Methods and Algorithms for Economic MPC in Power Production Planning

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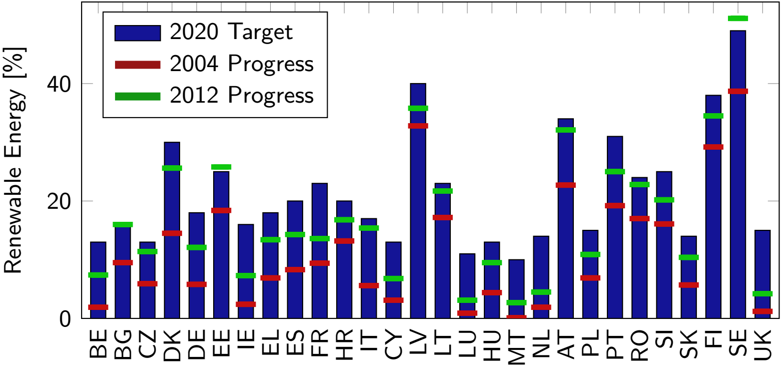
Presentation Outline

Background & Introduction

The Future Power Grid

- The penetration of wind, solar and hydro power is increasing significantly



- New planning methodologies are required to accommodate the intermittency of renewable energy resources

Control Hierarchy

Production Planning

- Hours-ahead unit commitment and economic dispatch of the system generators

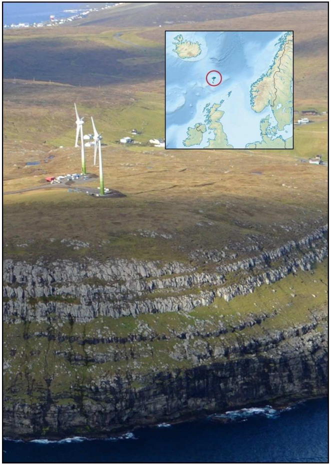
Balance Control

- Balancing of production and consumption in near real-time

Frequency Control

- Real-time activation of reserved generation capacity to maintain system stability

Case Study: The Faroe Islands



- Population of about 50,000 people
- No interconnectors to other countries (isolated power system)
- Some of the worlds best conditions for wind power
- Target: 100% renewable energy by 2030
- Flexibility on both the production and the consumption side of energy

Current challenges for the Faroe Islands are future challenges for larger interconnected power systems

Key Contributions

- Proof of concept for balance and frequency EMPC-based control schemes
- Mean-Variance EMPC accounts for the inherent uncertainty and variability of renewable energy sources
- Integrated planning and control using a hierarchical EMPC algorithm
- Computationally efficient algorithms overcome tractability issues of the proposed EMPC schemes
- An optimal reserve planning problem for unit commitment and economic dispatch in small isolated power systems

Economic MPC of Energy Systems

Economic MPC (EMPC)

Optimal Control Problem

$$\begin{aligned}
\min_{u,x,z}.\ & \phi(u,x,z) \\
\text{s.t.}\ & x_{k+1} = Ax_k + Bu_k, && k \in \mathcal{N}_0 \\
& z_k = C_z x_k, && k \in \mathcal{N}_1 \\
& (u,x,z) \in \mathbb{X}
\end{aligned}$$

- Prediction horizon $\mathcal{N}_i = \{0+i, 1+i, \ldots, N-1+i\}$
- Input vector $u = (u_0^T, u_1^T, u_2^T, \ldots, u_{N-1}^T)^T \in \mathbb{R}^{Nn_u}$
- State vector $x = (x_1^T, x_2^T, x_3^T, \ldots, x_N^T)^T \in \mathbb{R}^{Nn_x}$
- Output vector $z = (z_1^T, z_2^T, z_3^T, \ldots, z_N^T)^T \in \mathbb{R}^{Nn_z}$

