Model Predictive Control for smart energy systems

MPC for a portfolio of heat pumps - new results

Hjørdis Amanda Schlüter, Jozsef Gaspar, John Bagterp Jørgensen

CITIES MPC Workshop,

Technical University of Denmark,

Kgs. Lyngby, May 18, 2018

DTU Compute
Department of Applied Mathematics and Computer Science

Introduction

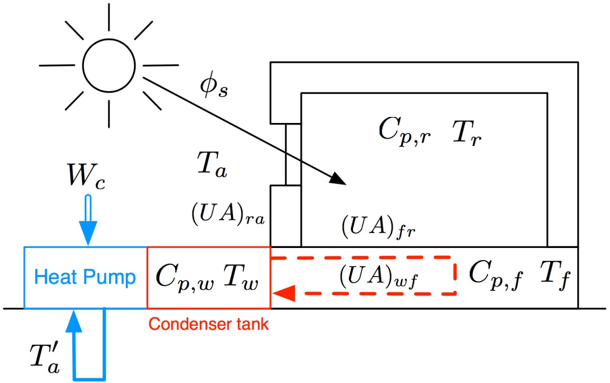
Heat pumps

- Efficient for heating (and cooling) buildings
- Use electricity efficiently
- Ideal for an energy system with significant stochastic energy sources such as wind and solar energy

Model Predictive Control (MPC)

- MPC has been suggested to control heat pumps.
- One can use direct control or control using prices. We investigate price based control.
- Use the thermal inertia (mass) of buildings to store heat when the electricity prices are low

The building and the heat pump



Building model

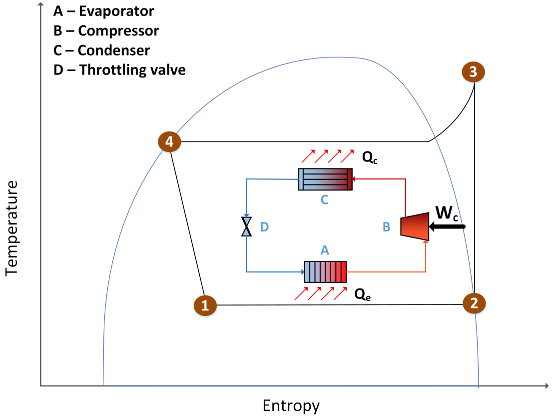
Energy balances

$$
\begin{aligned}
C_{p,r}\dot{T}_r &= Q_{fr} - Q_{ra} + (1-p)\phi_s \\
C_{p,f}\dot{T}_f &= Q_{wf} - Q_{fr} + p\phi_s \\
C_{p,w}\dot{T}_w &= Q_c - Q_{wf}
\end{aligned}
$$

Conductive heat transfer rates

$$
\begin{aligned}
Q_{ra} &= (UA)_{ra}(T_r - T_a) \\
Q_{fr} &= (UA)_{fr}(T_f - T_r) \\
Q_{wf} &= (UA)_{wf}(T_w - T_f)
\end{aligned}
$$

The heat pump - vapor compression cycle (VCC)



$$COP = \frac{h_3(T_3, P_4) - h_4(T_w + \Delta T, P_4)}{h_3(T_3, P_4) - h_2(T_{gr} - \Delta T, P_2)}$$

$$Q_c = \eta_e \cdot COP \cdot W_c$$

Nonlinear economic MPC

$$
\begin{aligned}
\min_{\hat{u}} \quad & \phi = \int_{t_0}^{t_f} (C(\hat{x}(t), \hat{u}(t), \hat{d}(t)) + V(\hat{y}(t)))dt, \\
\text{s.t.} \quad & \hat{x}(t_0) = \bar{x}_k, \;\; k \geq 0 \\
& \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + E\hat{d}(t), \;\; t \in [t_0, t_f] \\
& \hat{y}(t) = C\hat{x}(t), \;\; t \in [t_0, t_f] \\
& u_{min}(t) \leq \hat{u}(t) \leq u_{max}(t), \;\; t \in [t_0, t_f] \\
& \Delta u_{min}(t) \leq \Delta\hat{u}(t) \leq \Delta u_{max}(t), \;\; t \in [t_0, t_f] \\
& y_{min}(t) - v(t) \leq \hat{y}(t), \;\; t \in [t_0, t_f] \\
& y_{max}(t) + v(t) \geq \hat{y}(t), \;\; t \in [t_0, t_f] \\
& v(t) \geq 0, \;\; t \in [t_0, t_f]
\end{aligned}
$$

Energy cost:

