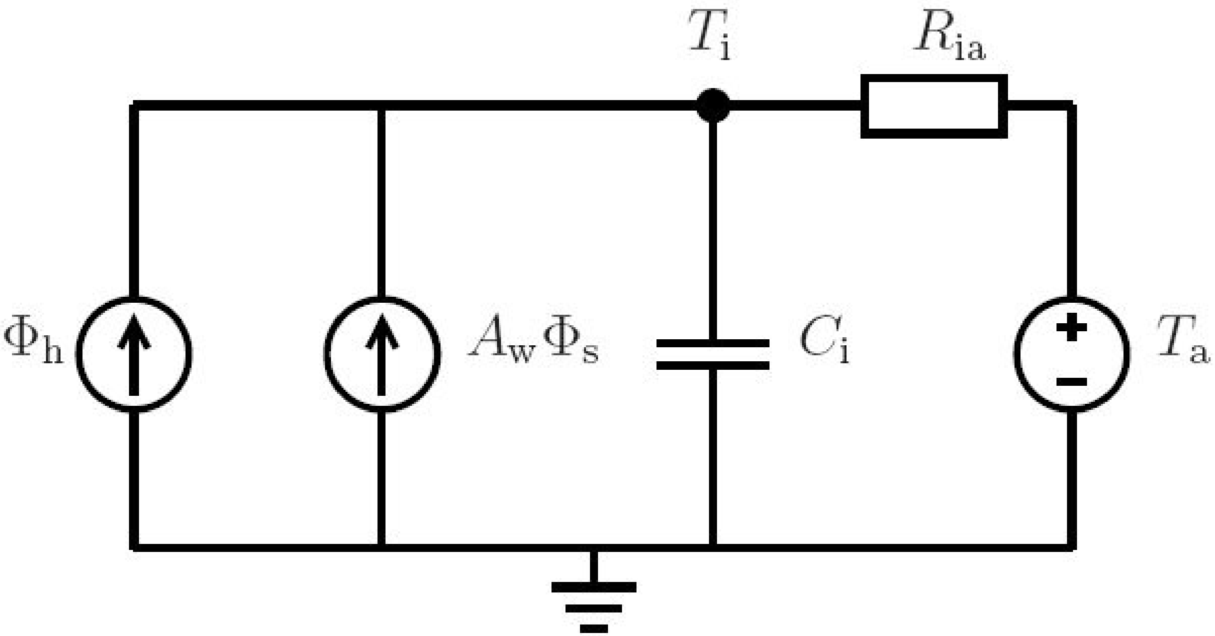
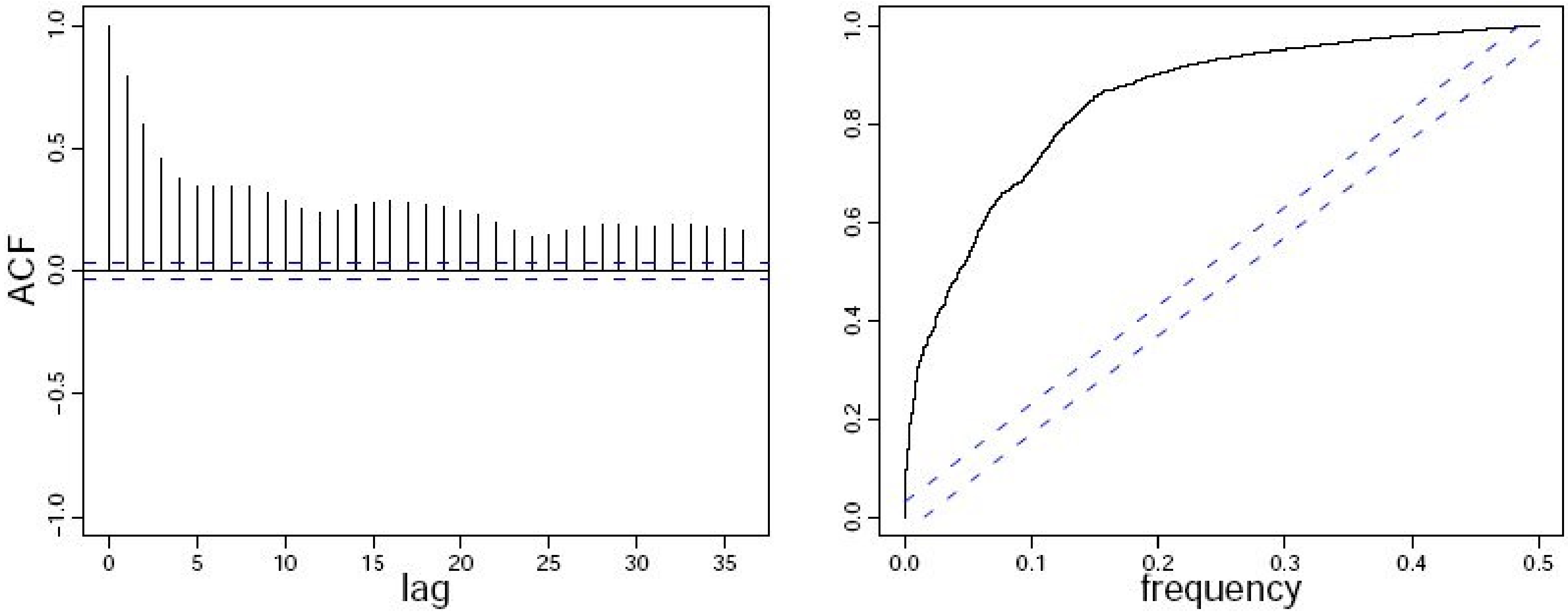
A first order model often used for simulation