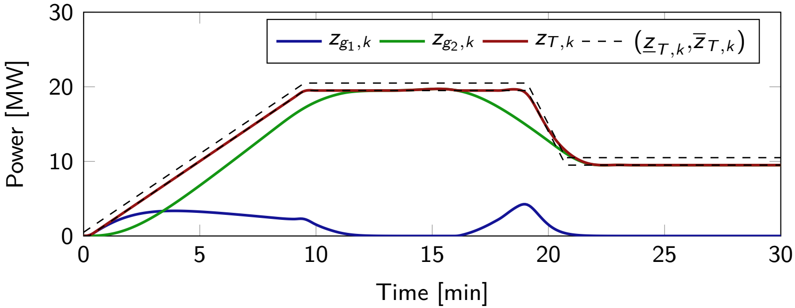
Assumption: Cost function ϕ is a convex function and constraint set $\mathbb{X}$ is a convex set

Two-Generator Case Study

Generator Specifications

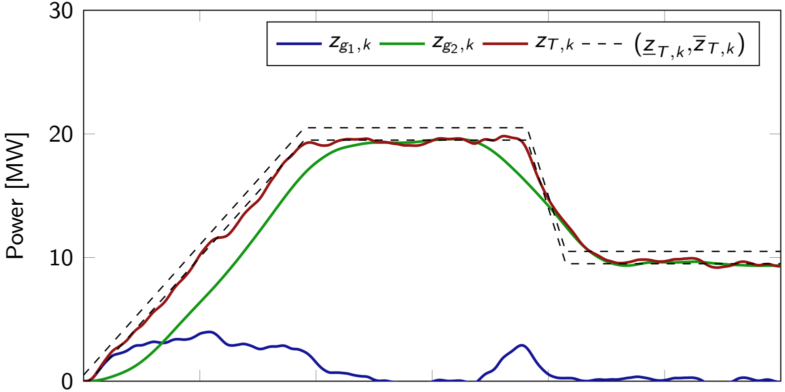
#Generator	Capacity	Response Time	Utilization Cost
1	Small	Fast	High
2	Large	Slow	Low

Closed-Loop Simulation (Deterministic)



Uncertainty Management

Closed-Loop Simulation (Stochastic)



Certainty-Equivalent EMPC does not perform well in the presence of uncertainty

Certainty-Equivalent EMPC (CE-EMPC)

- Linear stochastic system

$$
\begin{aligned}
\boldsymbol{x}_{k+1} &= A\boldsymbol{x}_k + Bu_k + \boldsymbol{w}_k, && k \in \mathcal{N}_0 \\
\boldsymbol{y}_k &= C_y\boldsymbol{x}_k + \boldsymbol{v}_k, && k \in \mathcal{N}_1 \\
\boldsymbol{z}_k &= C_z\boldsymbol{x}_k, && k \in \mathcal{N}_1
\end{aligned}
$$

- Affine functions

$$
\begin{aligned}
\boldsymbol{x} &= L_x(u; x_0, \boldsymbol{w}) \\
\boldsymbol{z} &= L_z(u; x_0, \boldsymbol{w})
\end{aligned}
$$

- Cost function

$$
\psi(u; x_0, \boldsymbol{w}) = \phi\left(u, L_x(u; x_0, \boldsymbol{w}), L_z(u; x_0, \boldsymbol{w})\right)
$$

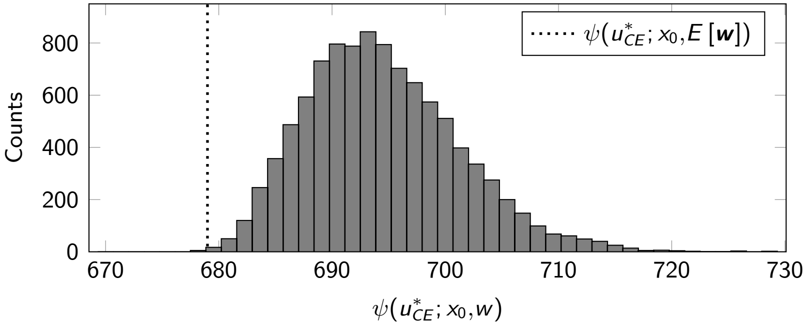
- Optimal control problem

$$
\min_{u \in \mathcal{U}} \; \Psi_{CE} = \psi(u; x_0, E\left[\boldsymbol{w}\right])
$$

