# WP 5
# Control and Forecasting for Smart Energy Systems

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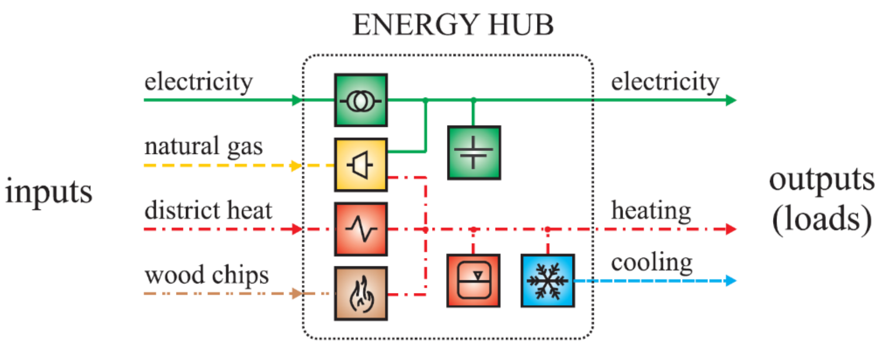
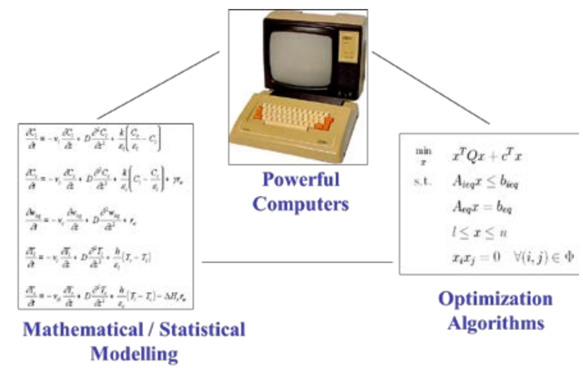
**CITIES Annual Meeting**
**May 30-31, 2017**
**Danish Technological Institute, Aarhus**

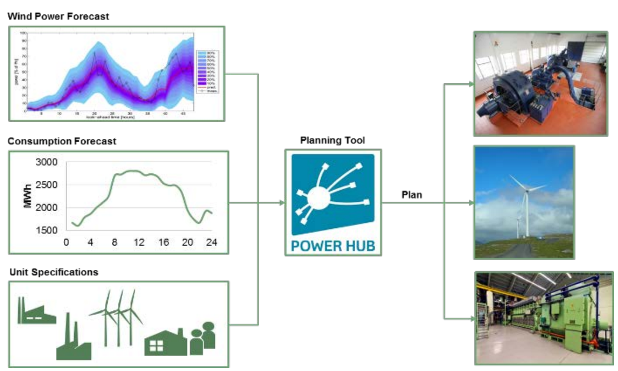
**DTU Compute**
Institut for Matematik og Computer Science

# Main Objectives

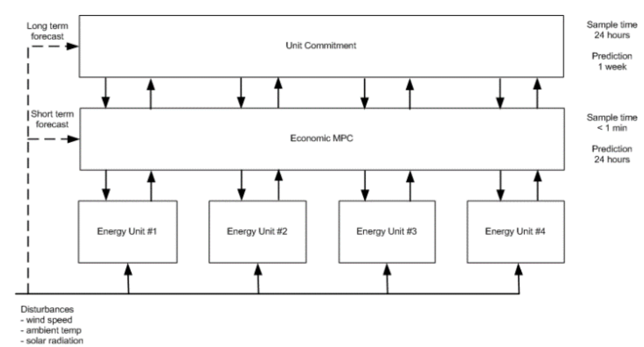
**Develop and test optimization and forecast based predictive control systems for more efficient and flexible operation of integrated energy systems.**