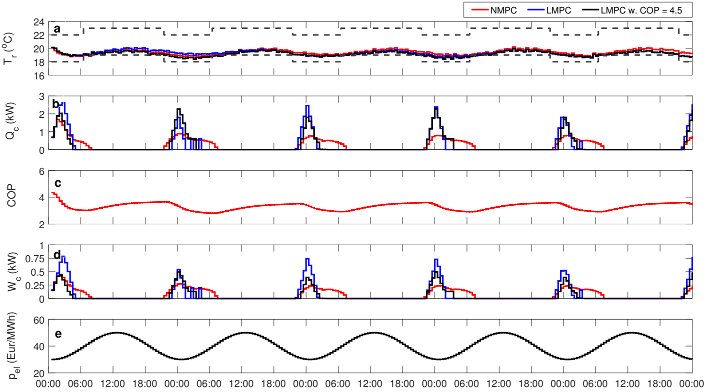
$$C(\hat{x}(t), \hat{u}(t), \hat{d}(t)) = p_{el}(t) \cdot W_c$$

Comfort penalty:

$$V(\hat{y}) = \rho(T_r - T_{min})_{min} + \rho(T_r - T_{max})_{max}$$

Nonlinear economic MPC - results

Gaspar et al (2017):



Control algorithm	W_c (kWh)	C_{total} $(Euro)$	$W_{c,avg}$ $(kWh/month)$	C_{avg} $Euro/month$	TVU (W)	TVT (K)	C_{rel} $(\%)$
Reference MPC	9.49	1.602	56.9	9.62	289.0	-	-
Linear MPC, $COP = 4.5$	8.15	0.945	48.9	5.67	389.2	0.92	41.0
Linear MPC	7.84	0.976	47.0	5.86	455.9	0.95	39.0
Nonlinear MPC	7.60	0.943	45.6	5.66	452.5	0.69	41.2

Heat pump model

Nonlinear model:

$$COP = \frac{h_3(T_3, P_4) - h_4(T_w + \Delta T, P_4)}{h_3(T_3, P_4) - h_2(T_{gr} - \Delta T, P_2)}$$

$$Q_c = \eta_e \cdot COP \cdot W_c$$

Linear model with constant coefficient of performance, COP, and electrical efficiency, η_e:

$$Q_c = \eta W_c$$

where

$$\eta = \eta_e \cdot COP$$

Model

Energy balances

$$C_{p,r}\dot{T}_r = Q_{fr} - Q_{ra} + (1-p)\phi_s$$
$$C_{p,f}\dot{T}_f = Q_{wf} - Q_{fr} + p\phi_s$$
$$C_{p,w}\dot{T}_w = Q_c - Q_{wf}$$

Conductive heat transfer rates

$$Q_{ra} = (UA)_{ra}(T_r - T_a)$$
$$Q_{fr} = (UA)_{fr}(T_f - T_r)$$
$$Q_{wf} = (UA)_{wf}(T_w - T_f)$$

Heat pump

$$Q_c = \eta W_c$$

Linear discrete time state space model

Linear model

$$x_{k+1} = Ax_k + Bu_k + Ed_k$$
$$y_k = Cx_k$$

States (x), manipulated variable (u), disturbances (d), and output (y):

$$x = \begin{bmatrix} T_r \\ T_f \\ T_w \end{bmatrix} \qquad u = W_c \qquad d = \begin{bmatrix} T_a \\ \phi_s \end{bmatrix} \qquad y = T_r$$

Linear program - linear economic MPC

$$
\begin{aligned}
\min \quad & \phi \\
s.t. \quad & x_{k+1} = Ax_k + Bu_k + Ed_k, && k = 0, 1, \dots, N-1, \\
& y_k = Cx_k, && k = 1, \dots, N, \\
& u_{\min} \leq u_k \leq u_{\max}, && k = 0, 1, \dots, N-1, \\
& \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\min}, && k = 0, 1, \dots, N-1,
\end{aligned}
$$

Soft constraints $(k = 1, 2, \dots, N)$

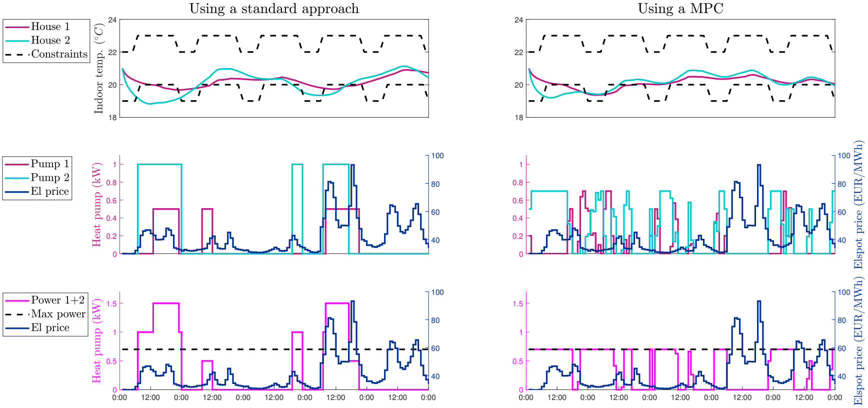
$$
\begin{aligned}
& y_{\min,k} - s_{1,k} - s_{2,k} \leq y_k, && y_k \leq y_{\max,k} + t_{1,k} + t_{2,k}, \\
& 0 \leq s_{1,k} \leq s_{1,\max}, && 0 \leq t_{1,k} \leq t_{1,\max}, \\
& 0 \leq s_{2,k} \leq \infty, && 0 \leq t_{2,k} \leq \infty.
\end{aligned}
$$

