Robust optimization for contingency-constrained models in power system operation and planning

José M. Arroyo*

E-mail: JoseManuel.Arroyo@uclm.es
Departamento de Ingeniería Eléctrica, Electrónica, Automática y Comunicaciones
Universidad de Castilla – La Mancha

*In collaboration with Alexandre Street and Alexandre Moreira, Pontifical Catholic University of Rio de Janeiro, Brazil

Contents

- Introduction
- Energy and reserve scheduling
- Transmission network expansion planning
- Numerical results
- Conclusions

Security in operation and planning

- Capability to withstand a set of credible contingencies
- Industry practice $\Rightarrow$ Deterministic criteria such as n – 1 and n – 2
- Recent major blackouts worldwide question the suitability of standard security criteria
- Need for tighter criteria $\Rightarrow$ n – K, n – K^G – K^L

Contingency-constrained models

- Consideration of a pre-contingency state (normal state) and a set of credible contingencies (security criterion) $\Rightarrow$ Replication of operational constraints
- Problem dimension depends on the number of contingencies $\Rightarrow$ Reduction of the contingency set for tractability purposes
- This is a critical issue if tighter security criteria (n – K) are imposed!!

Contingency-constrained models
Conventional approach

$Minimize\ c(x)$

$subject\ to$:

$g(x) \leq 0$

$g_k(x, x_k) \leq 0 \quad \forall k \in \mathcal{C}$

Contingency-constrained models Conventional approach

- Each contingency k represents a combination of unavailable assets

- Asset availability is characterized by binary parameters A_i^k (generator outages) and A_l^k (line outages)

  - 1 for available asset
  - 0 for unavailable asset

Contingency-constrained models Conventional approach

- Contingency set $\mathcal{C}$ determined by the security criterion being adopted

$$\boldsymbol{f}\left(\left\{A_i^k\right\}_{i\in I},\left\{A_l^k\right\}_{l\in\mathcal{L}}\right) \geq \mathbf{0};\ \forall\ k \in \mathcal{C}$$

Robust optimization

- Optimization under a pre-specified uncertainty set
- Uncertain variables are allowed to vary within certain limits
- Uncertainty budget $\Rightarrow$ Limit on the conservativeness of the solution
- Worst-case optimization

Robust optimization for contingency-constrained models

- A_i^k and A_l^k represent "uncertainty" and are respectively replaced by 0/1 decision variables a_i^G and a_l^L $\Rightarrow$ Polyhedral uncertainty set

- Uncertainty budget $\Rightarrow$ Security criterion:

$$\boldsymbol{f}\left(\{a_i^G\}_{i\in I}, \{a_l^L\}_{l\in\mathcal{L}}\right) \geq \mathbf{0}$$

- Non-dependent on index k!!

Robust optimization for contingency-constrained models

- Contingency constraints are replaced by an optimization problem to characterize the worst case
- Worst case $\Rightarrow$ Maximum damage (system power imbalance) associated with <u>ALL</u> contingencies implicitly modeled by a_i^G and a_l^L
- Robust counterpart $\Rightarrow$ Multilevel optimization (bilevel programming, trilevel programming)

Robust optimization for contingency-constrained models

- Solution approaches:
  - Single-level equivalent based on duality theory
  - Benders decomposition
  - Combination of both

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Problem definition

- Minimize the system cost (energy and reserves)
- Determination of a pre-contingency schedule (on/off statuses of generating units, generation levels, and reserve contributions)
- Feasible redispatch for all contingencies under the security criterion (no power balance violations) $\Rightarrow$ Operational contraints under all contingency states

Operational constraints

- Generation limits
- Reserve limits
- Network-related constraints $\Rightarrow$ dc load flow

$$f_l^k = \frac{A_l^k}{x_l}(\theta_{fr(l)}^k - \theta_{to(l)}^k)$$

$$-\bar{F}_l \leq f_l^k \leq \bar{F}_l$$

- Mixed-integer linear programming

Conventional contingency-dependent model

$$\text{Minimize } c(x)$$

System cost (energy and reserves)

$$\text{subject to}:$$

$$g(x) \leq 0$$

Pre-contingency constraints

$$g_k(x, x_k) \leq 0 \quad \forall k \in \mathcal{C}$$

Post-contingency constraints

- Large-scale mixed-integer program

Equivalent penalized model

- Nodal power imbalance is allowed $\Rightarrow$ Slack variables

- Penalty term in the objective function associated with the worst-case system power imbalance