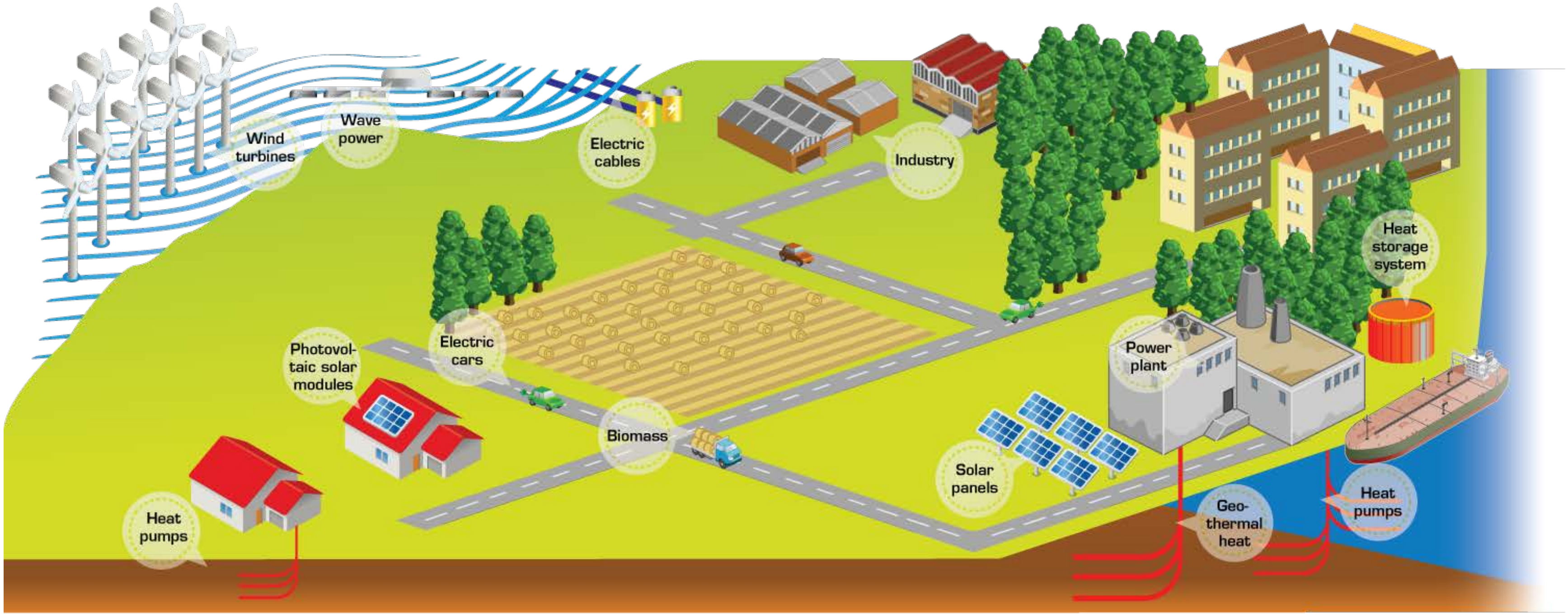




## Hierarchical Control Structure



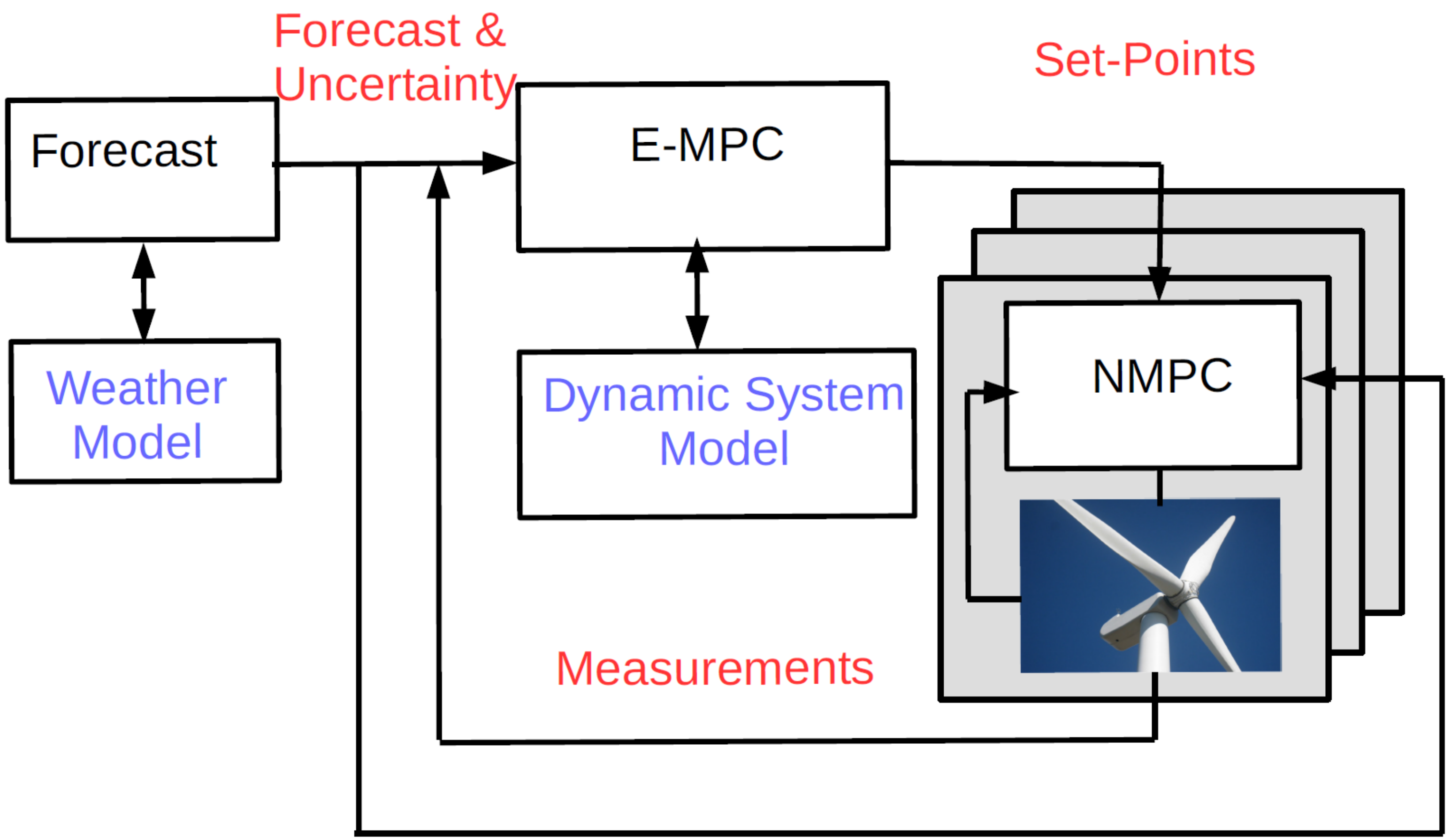
# ENERGY SYSTEMS

# Smart Energy Systems

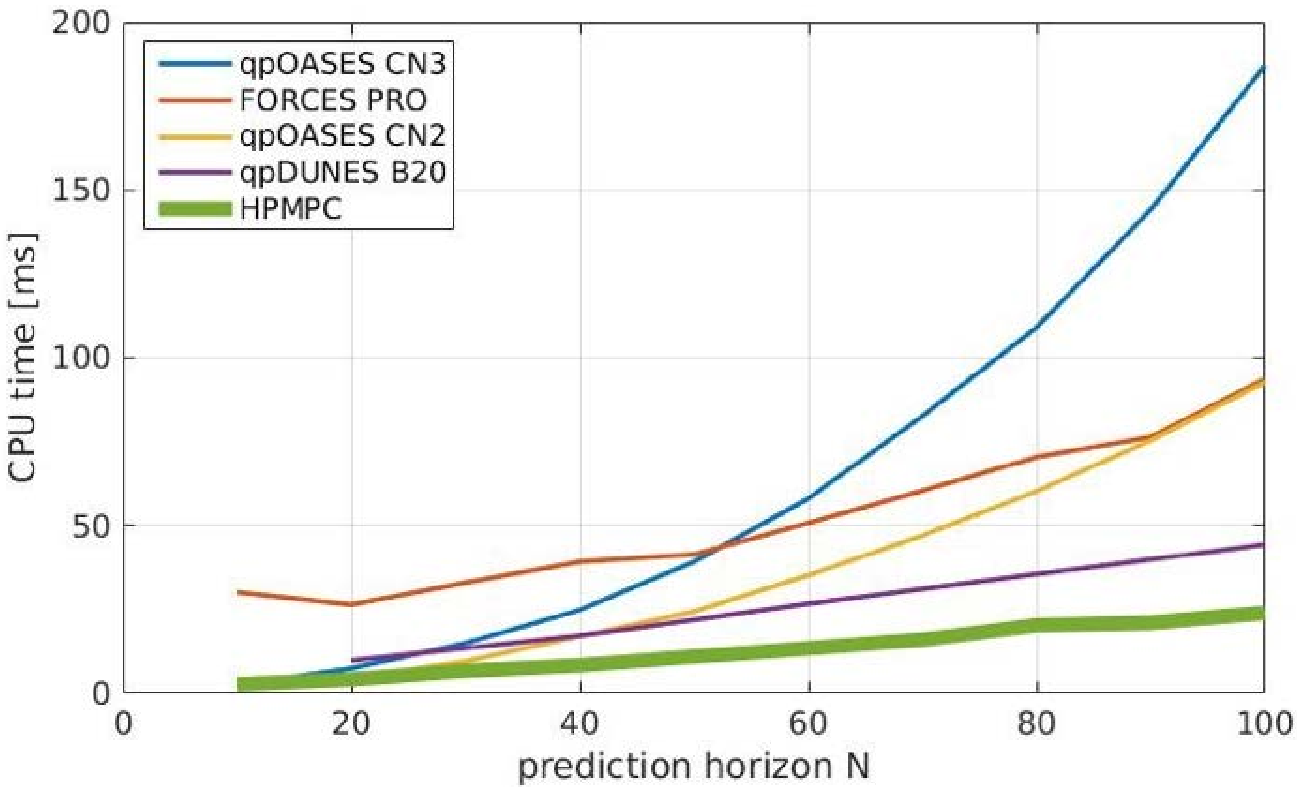



- **Thermal Storage**
  - Heating of floors etc
  - Heating of water accumulation tanks
  - Refrigeration Systems
- **Power / Heat Producers**
  - Wind Turbines
  - Photovoltaic Solar Modules
  - Solar Panels
  - CHP Plants
  - Fuel Cells