Mean-Variance EMPC (MV-EMPC)

- CE-EMPC does not minimize the expected cost

$$\psi(u; x_0, E[\boldsymbol{w}]) \neq E[\psi(u; x_0, \boldsymbol{w})]$$



- MV-EMPC

$$\min_{u \in \mathcal{U}} \; \Psi_{MV} = \alpha E[\psi(u; x_0, \boldsymbol{w})] + (1-\alpha) V[\psi(u; x_0, \boldsymbol{w})]$$

with risk-aversion parameter $\alpha \in [0; 1]$

Monte-Carlo Approximation

- Uncertainty scenarios $\mathcal{S} = \{1, 2, \ldots, S\}$

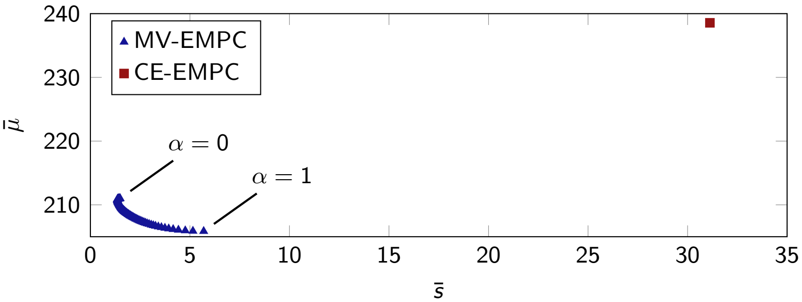
- Optimal control problem

$$
\begin{aligned}
\min_{u \in \mathcal{U}, \{x^s, z^s, \psi^s\}_{s \in \mathcal{S}}, \mu} \quad & \alpha\mu + \tfrac{1-\alpha}{S-1} \sum_{s \in \mathcal{S}} \left(\psi^s - \mu\right)^2 , \\
\text{s.t.} \quad & x_{k+1}^s = A x_k^s + B u_k + w_k^s, \qquad k \in \mathcal{N}_0, \ \ s \in \mathcal{S} \\
& z_k^s = C_z x_k^s, \qquad k \in \mathcal{N}_1, \ \ s \in \mathcal{S} \\
& \psi^s = \phi(u, x^s, z^s), \qquad s \in \mathcal{S} \\
& \mu = \tfrac{1}{S} \sum_{s \in \mathcal{S}} \psi^s
\end{aligned}
$$

- Two-stage extension with non-anticipative constraints can be applied for less conservative closed-loop performance

- Large-scale optimization problem even for small systems

Performance of MV-EMPC



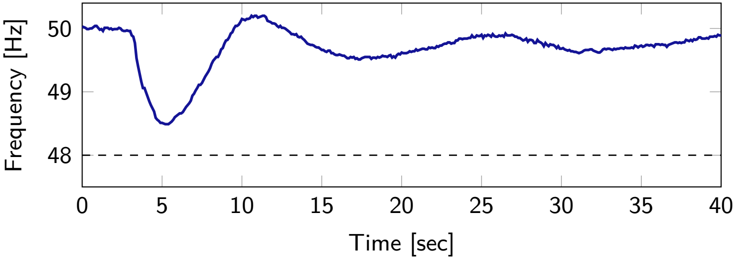
Computationally Attractive Alternatives

- Safety margin using constraint back-off
- Augmented objective function, e.g. setpoint-based penalty terms and/or regularization terms

MV-EMPC provides a baseline for performance evaluation

Frequency Control via EMPC

- **Objective 1**: Avoid critical frequency fluctuations



- **Objective 2**: Minimize cost of operations

Optimal Control Problem

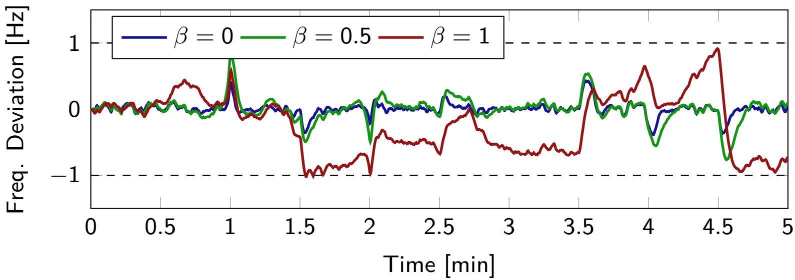
Objective Function

$$\phi(u, z) = \beta \phi^{\mathrm{eco}}(u, z) + (1 - \beta)\phi^{\mathrm{sp}}(u, z)$$

with risk-aversion parameter $\beta \in [0; 1]$

- ϕ^{eco}: Operate system at minimum cost
- ϕ^{sp}: Restore the frequency to the nominal frequency

