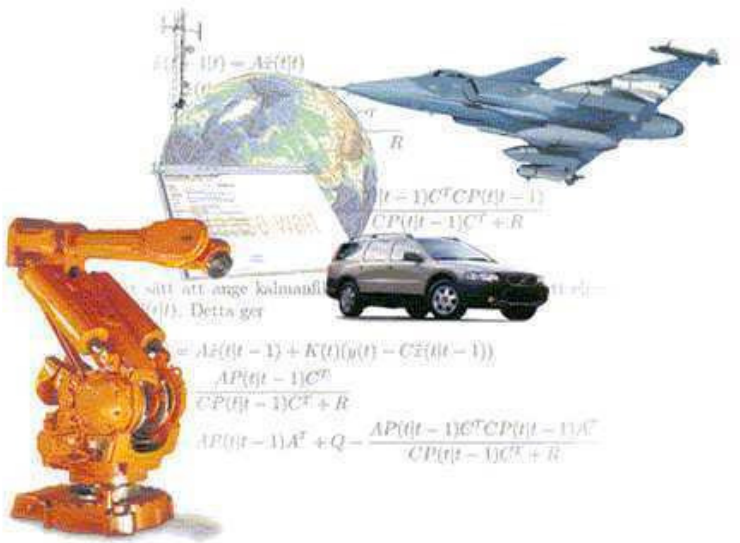
System Identification for Control and Simulation.



Lennart Ljung

Automatic Control, ISY, Linköpings Universitet

Outline

Two goals

- A general overview of basic steps in system identification

Outline

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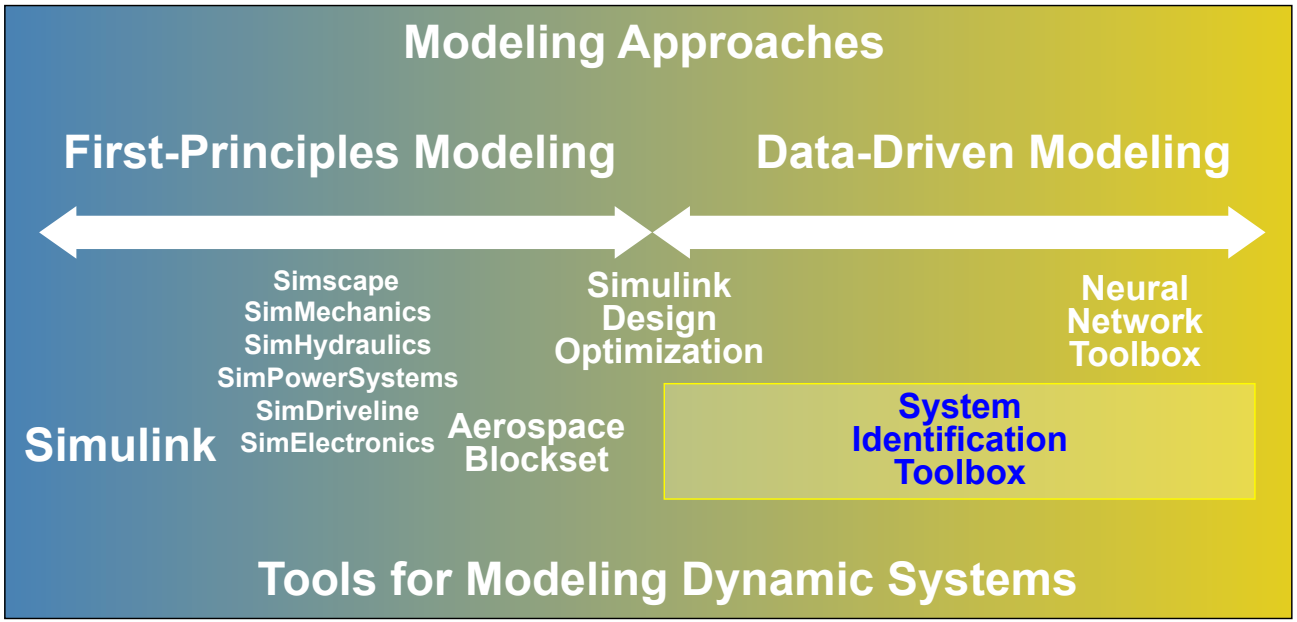
- A general overview of basic steps in system identification
- A more technical account of modeling goals for linear systems