# Forecast Based Hierarchical MPC



# Computational Performance



# Drivers of MPC

**Powerful Computers**

$$\frac{\partial C_l}{\partial t} = -v_l \frac{\partial C_l}{\partial z} + D\frac{\partial^2 C_l}{\partial z^2} + \frac{k}{\varepsilon_l}\left(\frac{C_c}{\varepsilon_l} - C_l\right)$$

$$\frac{\partial C_c}{\partial t} = -v_c \frac{\partial C_c}{\partial z} + D\frac{\partial^2 C_c}{\partial z^2} + \frac{k}{\varepsilon_c}\left(C_l - \frac{C_c}{\varepsilon_l}\right) + \gamma r_w$$

$$\frac{\partial w_{lig}}{\partial t} = -v_c \frac{\partial w_{lig}}{\partial z} + D\frac{\partial^2 w_{lig}}{\partial z^2} + r_w$$

$$\frac{\partial T_l}{\partial t} = -v_l \frac{\partial T_l}{\partial z} + D\frac{\partial^2 T_l}{\partial z^2} + \frac{h}{\varepsilon_l}(T_c - T_l)$$

$$\frac{\partial T_c}{\partial t} = -v_{cl} \frac{\partial T_c}{\partial z} + D\frac{\partial^2 T_c}{\partial z^2} + \frac{h}{\varepsilon_c}(T_l - T_c) - \Delta H_r r_w$$

**Mathematical / Statistical Modelling**

$$\begin{aligned} \min_x \quad & x^T Q x + c^T x \\ \text{s.t.} \quad & A_{ieq} x \le b_{ieq} \\ & A_{eq} x = b_{eq} \\ & l \le x \le u \\ & x_i x_j = 0 \quad \forall (i,j) \in \Phi \end{aligned}$$

**Optimization Algorithms**

# Economic MPC
# Mathematical Optimization

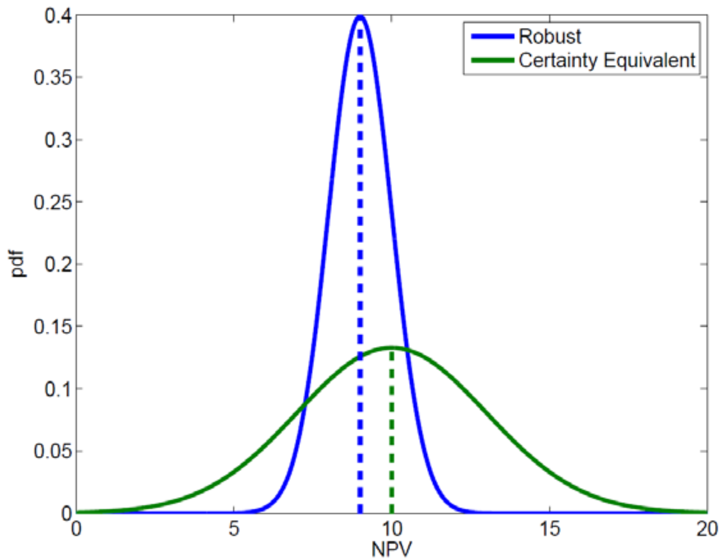
The portfolio power generation problem can be stated as