Objective function

$$
\phi = \overbrace{\sum_{k=0}^{N-1} c_k' u_k}^{\text{Energy cost}} + \overbrace{\sum_{k=1}^{N} \rho_{s1}' s_{1,k} + \rho_{s2}' s_{2,k} + \rho_{t1}' t_{1,k} + \rho_{t2}' t_{2,k}}^{\text{Temperature violations}}
$$

Heat pump performance using Model Predictive Control

Heat pump using a standard approach vs. using a MPC over a 5 day period



Heat pump performance using Model Predictive Control

Performance improval by using a MPC

As illustrated by the figures on the previous slide, the MPC is able to:

- Incorporate an upper limit on the input power of a portfolio of heat pumps
- Regulate the heat flow such that the indoor temperature is kept between an upper and a lower limit
- React on the given electricity prices by only turning the pump on when the electricity price is low

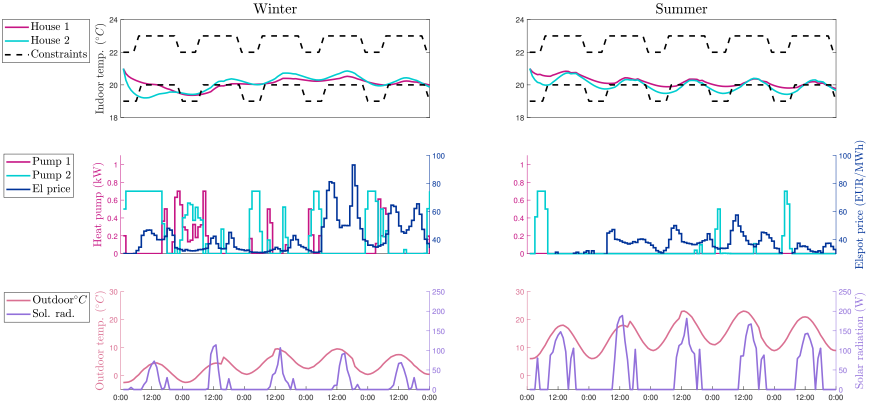
	House 1	House 2
Price (Eur) - standard approach	0.68	1.41
Price (Eur) - MPC	0.35	0.95
Savings by using MPC	48.9%	32.8%

Table: Price saving over the 5 day period of the previous example using a MPC

Heat pump for a good and a poorly insulated house

How the seasons affect performance of the heat pump of a good and a poorly insulated house

In the following "House 1" refers to a good insulated house and "House 2" refers to a poorly insulated house.



Relation between the magnitude of the electricity prices and the penalty for violating constraints

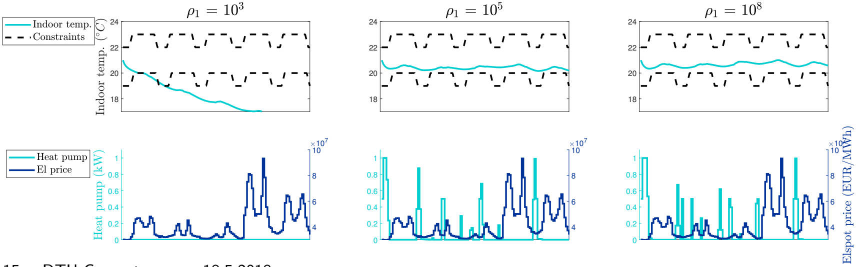
Importance of choosing the right penalty parameter

The lower soft constraint s_1 for the indoor temperature y and the lower constraint $y_{\min}$ is defined as

$$y \geq y_{\min} - s_1, \qquad \text{with } 0 \leq s_1 \leq \infty.$$

For the temperature to be as close as possible to the desired $y_{\min}$ there is a penalty ρ_1 associated with this soft constraint. However, for heat pumps there is a relation between this penalty parameter and the magnitude of the electricity prices.