Modeling Approaches: View from the Mathworks

MathWorks MATLAB&SIMULINK

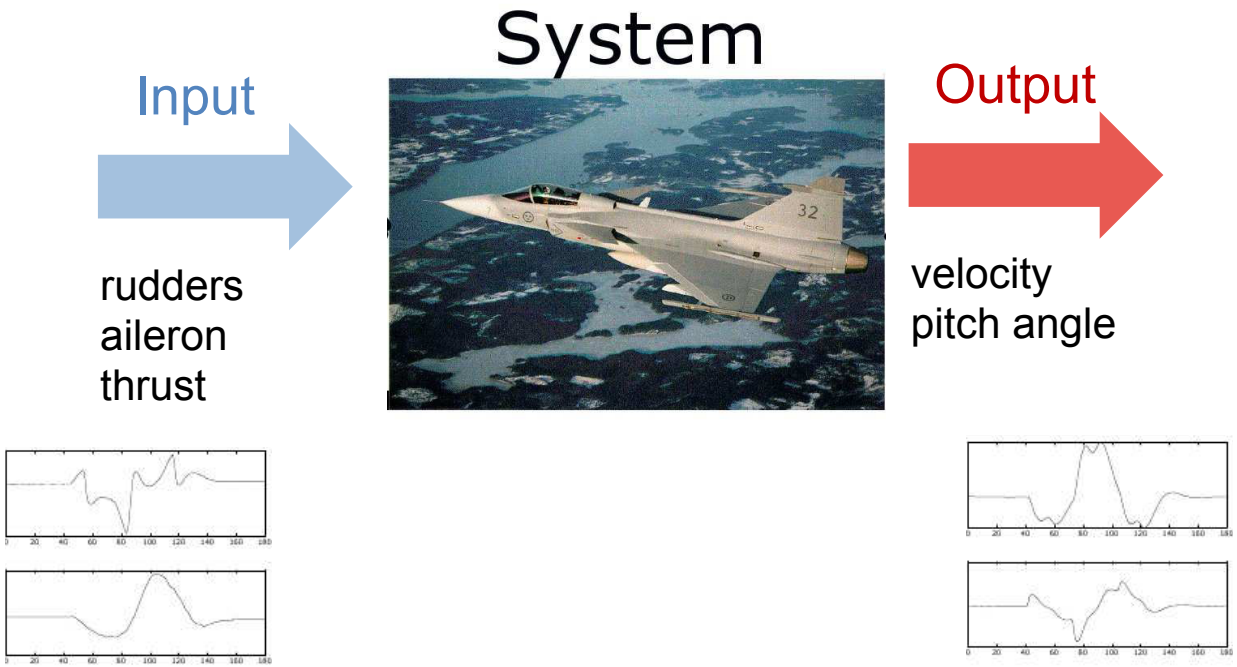
Modeling Dynamic Systems



2

An Introductory Example: System

The System

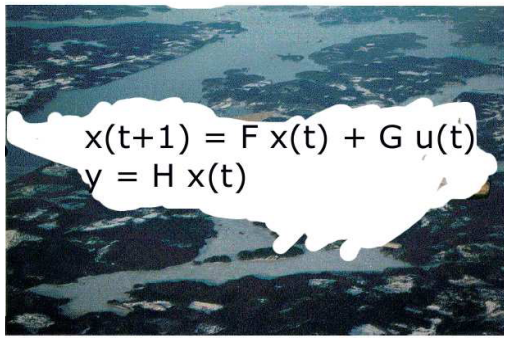


An Introductory Example 2: Model

The Model

u
Input

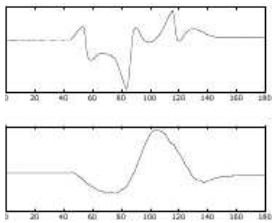
MODEL



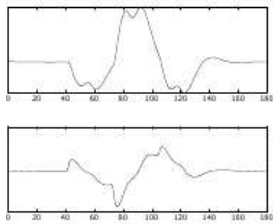
y
Output

rudders
aileron
thrust