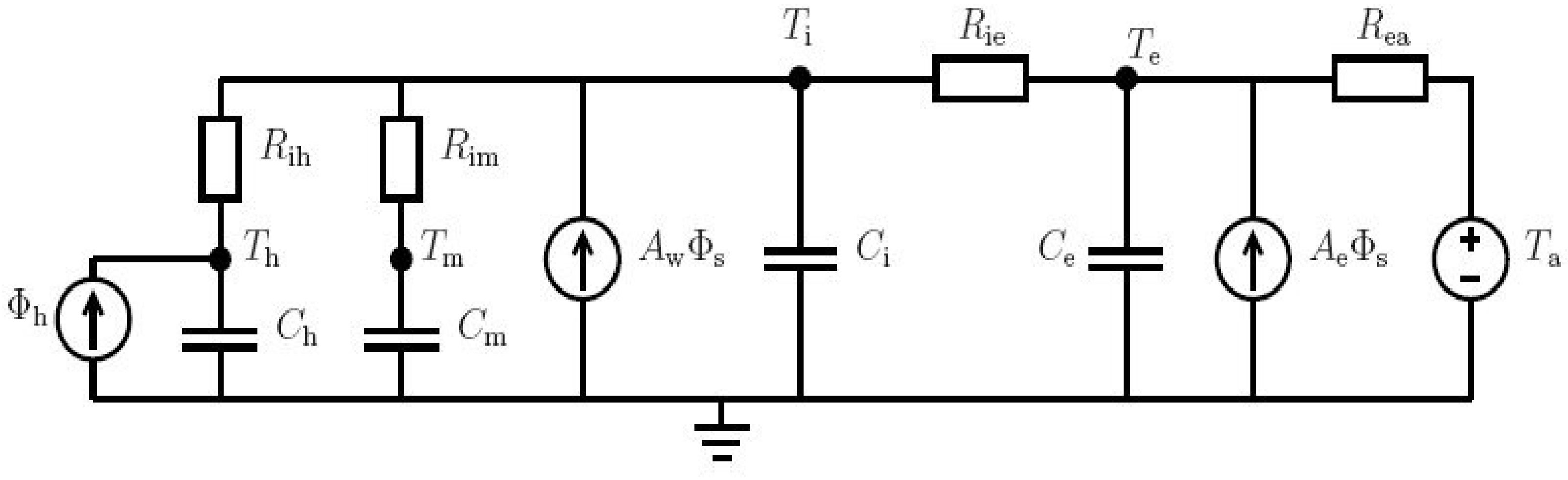


Model evaluation of the first order model

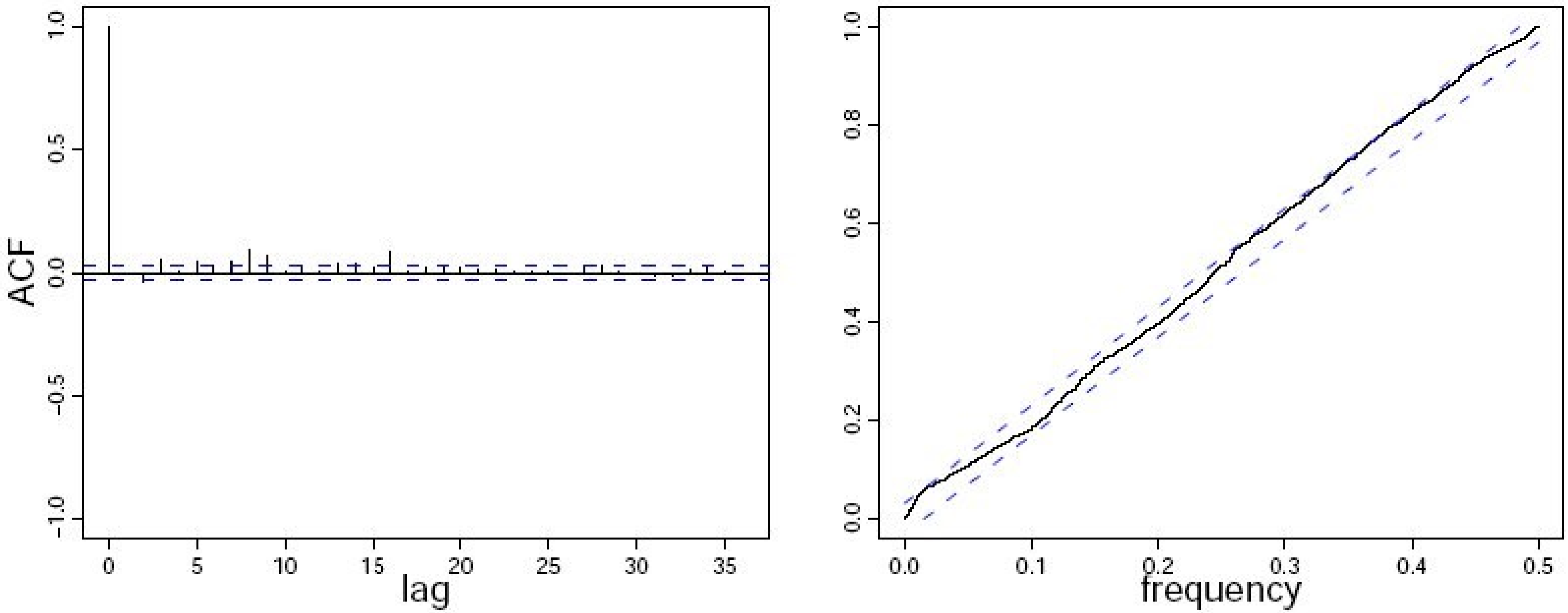


- Model is not adequate since residuals are not white noise