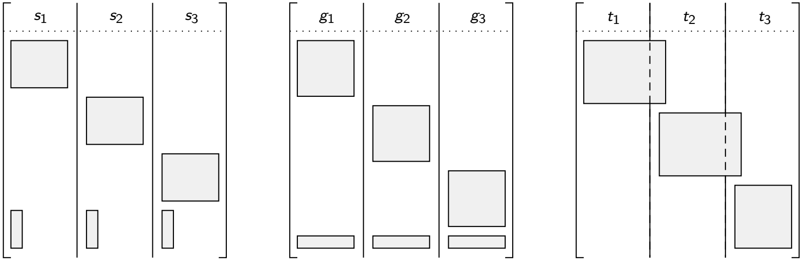
Closed-Loop Simulation



Optimization Algorithms

Computational Aspects of EMPC

Problem structure is utilized for real-time solution of the OCPs

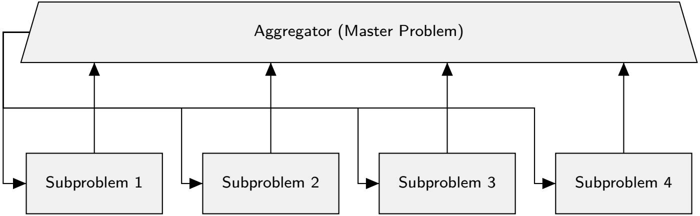


(a) Scenario Coupling (b) Generator Coupling (c) Temporal Coupling

- Case (a) and (b) are handled by decomposition methods
- Case (c) is handled using Riccati-based methods
- Nested structures occur (c)$\rightarrow$(b)$\rightarrow$(a)

EMPC Decomposition Algorithms

Schematic Diagram



Subproblems can be solved in parallel and warm-start is applicable

Methods

Method	Problem Class	Iterations	Accuracy	Dimensions
DWD	LPs	Few	High	Increasing
ADMM	CPs	Many	Low	Constant

Example: Block-Angular LPs

- Problem formulation

$$\min_{t} \left\{ \sum_{j \in \mathcal{J}} c_j^T t_j \mid G_j t_j \leq g_j,\, j \in \mathcal{J},\, \sum_{j \in \mathcal{J}} H_j t_j \leq h \right\}$$

- **DWD**: Extreme point representation

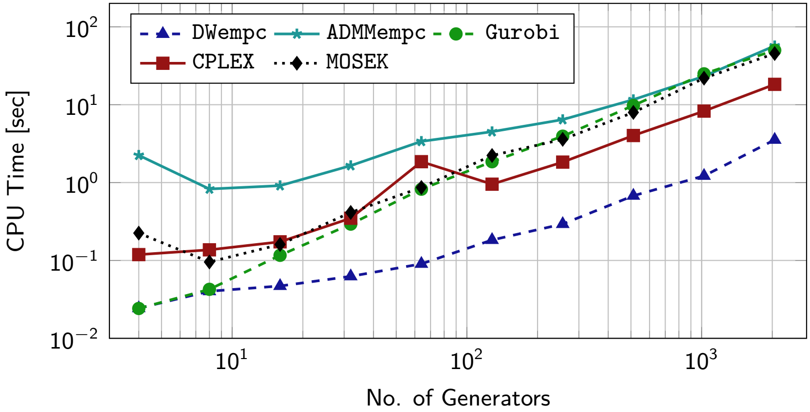
- **ADMM**: Problem splitting using auxiliary variables

$$v_j = H_j t_j, \qquad\qquad j \in \mathcal{J}$$

Formulation of modified problem and simplified recursion is challenging

Benchmark

CPU Time to Solve the OCP



- Memory issue around $M = 3000$ for centralized solves
- The performance of ADMM is very problem dependent