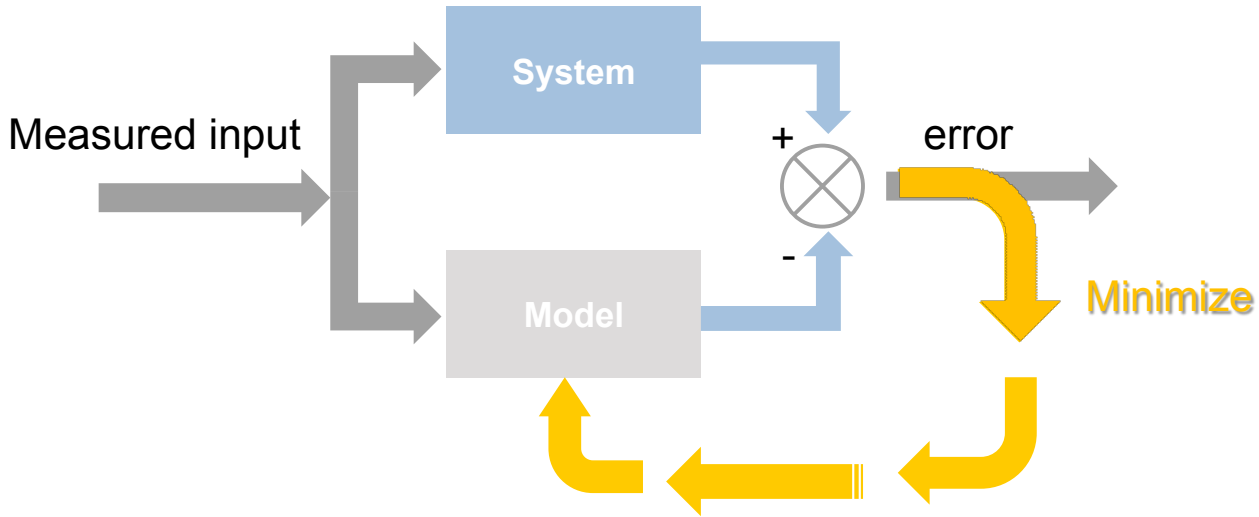
velocity
pitch angle



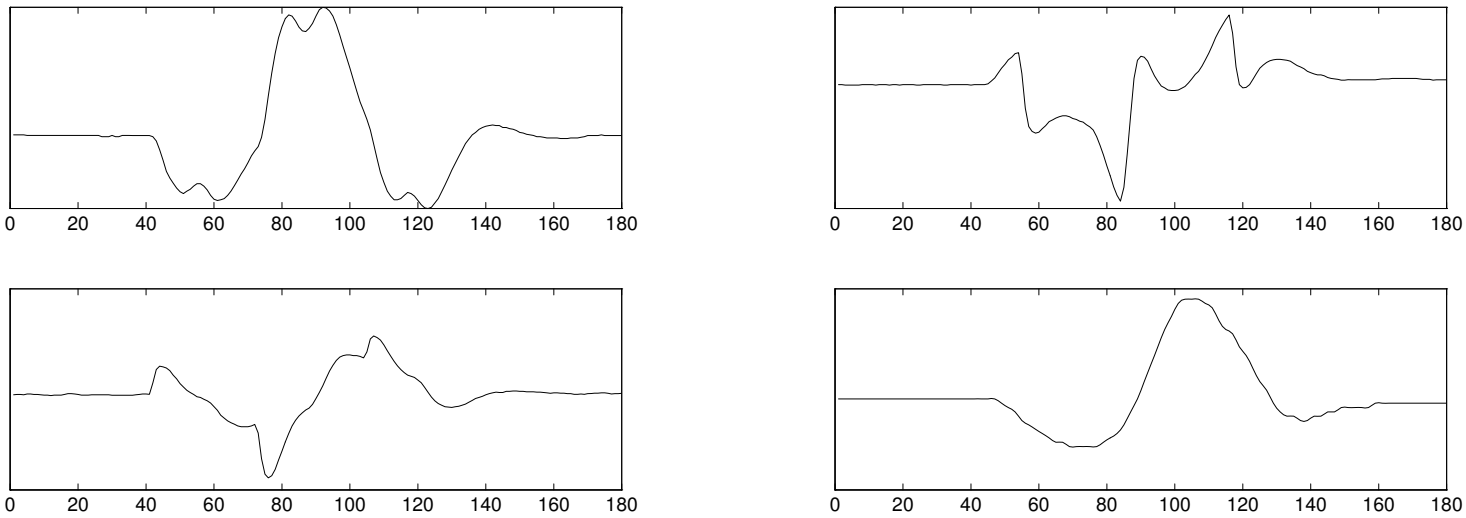
u, y: measured time or frequency domain signals



The System and the Model



Data from the Gripen Aircraft



Pitch rate, Canard,
Elevator, Leading Edge Flap

- How do the control surface angles affect the pitch rate?

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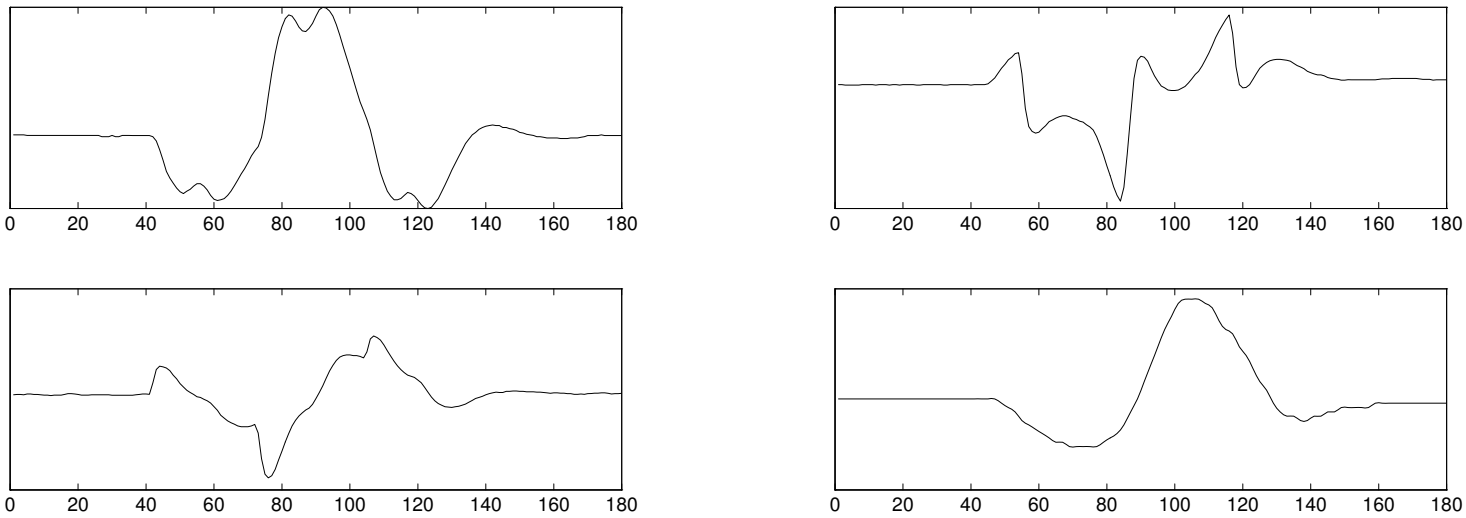