Final model found using Grey-box techniques



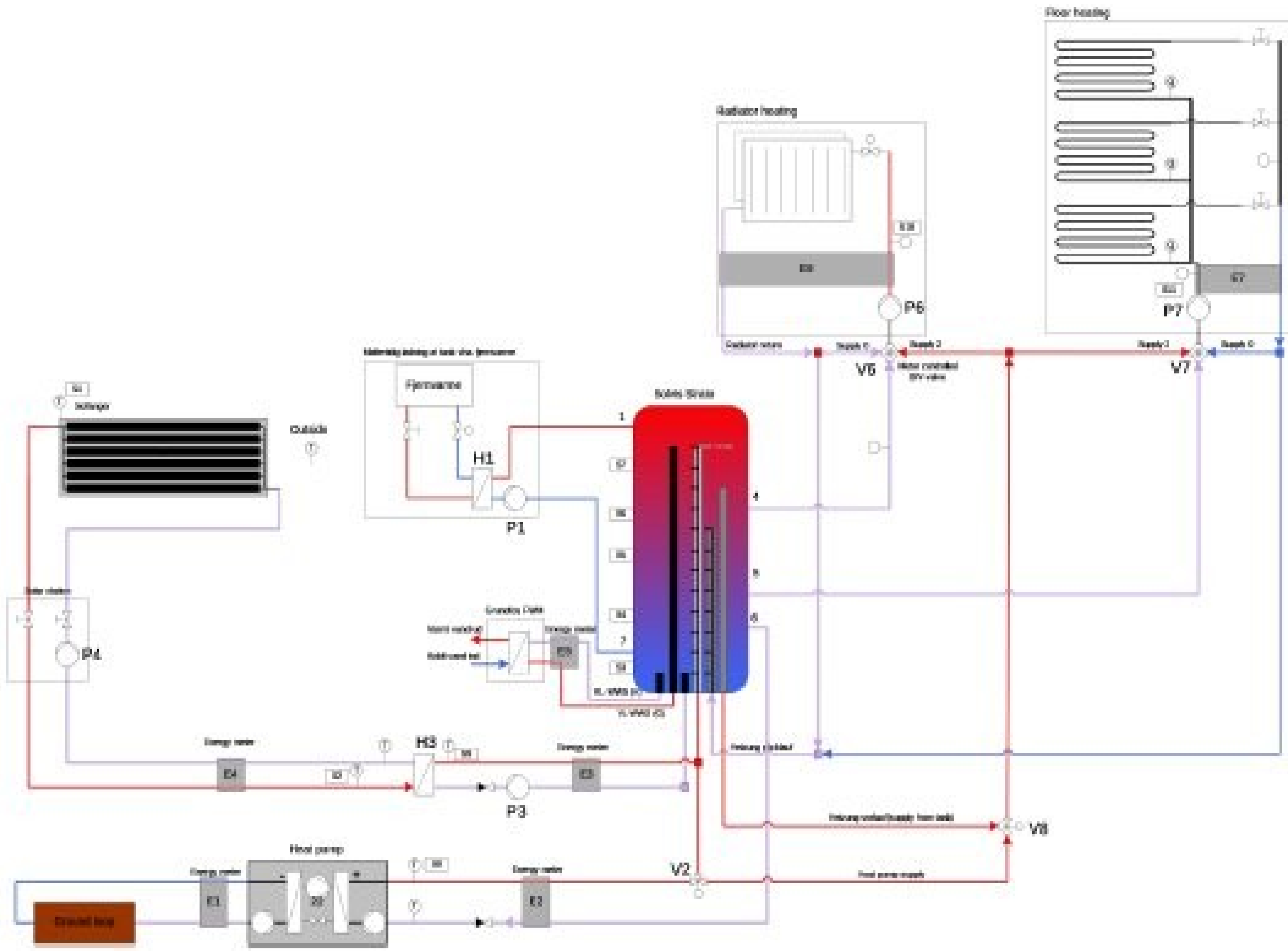
Model evaluation – Extended model



- This model is OK, since residuals are uncorrelated (white noise)