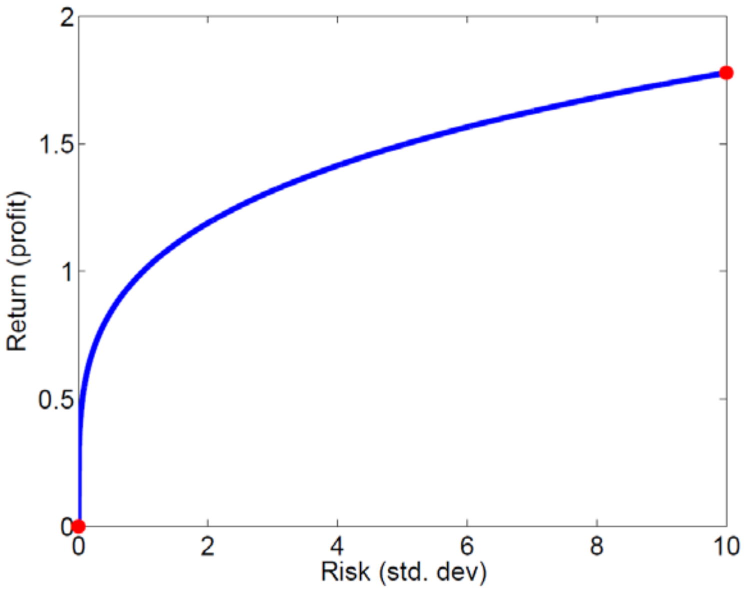
$$
\begin{aligned}
\min_{\{u_k\}_{k=0}^{N-1}} \quad & \phi = \sum_{k=0}^{N-1} c' u_k \\
s.t. \quad & x_{k+1} = A x_k + B u_k + E d_k && k = 0, 1, \ldots, N-1 \\
& y_k = C x_k && k = 1, 2, \ldots, N \\
& u_{\min} \leq u_k \leq u_{\max} && k = 0, 1, \ldots, N-1 \\
& \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} && k = 0, 1, \ldots, N-1 \\
& y_k \geq r_k && k = 1, 2, \ldots, N
\end{aligned}
$$

The parameters for this problem are

- Initial state, $x_0$, and previous decision, $u_{-1}$
- Predicted loads on non-controllable generators (e.g. wind speed on wind turbines): $\{d_k\}_{k=0}^{N-1}$
- Predicted power demand: $\{r_k\}_{k=1}^{N}$