Further ADMM Results

MV-EMPC

Step	Description
1	Solve a single OCP for each uncertainty scenario
2	Minimize variance s.t. non-anticipative constraints

Input-Constrained EMPC

Step	Description
1	Solve unconstrained OCP
2	Solve input-constrained OCP with no dynamics

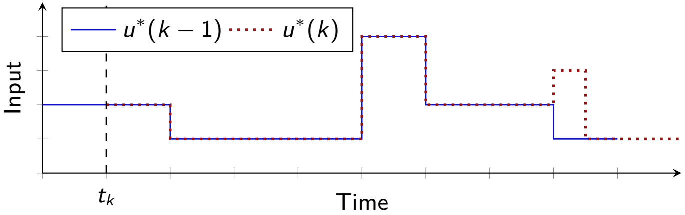
A speedup in computational speed of more than an order of magnitude is achieved for both cases

Homogeneous and Self-Dual Interior-Point Method

- Solution of the OCP $\min_x \{g^T x \mid Ax = b,\ Cx \leq d\}$ is obtained from solution of $(\tilde{z}, \tilde{s}, \tau, \kappa) \geq 0$ and

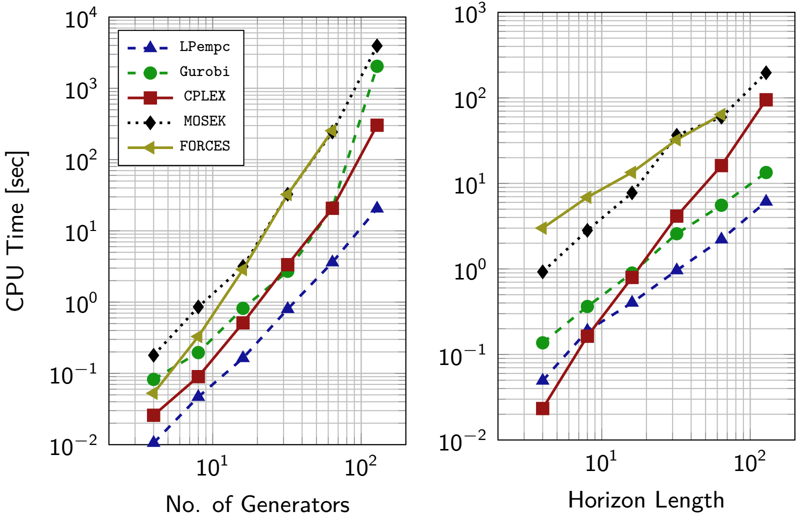
$$A^T \tilde{y} + C^T \tilde{z} + g\tau = 0, \qquad A\tilde{x} - b\tau = 0$$
$$C\tilde{x} - d\tau + \tilde{s} = 0, \qquad -g^T \tilde{x} - b^T \tilde{y} - d^T \tilde{z} + \kappa = 0$$