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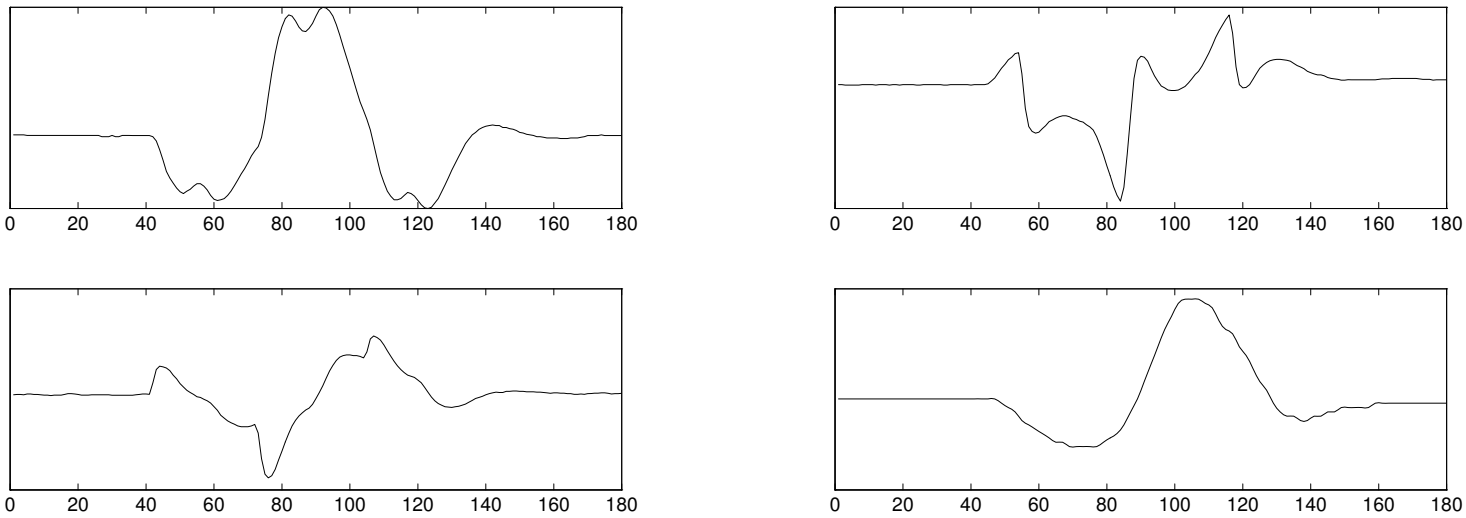


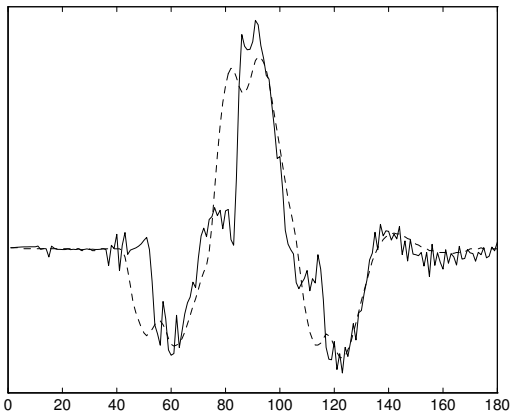
Pitch rate, Canard,
Elevator, Leading Edge Flap

- How do the control surface angles affect the pitch rate?
- Simulation of the aircraft
- Design of autopilot (regulator)

Aircraft Dynamics: From input 1

$y(t)$ pitch rate at time t. $u_1(t)$ canard angle at time t. $T = 1/60$.
Try

$$y(t) = +b_1 u_1(t-T) + b_2 u_1(t-2T) + b_3 u_1(t-3T) + b_4 u_1(t-4T)$$

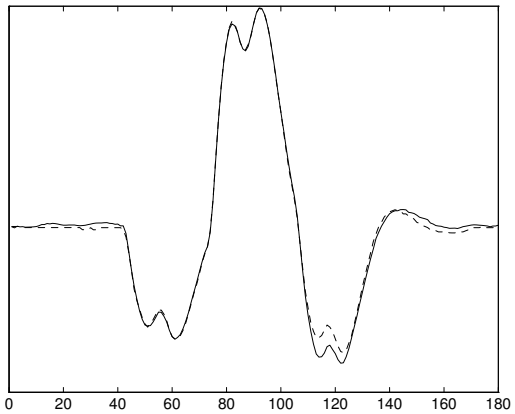


Dashed line: Measured output (Pitch rate). Solid line: Model output, simulated from the fourth order model from canard angle only.