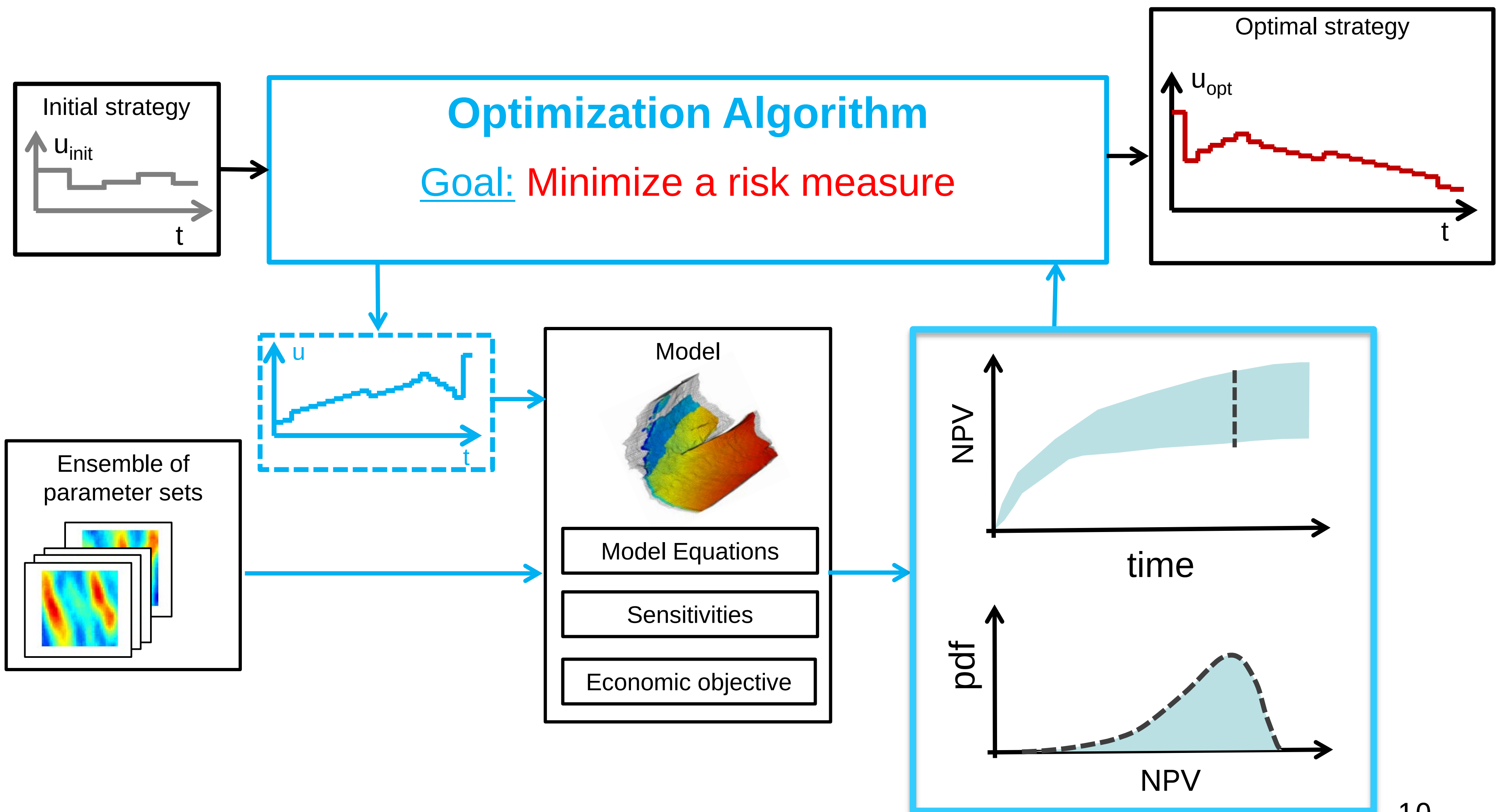
# Economic MPC for Uncertain Systems



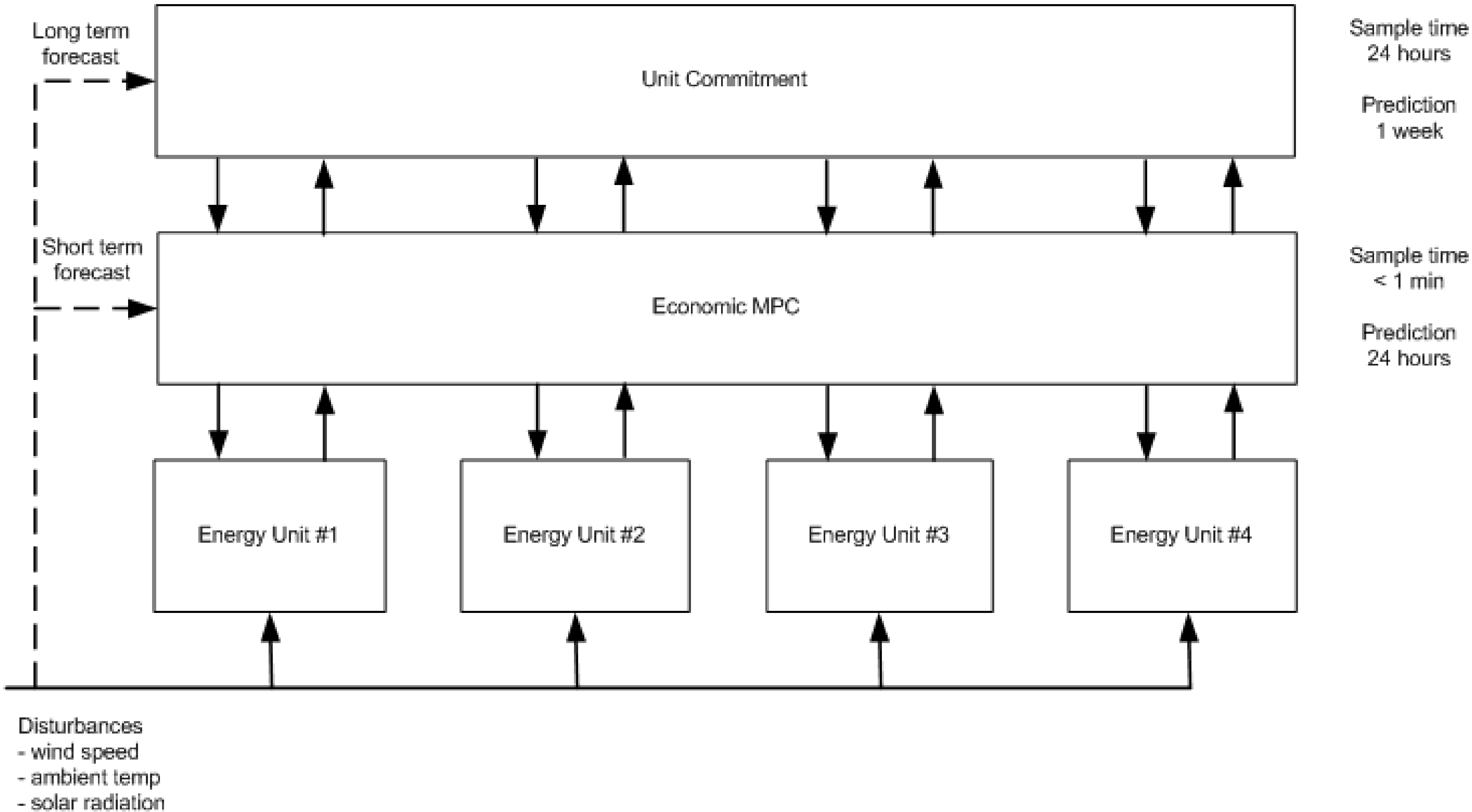




$$\max_{x \in \mathbb{R}^n} \quad \psi(x) = \alpha \overbrace{E_\theta \left\{ R(x,\theta) \right\}}^{\text{Mean profit}} - (1-\alpha) \overbrace{V_\theta \left\{ R(x,\theta) \right\}}^{\text{Profit variance}}$$