- Equivalence guaranteed by a sufficiently large penalty coefficient

Equivalent penalized model

$$Minimize\ c(x) + C^I max_{k \in \mathcal{C}}(e^T \Delta P_k)$$

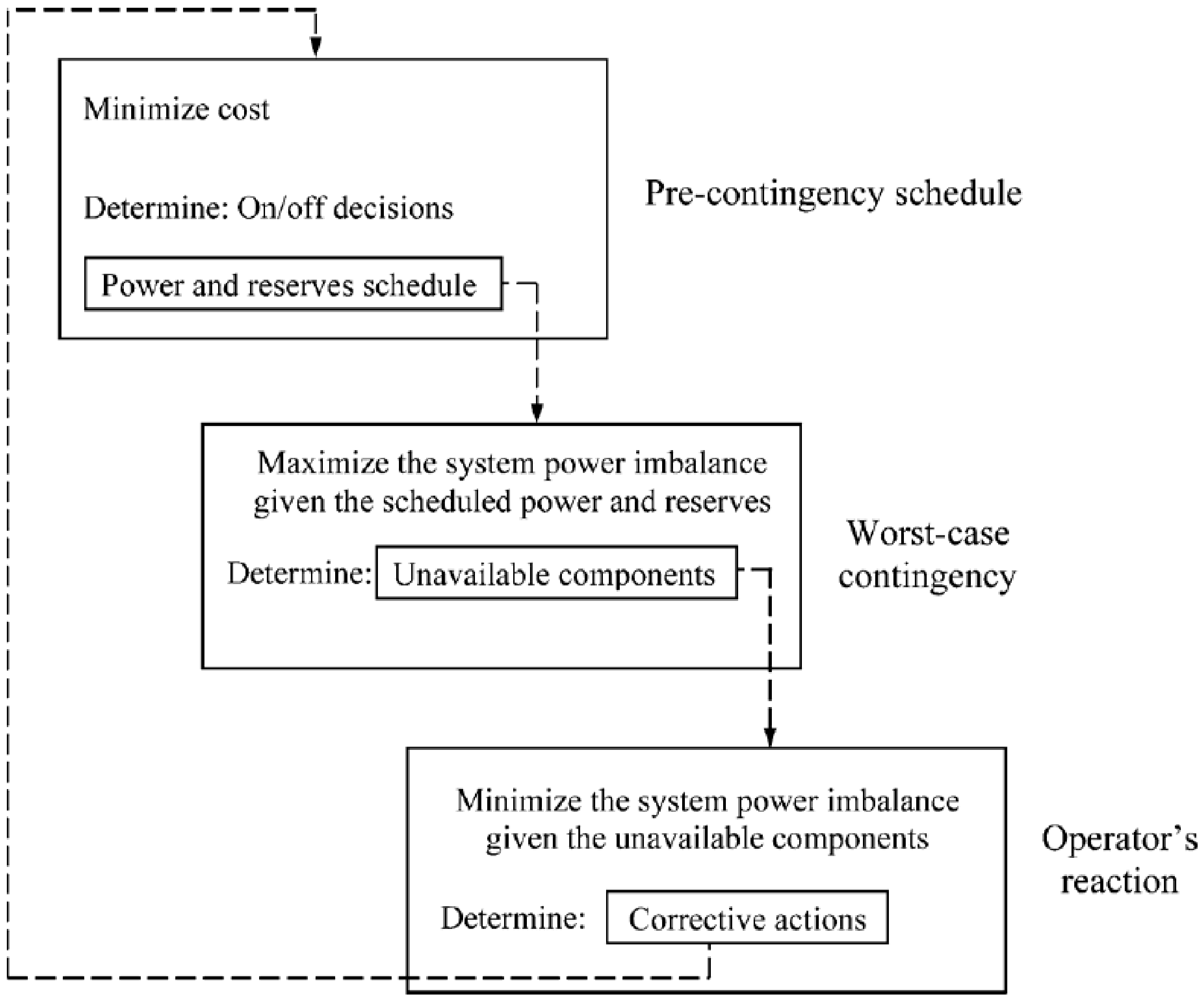
$$subject\ to:$$

$$g(x) \leq 0$$

$$g_k(x, x_k, \Delta P_k) \leq 0 \quad \forall k \in \mathcal{C}$$

- Also a large-scale mixed-integer program
- Useful to understand how the trilevel robust counterpart is devised