Using All Inputs

u_1 canard angle; u_2 Elevator angle; u_3 Leading edge flap;

$$
\begin{aligned}
y(t) = & -a_1 y(t-T) - a_2 y(t-2T) - a_3 y(t-3T) - a_4 y(t-4T) \\
& + b_1^1 u_1(t-T) + \ldots + b_1^4 u_1(t-4T) \\
& + b_2^1 u_2(t-T) + \ldots + b_1^3 u_3(t-T) + \ldots + b_4^3 u_3(t-4T)
\end{aligned}
$$



Dashed line: Measured output (Pitch rate). Solid line: Model output, simulated from the fourth order model with all three inputs

System Identification: Issues

- Select a class of candidate models
- Select a member in this class using the observed data
- Evaluate the quality of the obtained model
- Design the experiment so that the model will be “good”.

A Typical Problem

Given Observed Input-Output Data: Find a Description of the System that Generated the Data.

System Identification: State-of-the-Art Setup

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Basic Approach

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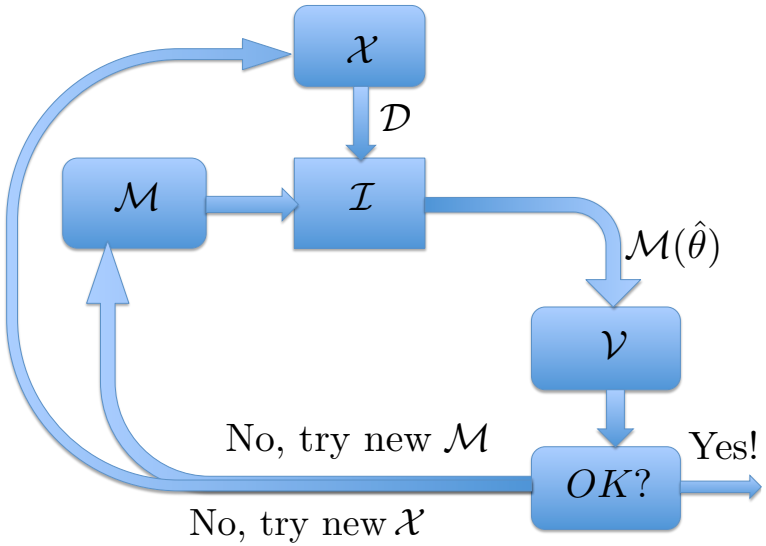
Techniques

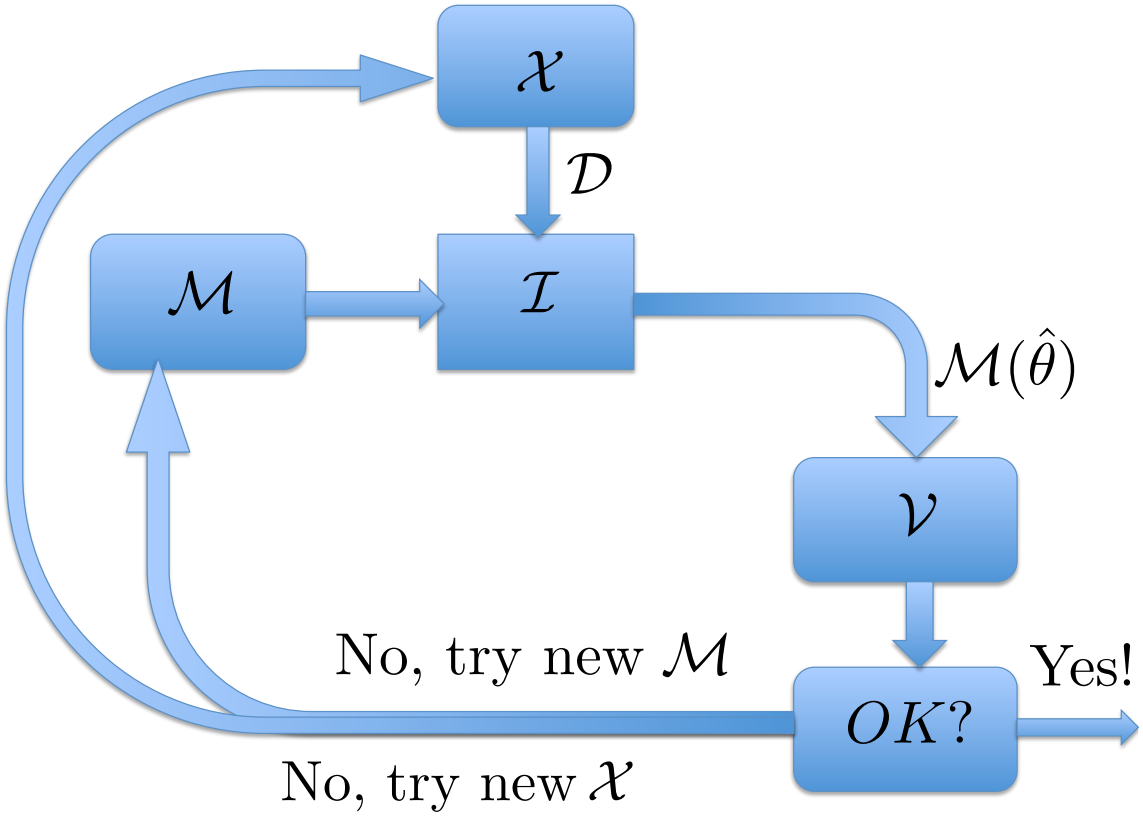
Estimate the parameters by ML techniques/PEM (prediction error methods). Find the model structure by Cross Validation or other validation techniques

The SI Flow



$\mathcal{X}$: The Experiment
$\mathcal{D}$: The Measured Data
$\mathcal{M}$: The Model Set
$\mathcal{I}$: The Identification Method
$\mathcal{V}$: The Validation Procedure

The SI Flow; Model Structures $\mathcal{M}$



Models: General Aspects

- A model is a mathematical expression that describes the connections between measured inputs and outputs, and possibly related noise sequences.