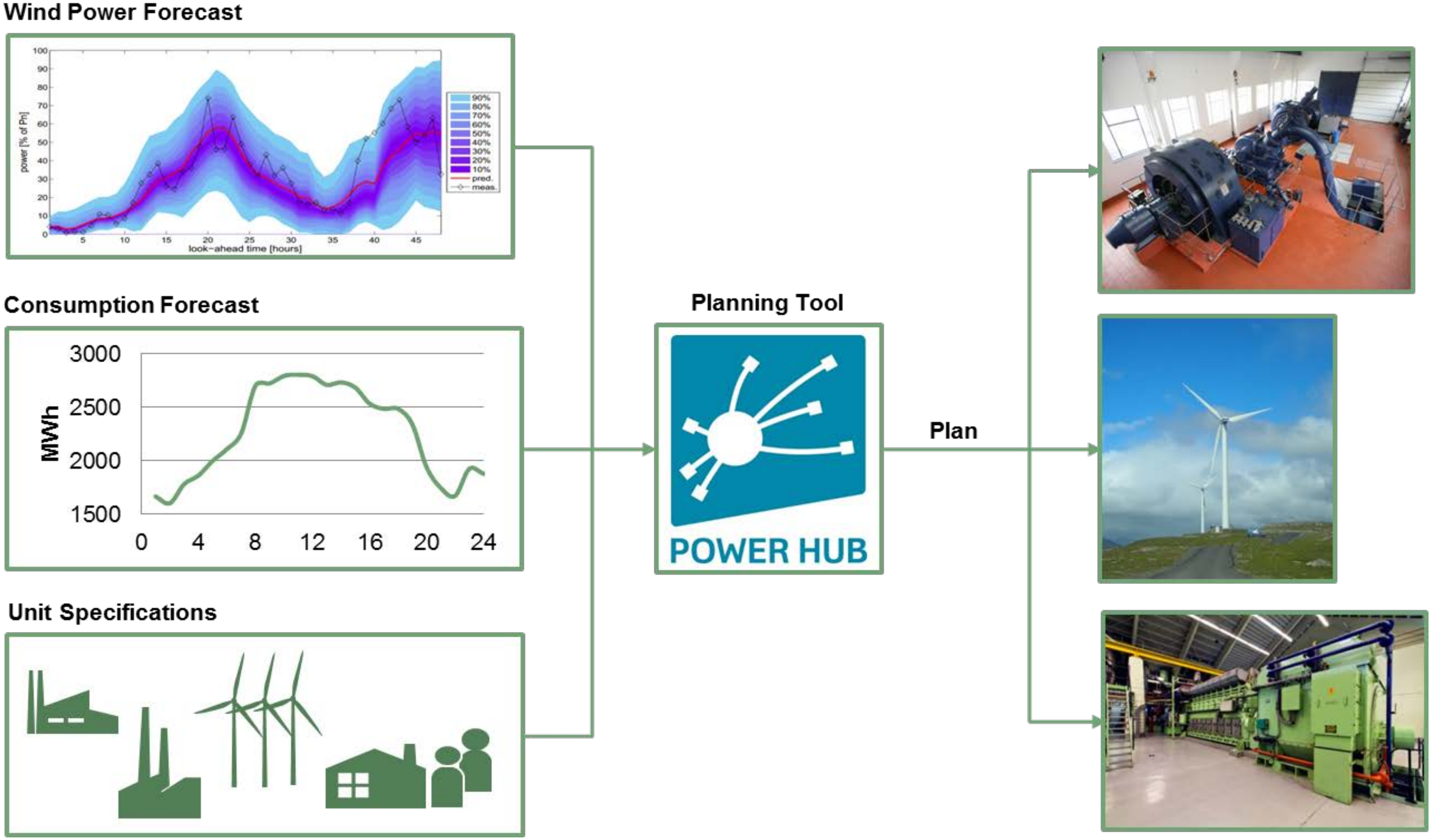
# Conditional Value at Risk (CVaR)



