System Identification for Control and Simulation.

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Two goals

- A general overview of basic steps in system identification
Outline

Two goals

- A general overview of basic steps in system identification
- A more technical account of modeling goals for linear systems
Modeling Dynamic Systems

Modeling Approaches

First-Principles Modeling

Data-Driven Modeling

Tools for Modeling Dynamic Systems

MathWorks
MATLAB & SIMULINK

Simulink
Simscape
SimMechanics
SimHydraulics
SimPowerSystems
SimDriveline
SimElectronics
Aerospace Blockset

Simulink Design Optimization

Neural Network Toolbox

System Identification Toolbox
An Introductory Example: System

The System

Input
rudders
aileron
thrust

System
velocity
pitch angle

Output

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System Identification for Control and Simulation
The Model

\[ x(t+1) = F x(t) + G u(t) \]
\[ y = H x(t) \]

\( u, y \): measured time or frequency domain signals
The System and the Model

System

Model

Measured input

Minimize error

Measured input

Minimize

error

+ -
Pitch rate, Canard, Elevator, Leading Edge Flap

How do the control surface angles affect the pitch rate?
How do the control surface angles affect the pitch rate?

Simulation of the aircraft
Data from the Gripen Aircraft

Pitch rate, Canard, Elevator, Leading Edge Flap

- How do the control surface angles affect the pitch rate?
- Simulation of the aircraft
- Design of autopilot (regulator)
$y(t)$ pitch rate at time $t$. $u_1(t)$ canard angle at time $t$. $T = 1/60$.

Try

$$y(t) = + b_1 u_1(t - T) + b_2 u_1(t - 2T) + b_3 u_1(t - 3T) + b_4 u_1(t - 4T)$$

Dashed line: Measured output (Pitch rate). Solid line: Model output, simulated from the fourth order model from canard angle only.
Using All Inputs

\( u_1 \) canard angle; \( u_2 \) Elevator angle; \( u_3 \) Leading edge flap;

\[
y(t) = -a_1 y(t - T) - a_2 y(t - 2T) - a_3 y(t - 3T) - a_4 y(t - 4T) \\
+ b_1^1 u_1(t - T) + \ldots + b_4^1 u_1(t - 4T) \\
+ b_2^1 u_2(t - T) + \ldots + b_3^1 u_3(t - T) + \ldots + b_4^4 u_3(t - 4T)
\]

Dashed line: Measured output (Pitch rate). Solid line: Model output, simulated from the fourth order model with all three inputs.
- Select a class of candidate models
- Select a member in this class using the observed data
- Evaluate the quality of the obtained model
- Design the experiment so that the model will be “good”.
A Typical Problem

Given Observed Input-Output Data: Find a Description of the System that Generated the Data.
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Basic Approach

Find a suitable Model Structure, Estimate its parameters, and compute the response of the resulting model.
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Find a suitable Model Structure, Estimate its parameters, and compute the response of the resulting model.

Techniques

Estimate the parameters by ML techniques/PEM (prediction error methods). Find the model structure by Cross Validation or other validation techniques.
The SI Flow

$\mathcal{X}$: The Experiment
$\mathcal{D}$: The Measured Data
$\mathcal{M}$: The Model Set
$\mathcal{I}$: The Identification Method
$\mathcal{V}$: The Validation Procedure

No, try new $\mathcal{M}$

Yes!

No, try new $\mathcal{X}$

$\mathcal{M}(\hat{\theta})$
The SI Flow; Model Structures $\mathcal{M}$

No, try new $\mathcal{A}$  
Yes!  
No, try new $\mathcal{M}$

No, try new $\mathcal{X}$

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System Identification for Control and Simulation
A model is a mathematical expression that describes the connections between measured inputs and outputs, and possibly related noise sequences.
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Individual models in the structure are labeled with a parameter vector $\theta$.

A common framework is to describe the model as a predictor of the next output, based on observations of past input-output data.