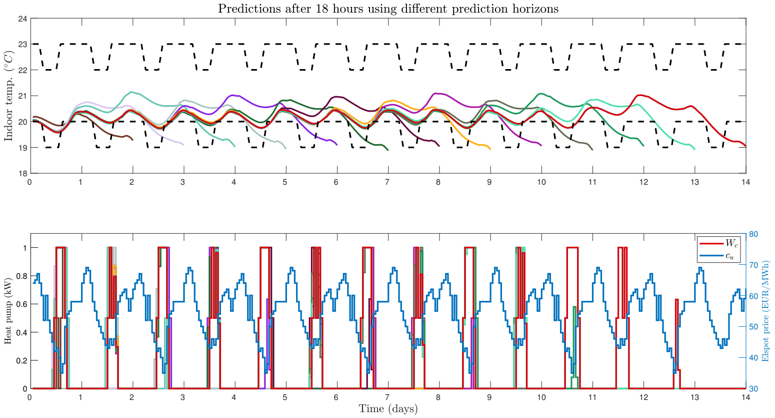
With a prediction horizon of 2 days this relation is visible:



How the prediction horizon contributes to an offset between the temperature and the lower constraint

Prediction horizons

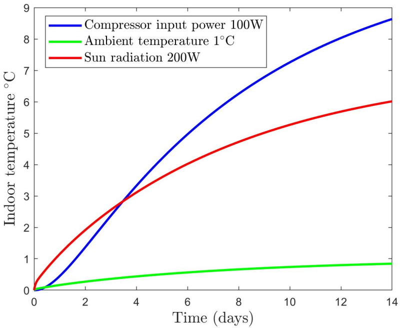
Investigations of different prediction horizons in open loop simulation:



How the prediction horizon contributes to an offset between temperature and constraint

Step responses

To understand why the predictions vary a lot for different horizon lengths, one can investigate the step responses. These give an indicate of what amount of compressor input power of the heat pump leads to the following increase in the indoor temperature.

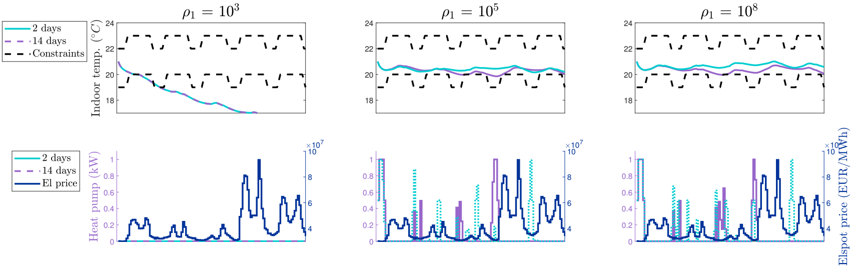


As illustrated, the heating process requires an interval of more than 14 days to arrive at a steady state.

Reducing the offset between temperature and constraint

Increasing the prediction horizon length

Using a prediction horizon of 2 days compared to 14 days, the offset between temperature and constraint is reduced:



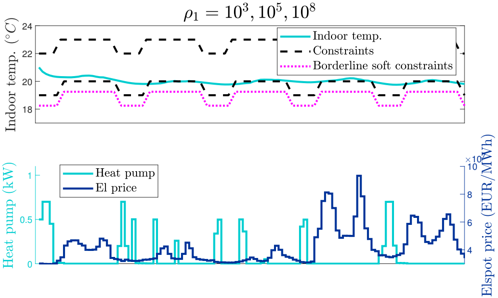
Choice of the right penalty parameter

Introducing another soft constraint

To avoid the search for a suited penalty parameter ρ_1 one could consider to introduce another soft constraint s_2 associated with a penalty parameter of large magnitude $\rho_2 = 10^{12}$:

$$y \geq y_{\min} - s_1 - s_2, \qquad \text{with } 0 \leq s_1 \leq 0.75, 0 \leq s_2 \leq \infty,$$

where the value 0.75 is called the borderline between these two soft constraints. Here the borderline parameter can be used to enforce the temperature up or down.



Conclusions

Observations

- Long control and prediction horizons necessary (N)
- Selection of the prices for soft constraints is non trivial

Recommendations

- Use tailored algorithms for MPC that scales well with the prediction horizon
- Use linear MPC with constant COP (surpricingly it seems that the benefits of NMPC are marginal)
- Use several penalty levels for the soft constraints (or a quadratic penalty function - it will however give a QP and not an LP)

Key future focus:

- Efficient algorithms to tackle challenging realistic problems that cannot be solved by off-the-shelf optimization software

Acknowledgements and Contact

This project is partially supported by

- CITIES
- SCA

Contact:
John Bagterp Jørgensen
jbjo@dtu.dk