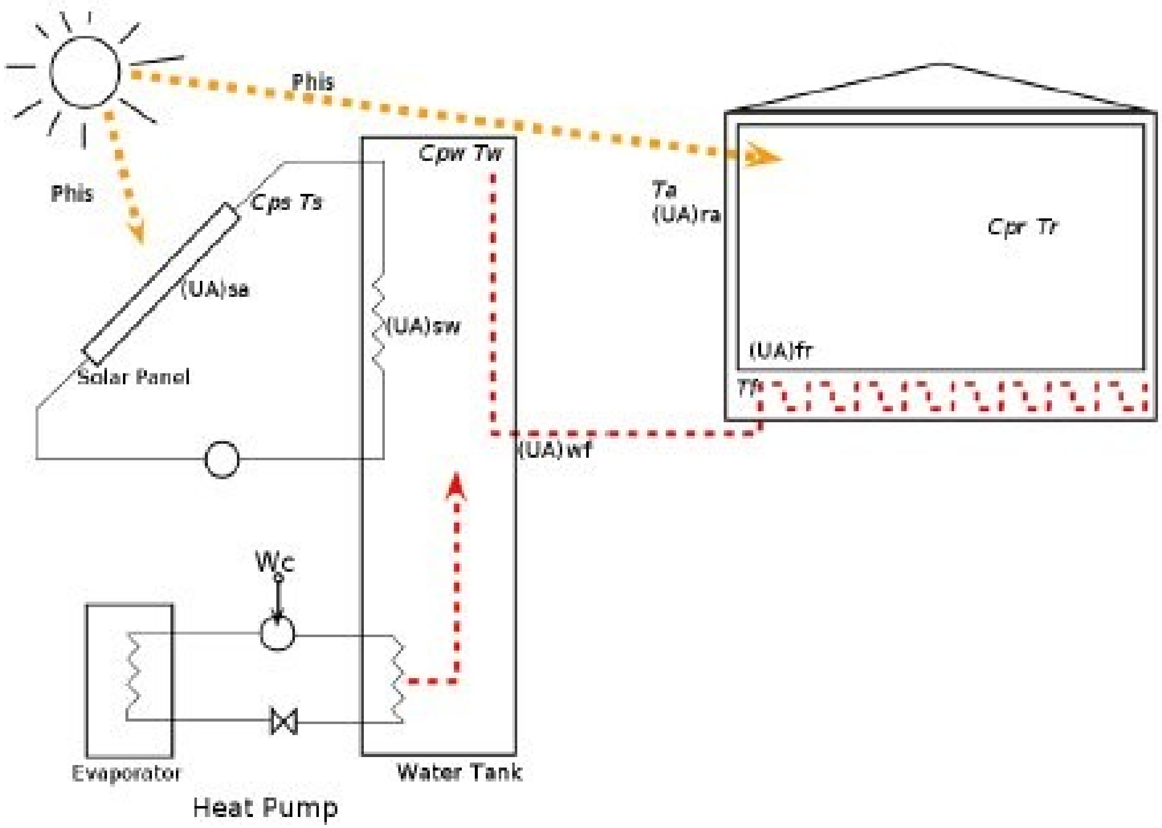
Grundfos Case Study

Schematic of the heating system



Modeling Heat Pump and Solar Collector

Simplified System



Avanced Controller

Economic Model Predictive Control

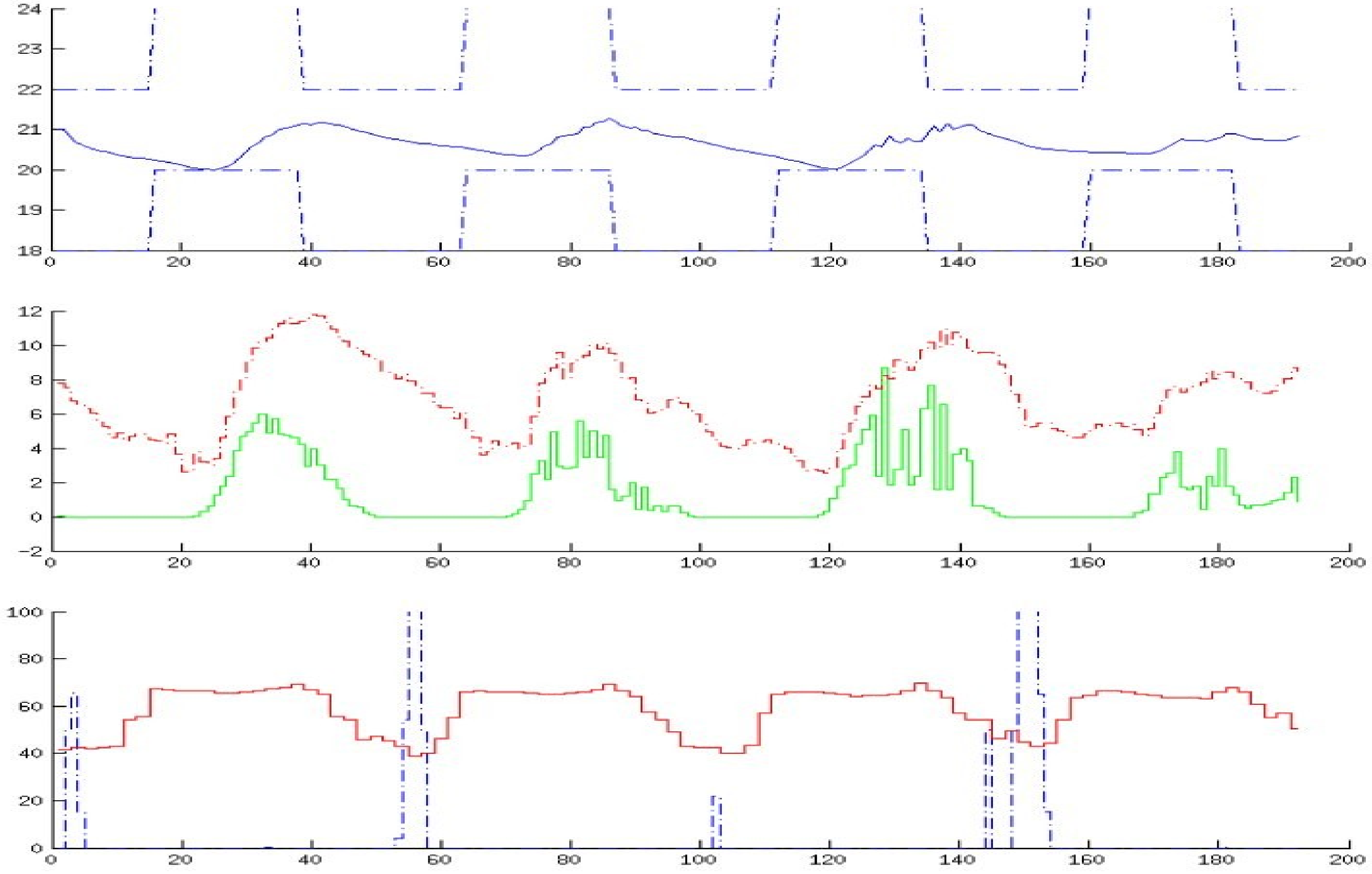
Formulation

The Economic MPC problem, with the constraints and the model, can be summarized into the following formal formulation:

$$\min_{\{u_k\}_{k=0}^{N-1}} \phi = \sum_{k=0}^{N-1} c' u_k \tag{4a}$$

$$\begin{aligned}