**Observed input–output $(u, y)$ data up to time $t$: $Z^t$**

**Model described by predictor:** $\mathcal{M}(\theta) : \hat{y}(t|\theta) = g(t, \theta, Z^{t-1})$. 
The SI Flow: Estimation: $\mathcal{I}$

$\mathcal{M} \xrightarrow{\mathcal{X}} \mathcal{I} \xrightarrow{\mathcal{D}} \mathcal{M}(\hat{\theta}) \xrightarrow{\mathcal{V}} OK? \xrightarrow{Yes!} \xrightarrow{No, \text{try new } \mathcal{M}} \xrightarrow{No, \text{try new } \mathcal{X}}$
If a model, $\hat{y}(t|\theta)$, essentially is a predictor of the next output, it is natural to evaluate its quality by assessing how well it predicts: Form the Prediction error and measure its size:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t|\theta), \quad \ell(\varepsilon(t, \theta))$$
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Typically \( \ell(x) = x^2 \). How has it performed historically?

\[
V_N(\theta) = \sum_{t=1}^{N} \epsilon^2(t, \theta)
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A Pragmatic Fit Criterion

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\[
V_N(\theta) = \sum_{t=1}^{N} \varepsilon^2(t, \theta)
\]

Which model in the structure performed best (Prediction Error Method, PEM)?

\[
\hat{\theta}_N = \arg \min_{\theta \in D_M} V_N(\theta)
\]

(This is often also the Maximum Likelihood Estimate (MLE).)
Model Estimate Properties $\mathcal{M}(\hat{\theta}_N)$
As the number of data, \( N \), tends to infinity

\[ \hat{\theta}_N \to \theta^* \sim \arg \min_{\theta} E \varepsilon^2(t, \theta) \]

\( M(\theta^*) \) is the best possible predictor in \( M \)
As the number of data, $N$, tends to infinity

- $\hat{\theta}_N \rightarrow \theta^* \sim \arg \min_{\theta} E\varepsilon^2(t, \theta)$
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$E$: Expectation. This is very nice approximation property:

- The model structure is not large enough: The ML/PEM estimate converges to the best possible approximation of the system.
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- The model structure is not large enough: The ML/PEM estimate converges to the best possible approximation of the system.
- “Best possible approximation” ...
- ... under the conditions of the experiment
- (If the model structure is large enough to contain a true description of the system, then the ML/PEM estimated model has (asymptotically) the best accuracy).
Experiment Design: $\mathcal{X}$

$\mathcal{M} \xrightarrow{I} \mathcal{X} \xrightarrow{D} \mathcal{M}(\hat{\theta}) \xrightarrow{\mathcal{V}} \text{OK?}$

- No, try new $\mathcal{M}$
- No, try new $\mathcal{X}$
- Yes!
$\mathcal{X}$: The design variables: Input, Sampling Interval, Feedback,...

Then we just saw

$$\hat{\theta}_N \rightarrow \theta^*(\mathcal{X})$$
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- The model $\mathcal{M}(\theta^*(\mathcal{X}))$ is the best approximation of the system under $\mathcal{X}$
\( \mathcal{X} \): The design variables: Input, Sampling Interval, Feedback,...

Then we just saw

\[
\hat{\theta}_N \rightarrow \theta^*(\mathcal{X})
\]

- The model \( M(\theta^*(\mathcal{X})) \) is the best approximation of the system under \( \mathcal{X} \)

- Let the experimental conditions resemble those under which the model is to be used!
Model Validation: \( \checkmark \)

\[
\begin{align*}
\mathcal{M} & \quad \mathcal{I} & \quad \mathcal{V} \\
\text{No, try new } \mathcal{M} & \quad \text{No, try new } \mathcal{X} & \quad \text{Yes!}
\end{align*}
\]
“Twist and turn” the model(s) to check if they are good enough for the intended application.
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1. Simulation (prediction) cross validation
   - Check how well the estimated model can reproduce new, validation data - Can compare different models in that way.
“Twist and turn” the model(s) to check if they are good enough for the intended application. Essentially a subjective decision. Several basic techniques are available:

1. **Simulation (prediction) cross validation**
   - Check how well the estimated model can reproduce new, validation data - Can compare different models in that way.