Trilevel robust counterpart



Trilevel robust counterpart

$$Minimize_x \; c(x) + C^I \Delta^{wc}(x)$$

$subject\ to$:

$$g(x) \leq 0$$

$$\Delta^{wc}(x) = max_{a \in \{0,1\}} \; \delta(x, a)$$

$subject\ to$:

$$f(a) \geq 0$$

$$\delta(x, a) = min_{x^{wc},\, \Delta P^{wc}} \; (e^T \Delta P^{wc})$$

$subject\ to$:

$$g^{wc}(x, a, x^{wc},\ \Delta P^{wc}) \leq 0$$

Trilevel robust counterpart

- Two lowermost optimization problems replace post-contingency constraints $\Rightarrow$ Mixed-integer trilevel program

- Penalty term in the original objective function is optimized in the two lowermost problems

- Two lowermost optimization problems $\Rightarrow$ Max-min optimization with a linear lower-level problem

Solution approach

- Two-step procedure:
  - Conversion of the trilevel program into an equivalent bilevel program
  - Application of Benders decomposition
- Guaranteed convergence to the optimal solution

Bilevel equivalent

- Two lowermost optimization levels (max-min problem) are transformed into an equivalent single-level optimization

- Dual of the lower-level problem renders the original max-min a max-max $\Rightarrow$ Maximization problem (strong duality theorem)

Dual of the lower-level problem

$$\delta(x,a) = max_{\pi}\, h^{dual}(\pi,x,a)$$

$$subject\ to:$$

$$g^{dual}(\pi,a) \leq 0$$

- At the optimum (strong duality theorem):

$$(e^{T}\Delta P^{wc}) = h^{dual}(\pi,x,a)$$

Equivalent of the two lowermost optimization levels

$$\Delta^{wc}(x) = max_{a \in \{0,1\}, \pi}\, h^{dual}(\pi, x, a)$$

subject to:

$$f(a) \geq 0$$

$$g^{dual}(\pi, a) \leq 0$$

- Bilinear terms in $h^{dual}(\pi, x, a)$ and $g^{dual}(\pi, a)$ $\Rightarrow$ Well-known linearization scheme

Resulting bilevel equivalent without bilinear terms

$$Minimize_x \; c(x) + C^I \Delta^{wc}(x)$$

$subject\ to$:

$$g(x) \leq 0$$

$$\Delta^{wc}(x) = max_{a\in\{0,1\},\pi,x^{aux}} \; h^{dual\_li}(\pi, x, x^{aux})$$

$$subject\ to:$$

$$f(a) \geq 0$$

$$g^{dual\_li}(\pi, a, x^{aux}) \leq 0$$

Nice features of the bilevel equivalent

- Lower-level problem $\Rightarrow$ Mixed-integer linear program parameterized in upper-level variables