- Warm-start works well for homogeneous and self-dual IPMs



- Search direction is computed using a Riccati-iteration procedure

LP Solver Comparison



Warm-start reduces the CPU time by further 40% on average

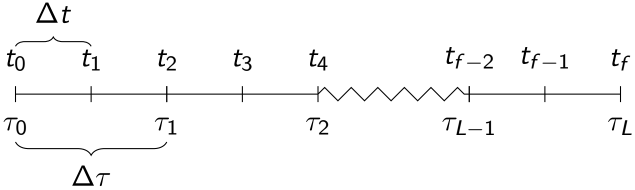
Integrated Planning and Control

Production Planning

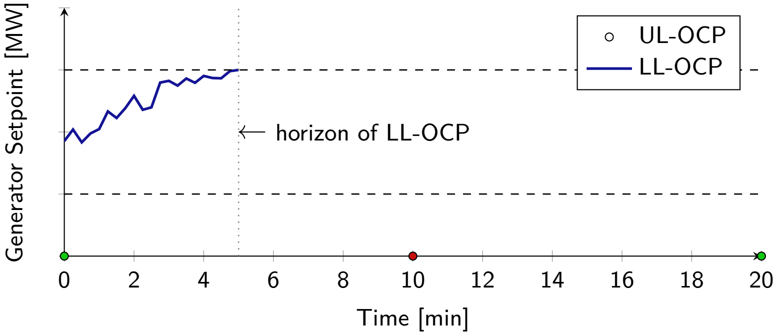
- Binary decisions $b = (b_0^T, b_1^T, \dots, b_L^T)^T$
- Problem formulation (simplified)

$$
\begin{aligned}
\min_{u,x,z,b} \quad & f_{\mathbb{R}}(u,x,z,b) + f_{\mathbb{Z}}(b) \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k, && k \in \mathcal{N}_0 \\
& z_k = C_z x_k + F_z d_k, && k \in \mathcal{N}_1 \\
& c_{\mathbb{R}}(u,x,z,b) \le 0 \\
& c_{\mathbb{Z}}(b) \le 0
\end{aligned}
$$

- Two time scales

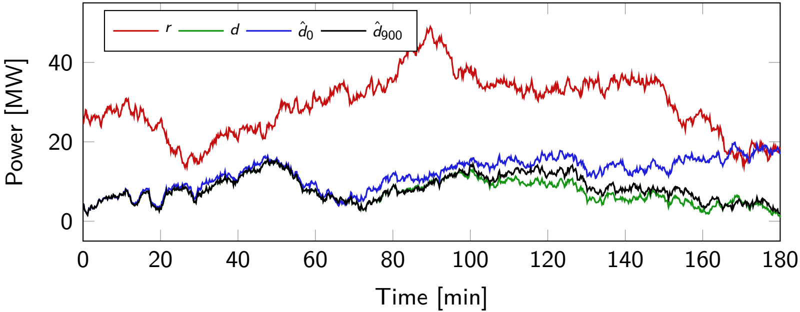


Hierarchical Algorithm



- The UL-OCP (MIQP/MILP) is closely related to the unit commitment problem
- The UL-OCP may be solved with a low frequency
- Tailored algorithms can solve the LL-OCP (QP/LP) efficiently

Three-Generator Example



- Direct solution of the full OCP is 15 minutes
- Solution times are 2s (UL-OCP) and 0.1s (LL-OCP)
- Single resolve of the UL-OCP is performed
- Cost increase is less than 1% for the hierarchical approach

Optimal Reserve Planning

Unit Commitment in Isolated Power Systems

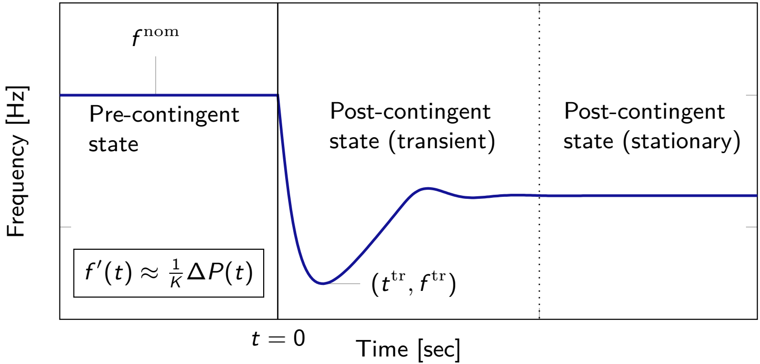
The conventional unit commitment and economic dispatch problem can be posed as an MILP

$$
\begin{aligned}
\min_{x,y}.\ & f^T x + g^T y \\
\text{s.t.}\ & Ax + By \leq b \\
& x \in \mathbb{R}^n \\
& y \in \{0,1\}^m
\end{aligned}
$$

- **Constraints**: Power balance, fixed reserves, production limits, ramping limits, etc.
- **Variables**: Production levels, reserve levels, on/off decisions, etc.