\text{Subject to} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k & k = 0, 1, \ldots, N-1 \quad & (4b) \\
& y_k = Cx_k & k = 1, 2, \ldots, N \quad & (4c) \\
& u_{min} \leq u_k \leq u_{max} & k = 0, 1, \ldots, N-1 \quad & (4d) \\
& \Delta u_{min} \leq \Delta u_k \leq \Delta u_{max} & k = 0, 1, \ldots, N-1 \quad & (4e) \\
& y_{min} \leq y_k \leq y_{max} & k = 0, 1, \ldots, N \quad & (4f)
\end{aligned}$$

EMPC for heat pump with solar collector (savings 35 pct)



The software ...

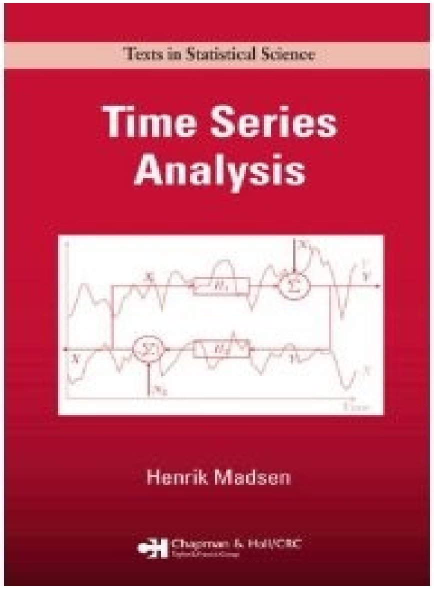
- CTSM-R – *Continuous Time Stochastic Modelling in R*
- Download from http://ctsm.info
- **User Guide** and **Math Guide** available
- For more information. Email to

  info@ctsm.info

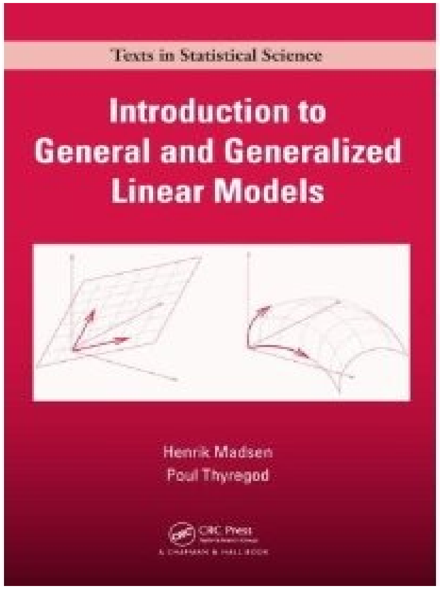
- Our intention is to provide relevant model examples and guidelines for at the homepage

Thanks for your time ...

Some 'randomly picked' books on modeling ....



2008



2011