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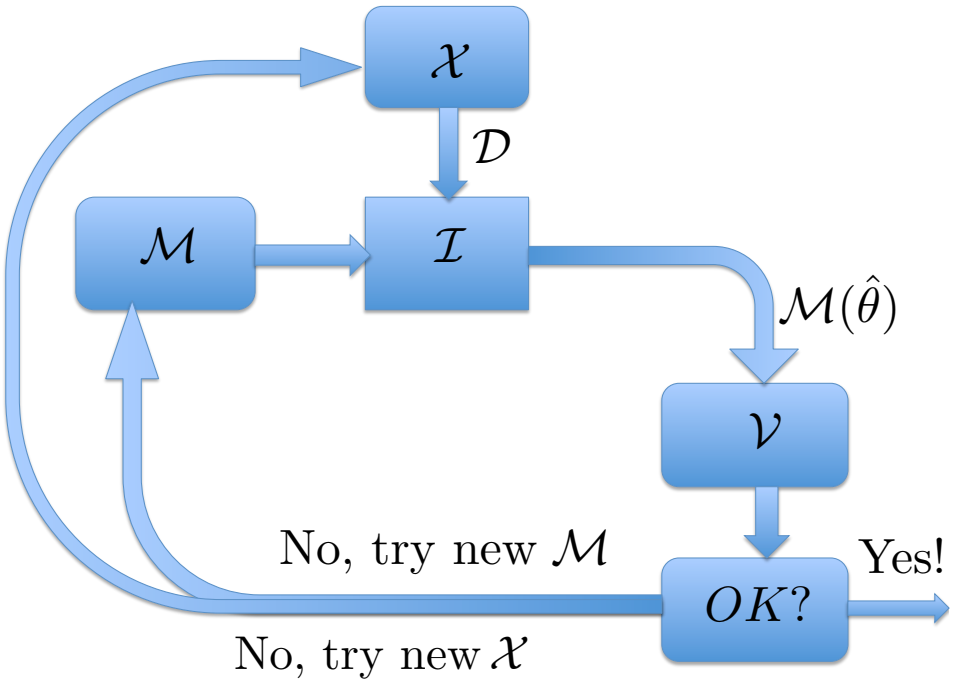
- A model is a mathematical expression that describes the connections between measured inputs and outputs, and possibly related noise sequences.
- They can come in many different forms
- Individual models in the structure are labeled with a parameter vector θ
- A common framework is to describe the model as a predictor of the next output, based on observations of past input-output data.
  Observed input–output (u, y) data up to time t: Z^t
  Model described by predictor: $\mathcal{M}(\theta) : \hat{y}(t|\theta) = g(t, \theta, Z^{t-1})$.

The SI Flow: Estimation: $\mathcal{I}$



A Pragmatic Fit Criterion

If a model, $\hat{y}(t|\theta)$, essentially is a predictor of the next output, is is natural to evaluate its quality by assessing how well it predicts: Form the *Prediction error* and measure its size:

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t|\theta), \quad \ell(\varepsilon(t,\theta))$$

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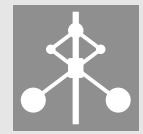
$$V_N(\theta) = \sum_{t=1}^{N} \varepsilon^2(t,\theta)$$

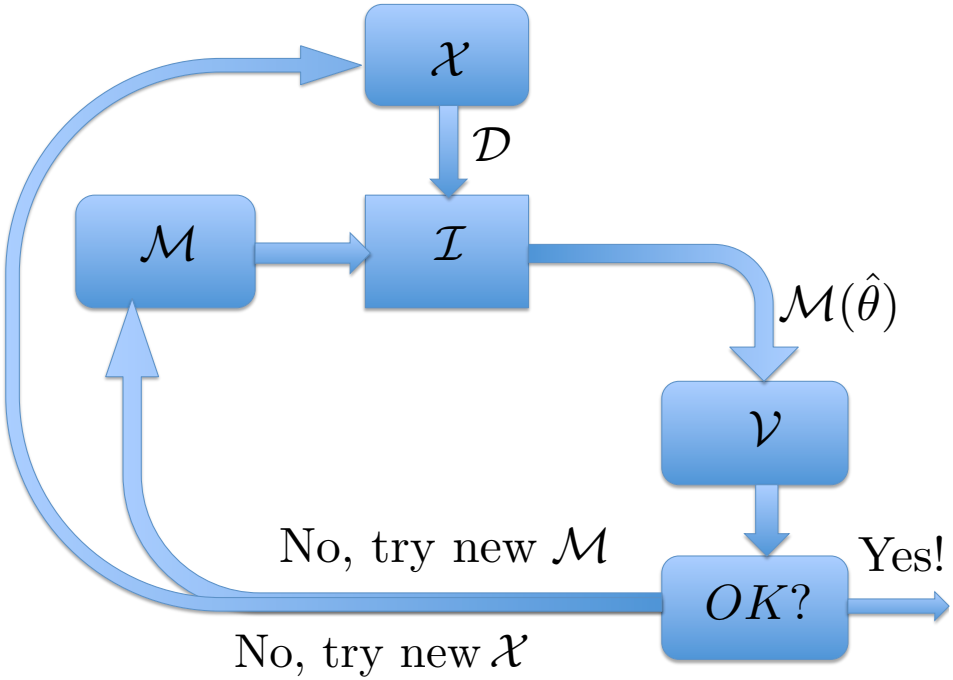
Which model in the structure performed best (Prediction Error Method, PEM)?