The solution of the MILP provides a ≈24-hours ahead production plan with a ≈15-minute resolution

Operational Reserves



- Primary reserves are critical to avoid power outages (blackouts) in the event of a contingency $\Delta P(t) \neq 0$
- Primary reserves are activated in direct proportion to the frequency deviation from the nominal frequency

Minimum Frequency Constraint

It is critical that $f(t) \geq \underline{f}$ for some cut-off frequency $\underline{f}$

- **Large interconnected systems**
  System inertia is large and approximately constant
  $\Rightarrow$ A fixed amount of primary reserve is sufficient

- **Small isolated power systems**
  System inertia is small and varies considerably
  $\Rightarrow$ Minimum frequency constraints are required

The constraint $f(t) \geq \underline{f}$ is intractable to handle using mixed-integer linear programming

Alternative Formulation

The minimum frequency constraint

$$f(t) \geq \underline{f}$$

may be expressed as

$$E^{\mathrm{PR}}(t) + \Delta E^{\mathrm{rot}} \geq P^{\mathrm{lost}} t$$

- $E^{\mathrm{PR}}(t) = \int_0^{\mathrm{t}} P^{\mathrm{PR}}(\tau) d\tau$ is the energy contribution from the activation of primary reserves
- ΔE^{rot} is the energy contribution from the system inertia
- $P^{\mathrm{lost}} t$ is the energy lost as a result of the contingency (generator trip)

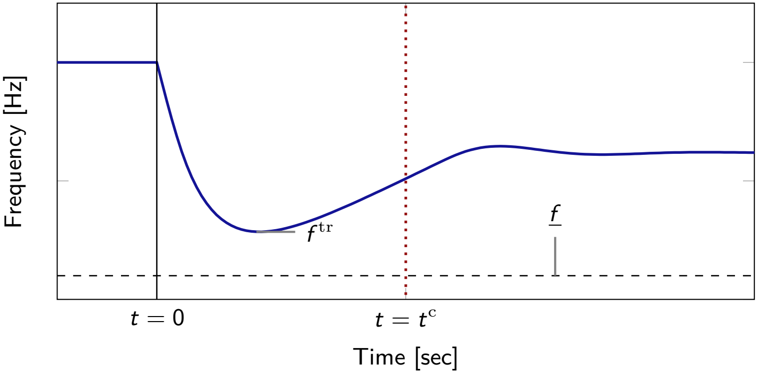
Sufficient Conditions

- Minimum frequency occurs no later than time t^{c}

$$P^{\mathrm{PR}}(t^{\mathrm{c}}) \geq P^{\mathrm{lost}}$$

- Satisfy $f(t) \geq \underline{f}$ for $t \leq t^{\mathrm{c}}$, i.e.

$$E^{\mathrm{PR}}(t) + \Delta E^{\mathrm{rot}} \geq P^{\mathrm{lost}} t, \quad t \leq t^{\mathrm{c}}$$



Optimal Reserve Planning Problem (ORPP)

- Unit commitment and economic dispatch problem with minimum frequency constraints
- Compared to a conventional production and reserve planning problem (BLUC)
- Simulations show that several potential blackouts are avoided at a cost increase of 3%
- Tested in the Faroe Islands in 2015

Conclusions & Future Work

Conclusions

Methods

- MV-EMPC overcomes performance issues of CE-EMPC in operation of uncertain systems
- MV-EMPC provides a baseline for approximate methods

Algorithms

- Tailored decomposition schemes significantly reduces computational requirements of the proposed EMPC methods
- Additional speedup is achieved using Riccati-based IPMs

Applications

- Simulations demonstrate that EMPC-based methods for balance and frequency control reduce cost and risk
- Unifying framework for balance control and unit commitment
- Frequency-constrained planning in isolated power systems

Future Work

Feedback From Experiments

- Use feedback from the Faroe Islands to improve the proposed planning and control methods

Risk Measures in MV-EMPC

- Employ other risk measures than the variance
- Increase sensitive to the tail shape of the cost distribution
- Develop algorithms to solve the resulting OCPs efficiently

Algorithms for EMPC

- Quadratic programming extensions of LP solvers
- Tuned and parallel implementations
- Scenario reduction in MV-EMPC

Thanks! Questions and Comments?