2. **Residual Analysis**
   - Are the residuals $\varepsilon(t, \hat{\theta}_N)$ (the "leftovers") unpredictable? They should not be correlated with anything we knew when estimating the model. Check correlation of the residuals with old inputs.
So this is the System Identification Flow or Loop
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Several essential choices that have to be made, and often revised.
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Let us now turn to some specific illustrations for linear models (Requires some more mathematical background.)
More Technical Details: Linear Models

\[ y(t) = G(q, \theta)u(t) + v(t); \quad G(q, \theta)u(t) = \sum_{k=1}^{\infty} g_k u(t - k), \]
More Technical Details: Linear Models

\[ y(t) = G(q, \theta)u(t) + \nu(t); \quad G(q, \theta)u(t) = \sum_{k=1}^{\infty} g_k u(t - k), \]

\[ \nu \text{ has spectrum } \Phi_\nu(\omega) = \lambda |H(e^{i\omega}, \theta)|^2 \]

\[ \nu(t) = H(q, \theta)e(t) \quad e(t) \text{ white noise} \]
**More Technical Details: Linear Models**

\[
y(t) = G(q, \theta)u(t) + v(t); \quad G(q, \theta)u(t) = \sum_{k=1}^{\infty} g_k u(t - k),
\]

\[
v \text{ has spectrum } \Phi_v(\omega) = \lambda |H(e^{i\omega}, \theta)|^2
\]

\[
v(t) = H(q, \theta)e(t) \quad e(t) \text{ white noise}
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More Technical Details: Linear Models

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General Description of a Linear Model

\[ y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \]

The Prediction Errors

\[ \varepsilon(t, \theta) = H^{-1}(q, \theta)[y(t) - G(q, \theta)u(t)] \]
Suppose the data is generated by a true linear system $G_0(q)$ and that the prediction errors are pre-filtered by a filter $L(q)$,

$$\hat{\theta}_N = \arg \min \sum (L(q)e(t, \theta))^2$$
Suppose the data is generated by a true linear system $G_0(q)$ and that the prediction errors are pre-filtered by a filter $L(q)$,

$$
\hat{\theta}_N = \arg \min \sum (L(q)\epsilon(t, \theta))^2
$$

Then

$$
\hat{\theta}_N \to \theta^* = \arg \min_{\theta} \int_{-\pi}^{\pi} |G(e^{i\omega}, \theta) - G_0(e^{i\omega})|^2 Q(\omega) d\omega
$$

$$
Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega}, \theta)|^2}
$$
Suppose the data is generated by a true linear system $G_0(q)$ and that the prediction errors are pre-filtered by a filter $L(q)$,

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\hat{\theta}_N \to \theta^* = \arg \min \theta \int_{-\pi}^\pi |G(e^{i\omega}, \theta) - G_0(e^{i\omega})|^2 Q(\omega) d\omega
$$

$$
Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega}, \theta)|^2}
$$

So the resulting model is closest to the true system in a norm defined by $Q$. 
Note that we can affect \( Q(\omega) = \frac{|L(e^{i\omega})|^2\Phi_u(\omega)}{|H(e^{i\omega},\theta)|^2} \) by choosing \( L = -(I) \), the input spectrum \( \Phi_u = -(\mathcal{X}) \) and the noise model \( H = -(\mathcal{M}) \), so it depends on all boxes in the identification chart.
Note that we can affect $Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega},\theta)|^2}$ by choosing $L = -\mathcal{I}$, the input spectrum $\Phi_u = -\mathcal{X}$ and the noise model $H = -\mathcal{M}$, so it depends on all boxes in the identification chart.

Identification for simulation with an input $u_\ast \Rightarrow$ Make $Q$ equal to the spectrum of $u_\ast$
Note that we can affect $Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega}, \theta)|^2}$ by choosing $L = -\mathcal{I}$, the input spectrum $\Phi_u = -\mathcal{X}$ and the noise model $H = -\mathcal{M}$, so it depends on all boxes in the identification chart.

- **Identification for simulation with an input $u_*$** ⇒ Make $Q$ equal to the spectrum of $u_*$

- **Identification for control** ⇒ Make $Q$ large at the intended cross-over frequency ($\approx$ the intended bandwidth)
Identification is a work-flow loop with nodes that contain essential user choices.
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These may (should) depend on the intended use of the final model
Conclusions

- Identification is a work-flow loop with nodes that contain essential user choices
- These may (should) depend on the intended use of the final model
- Concrete illustration for linear system models