$$\hat{\theta}_N = \arg\min_{\theta \in D_{\mathcal{M}}} V_N(\theta)$$

(This is often also the Maximum Likelihood Estimate (MLE).)



Model Estimate Properties $\mathcal{M}(\hat{\theta}_N)$



Model Estimate Properties

As the number of data, N, tends to infinity

- $\hat{\theta}_N \to \theta^* \sim \arg\min_\theta E\varepsilon^2(t,\theta)$
- $\mathcal{M}(\theta^*)$ is the best possible predictor in $\mathcal{M}$

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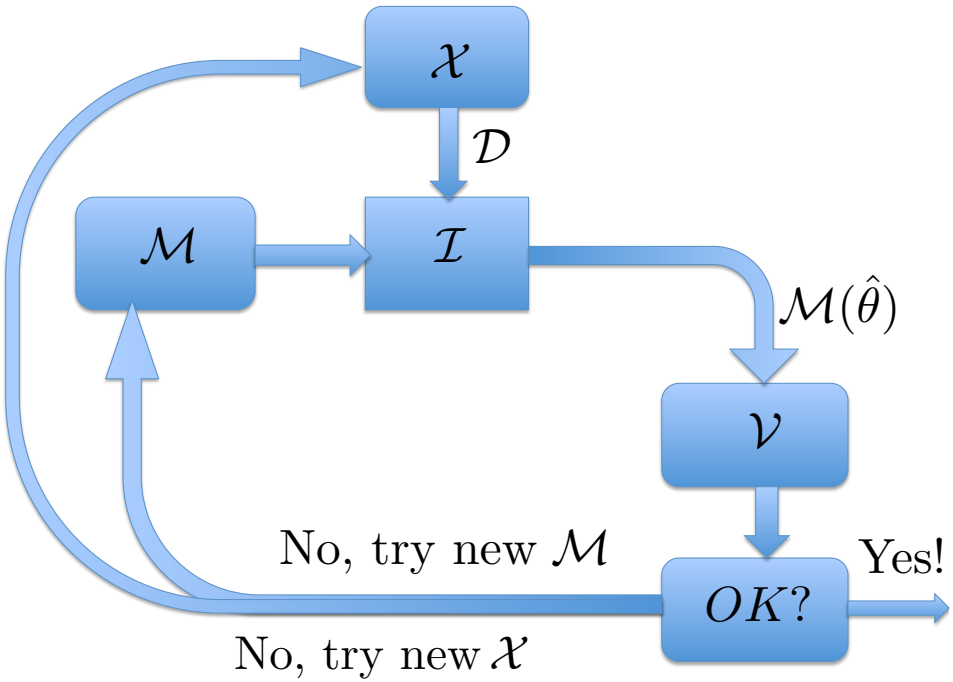
- The model structure is not large enough: The ML/PEM estimate converges to the best possible approximation of the system.
- "Best possible approximation" ...
- ... under the conditions of the experiment
- (If the model structure is large enough to contain a true description of the system, then the ML/PEM estimated model has (asymptotically) the best accuracy).

Experiment Design: $\mathcal{X}$



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Experiment Design - Basic Principle

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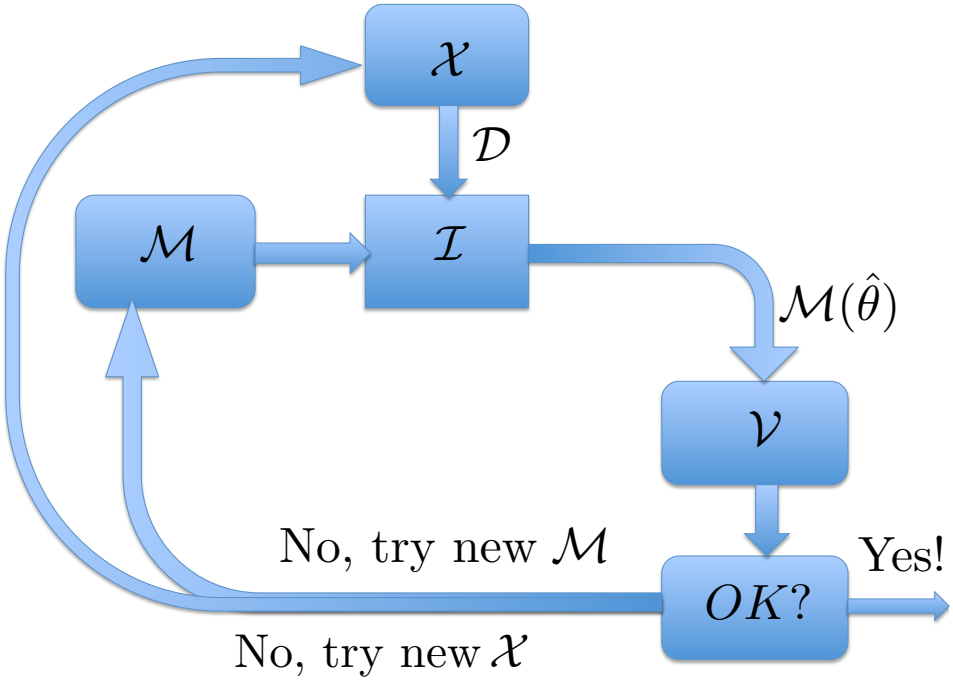
Then we just saw