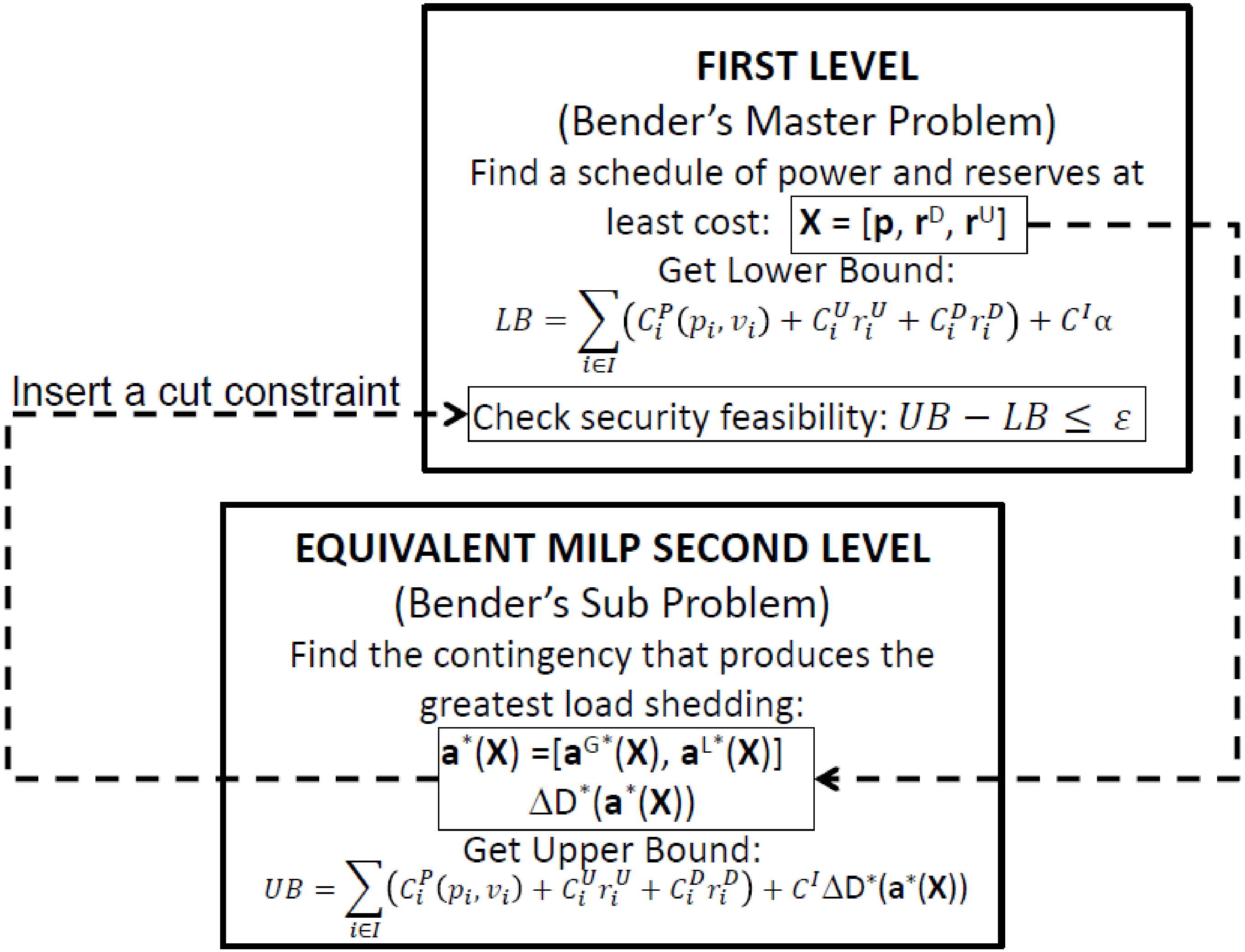
- Upper-level decision variables x only appear in linear terms in the lower-level objective function $\Rightarrow$ $\Delta^{wc}(x)$ is a convex function of upper-level variables

Nice features of the bilevel equivalent

- Suitable for Benders decomposition $\Rightarrow$ Worst-case power imbalance = Recourse function

- Partial subgradients of the recourse function straightforwardly available $\Rightarrow$ Approximation of the recourse function via cutting planes (Benders cuts)

Benders decomposition



Accelerating Benders decomposition

- Additional constraints in the master problem to narrow the search space without removing the optimal solution:

  - Generation outage constraints $\Rightarrow$ Sufficient levels of reserves at initial iterations

  - Redispatch constraints $\Rightarrow$ Previous infeasible schedules are cut off

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Problem definition

- Optimal location and sizing of candidate transmission assets
- Single stage $\Rightarrow$ Static model
- Joint generation and transmission security criterion for both existing and candidate assets
- Power imbalance below a pre-specified limit

Problem definition

- Structurally identical to the energy and reserve scheduling problem with penalty term

- Pre-contingency variables include on/off expansion decisions as well as generation and network-related variables (continuous)

- Post-contingency variables include generation and network-related variables

Problem definition
Distinctive features

- No unit commitment (planning problem)
- No scheduling of reserves (planning problem)
- Bounding constraint for the worst-case system power imbalance
- Products of binary pre-contingency variables (expansion decisions) and post-contingency variables

Problem definition
Distinctive features

- dc load flow model:

$$f_l^k = \frac{v_l A_l^k}{x_l}(\theta_{fr(l)}^k - \theta_{to(l)}^k)$$

$$-\bar{F}_l \leq f_l^k \leq \bar{F}_l$$

- Nonconvexity of the recourse function

Problem definition
dc model with disjunctive constraints

- dc load flow model:

$$f_l^k - \frac{1}{x_l}(\theta_{fr(l)}^k - \theta_{to(l)}^k) \leq M_l(1 - v_l A_l^k)$$

$$f_l^k - \frac{1}{x_l}\left(\theta_{fr(l)}^k - \theta_{to(l)}^k\right) \geq -M_l(1 - v_l A_l^k)$$

$$-v_l A_l^k \overline{F}_l \leq f_l^k \leq v_l A_l^k \overline{F}_l$$

Conventional problem formulation

- Contingency-dependent model:

  Minimize system cost (investment and production) + worst-case imbalance cost

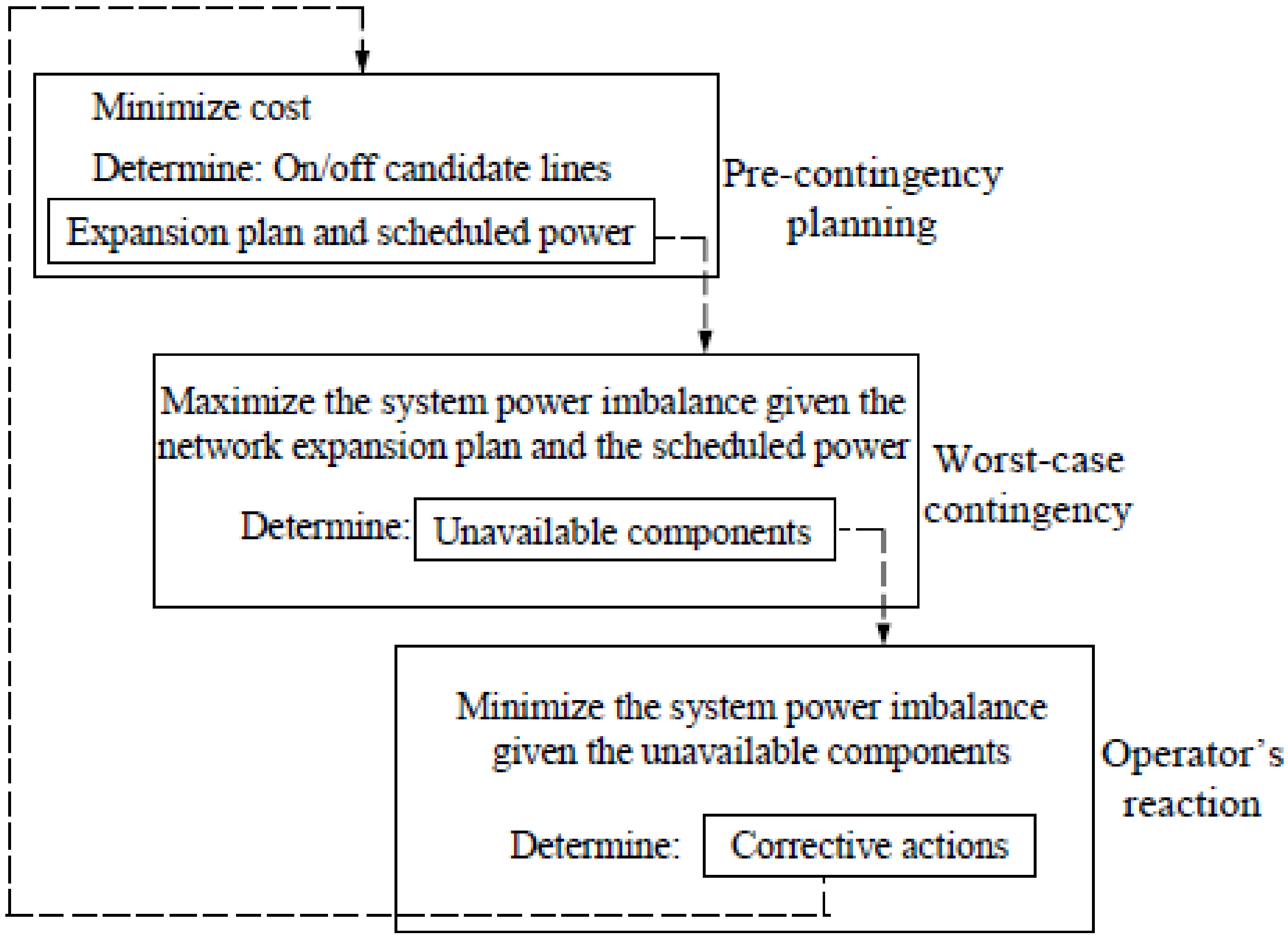
  Subject to:

  Pre-contingency constraints

  Post-contingency constraints

- Large-scale mixed-integer program

Trilevel robust counterpart



Solution approach

- Identical two-step procedure:
  - Conversion of the trilevel program into an equivalent bilevel program
  - Application of Benders decomposition
- Guaranteed convergence to the optimal solution

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Numerical results

- Energy and reserve scheduling:
  - n – K and n – K^G – K^L security criteria
  - IEEE RTS, IEEE 118-bus
- Transmission network expansion planning:
  - n – K security criterion
  - IEEE 118-bus, IEEE 300-bus
- Optimality gaps well below 1%

Scheduling. IEEE RTS case

K	Conventional		Robust	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	2068020	0.30	2068020	0.34
1	2688760	3.60	2688760	0.75
2	3751680	334.78	3751680	1.79
3	-	14400.00	5232740	16.77
4	Out of memory		Infeasible	1.31
5	Out of memory		Infeasible	2.45

Scheduling. IEEE 118-bus case

RESULTS FOR THE IEEE 118-BUS SYTEM

		T-BP		CD	
K^G	K^L	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	0	12826.6	0.17	12826.6	0.81
0	1	12826.6	2.31	12826.6	115.06
0	2	Infeasible	6.55	–	14400.00
1	0	15450.7	0.47	15450.7	15.77
1	1	15643.4	50.91	–	14400.00
1	2	Infeasible	14.13	–	Out of memory
2	0	17507.4	0.61	17507.4	8070.47
2	1	17641.6	276.66	–	Out of memory
2	2	Infeasible	8.42	–	Out of memory
3	0	18503.7	1.05	–	14400.00
3	1	18503.7	38.60	–	Out of memory
3	2	Infeasible	15.37	–	Out of memory
4	0	19356.5	1.22	–	Out of memory
4	1	19881.2	453.62	–	Out of memory
4	2	Infeasible	43.04	–	Out of memory
5	0	20343.5	2.17	–	Out of memory
5	1	21520.5	7897.73	–	Out of memory
5	2	Infeasible	96.47	–	Out of memory

Planning. IEEE 118-bus case

IEEE 118-Bus System—System Costs and Computing Times

K	PDBD		CD	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	1.08E+08	0.22	1.08E+08	0.52
1	1.10E+08	8.56	1.10E+08	291.54
2	4.18E+08	2261.99	4.66E+08*	269087.00*
3	1.60E+11	26.76	Out of Memory	Out of Memory
4	2.40E+11	46.00	Out of Memory	Out of Memory
5	2.40E+11	716.97	Out of Memory	Out of Memory

*Unfinished (Optimality gap = 32.34%).

Planning. IEEE 118-bus case

IEEE 118-Bus System—Levels of System Power Imbalance and Sizes of Expansion Plans for PDBD

K	$\overline{\Delta}$ (%)	System Power Imbalance (%)	Number of Circuits Built
0	0	0.00	2
1	0	0.00	3
2	0	0.00	33
3	5	3.10	3
4	10	4.65	3
5	15	4.65	8

Planning. IEEE 300-bus case

IEEE 300-Bus System—System Costs and Computing Times

K	PDBD		CD	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	2.06E+09	0.78	2.06E+09	1.01
1	2.08E+09	108.50	2.08E+09	30145.80
2	3.26E+09	7222.27	Out of Memory	Out of Memory
3	6.18E+11	1125.91	Out of Memory	Out of Memory
4	1.64E+12	188.49	Out of Memory	Out of Memory
5	2.28E+12	66.61	Out of Memory	Out of Memory

Planning. IEEE 300-bus case

IEEE 300-Bus System—Levels of System Power Imbalance and Sizes of Expansion Plans for PDBD

K	$\overline{\Delta}$ (%)	System Power Imbalance (%)	Number of Circuits Built
0	0	0.00	2
1	0	0.00	10
2	0	0.00	46
3	5	1.26	13
4	10	3.35	6
5	15	4.65	4

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Conclusions

- Robust optimization is suitable for deterministic contingency-constrained operational and planning models
- Implicit consideration of contingencies yields significant computational advantages over contingency-dependent models
- Robust optimization paves the way for the exploration of tighter security criteria

Further research

- Incorporation of uncertainty sources (demand, renewable power generation) $\Rightarrow$ Impact of correlation
- Consideration of more sophisticated operational models (ac load flow, line switching)
- Analysis of alternative solution approaches to avoid the dual-based transformation

References

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Thanks for your attention!

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