$$\hat{\theta}_N \to \theta^*(\mathcal{X})$$

- The model $\mathcal{M}(\theta^*(\mathcal{X}))$ is the best approximation of the system under $\mathcal{X}$
- Let the experimental conditions resemble those under which the model is to be used!

Model Validation: $\mathcal{V}$



Model Validation Techniques

"Twist and turn" the model(s) to check if they are good enough for the intended application.

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Essentially a subjective decision. Several basic techniques are available:

1. Simulation (prediction) cross validation
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1. Simulation (prediction) cross validation
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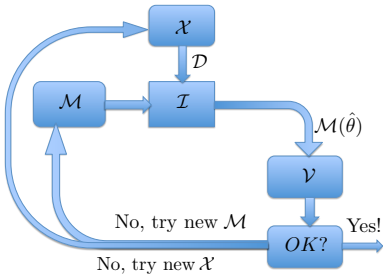
2. Residual Analysis
   - Are the residuals $\varepsilon(t, \hat{\theta}_N)$ (the "leftovers") unpredictable? They should not be correlated with anything we knew when estimating the model. Check correlation of the residuals with old inputs.

Rest Point



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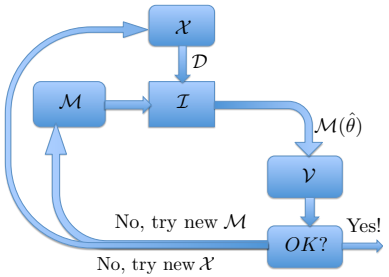

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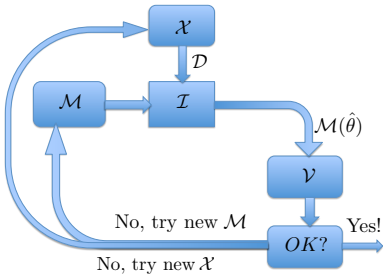

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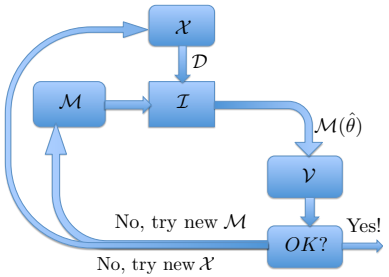

- So this is the System Identification Flow or Loop
- Several essential choices that have to be made, and often revised.
- Many of the choices have to be taken with the intended model use in mind and thus have a subjective flavour.
- Let us now turn to some specific illustrations for linear models (Requires some more mathematical background.)

More Technical Details: Linear Models

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General Description of a Linear Model

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The Prediction Errors

$$\varepsilon(t,\theta) = H^{-1}(q,\theta)[y(t) - G(q,\theta)u(t)]$$

Suppose the data is generated by a true linear system $G_0(q)$ and that the prediction errors are pre-filtered by a filter $L(q)$,

$$\hat{\theta}_N = \arg\min \sum (L(q)\varepsilon(t,\theta))^2$$

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$$\hat{\theta}_N \to \theta^* = \arg\min_\theta \int_{-\pi}^{\pi} |G(e^{i\omega},\theta) - G_0(e^{i\omega})|^2 Q(\omega) d\omega$$

$$Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega},\theta)|^2}$$

Asymptotic Properties for Linear Models

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$$Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega},\theta)|^2}$$

So the resulting model is closest to the true system in a norm defined by Q.

Note that we can affect $Q(\omega) = \frac{|L(e^{i\omega})|^2 \Phi_u(\omega)}{|H(e^{i\omega},\theta)|^2}$ by choosing $L - -(\mathcal{I})$, the input spectrum $\Phi_u - -(\mathcal{X})$ and the noise model $H - -(\mathcal{M})$, so it depends on all boxes in the identification chart.

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- Identification for simulation with an input u_* $\Rightarrow$ Make Q equal to the spectrum of u_*
- Identification for control $\Rightarrow$ Make Q large at the intended cross-over frequency ($\approx$ the intended bandwidth)

Conclusions

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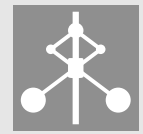

Conclusions

- Identification is a work-flow loop with nodes that contain essential user choices
- These may (should) depend on the intended use of the final model
- Concrete illustration for linear system models