# Hierarchical Control Structure



# Control of Smart Energy Systems = Economic MPC

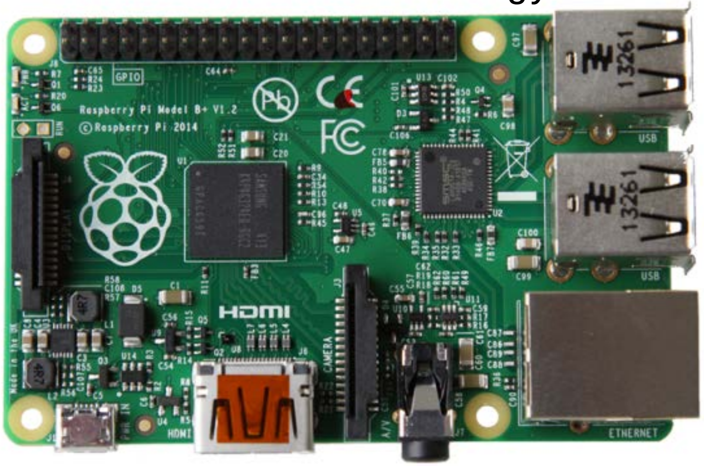


# ENERGY UNITS

# Control of Individual Energy Units

**Raspberry Pi**

Embedded MPC Algorithms for control of individual energy units



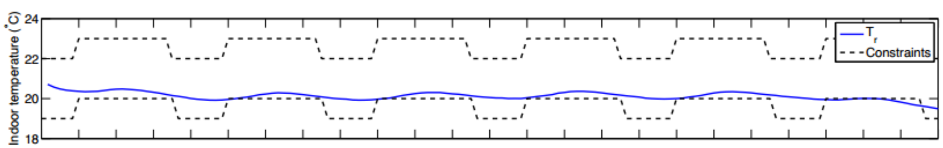

$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \sum_{k=0}^{N-1} l_k(x_k, u_k) + l_N(x_N) \tag{1a}$$

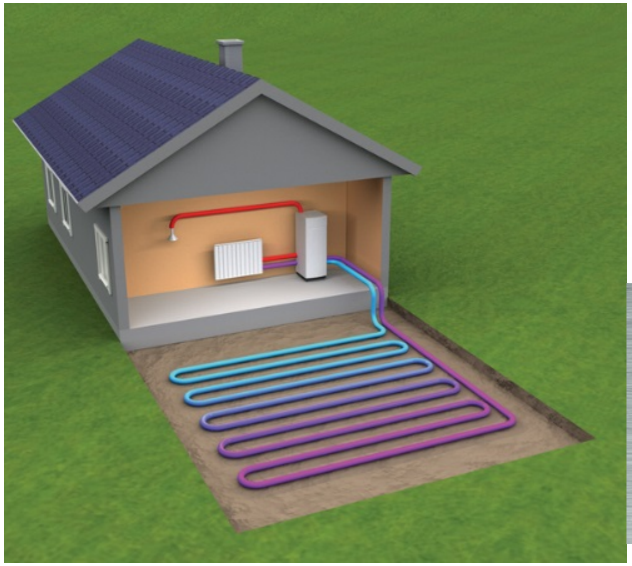
$$s.t. \quad x_{k+1} = A_k x_k + B_k u_k + b_k \quad k \in \mathcal{N} \tag{1b}$$

with $\mathcal{N} = \{0, 1, \ldots, N-1\}$ and stage costs defined by

$$l_k(x_k, u_k) = \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}' \begin{bmatrix} Q_k & M_k' \\ M_k & R_k \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \begin{bmatrix} q_k \\ s_k \end{bmatrix}' \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \rho_k \tag{2a}$$

$$l_N(x_N) = \frac{1}{2} x_N' P_N x_N + p_N' x_N + \gamma_N \tag{2b}$$

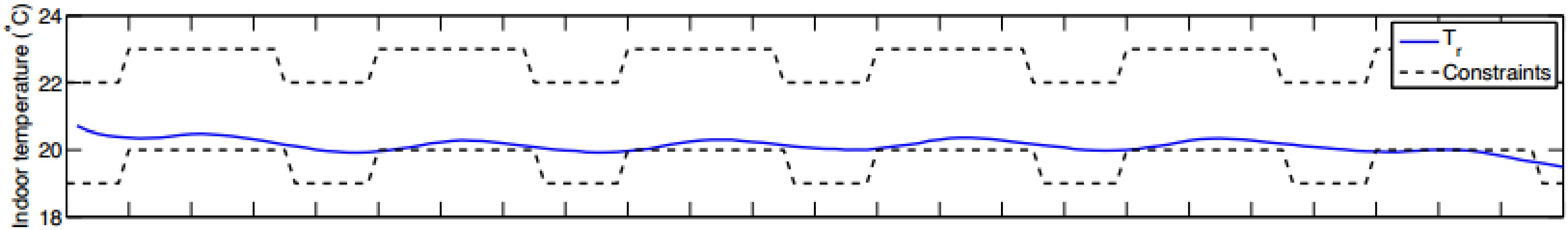



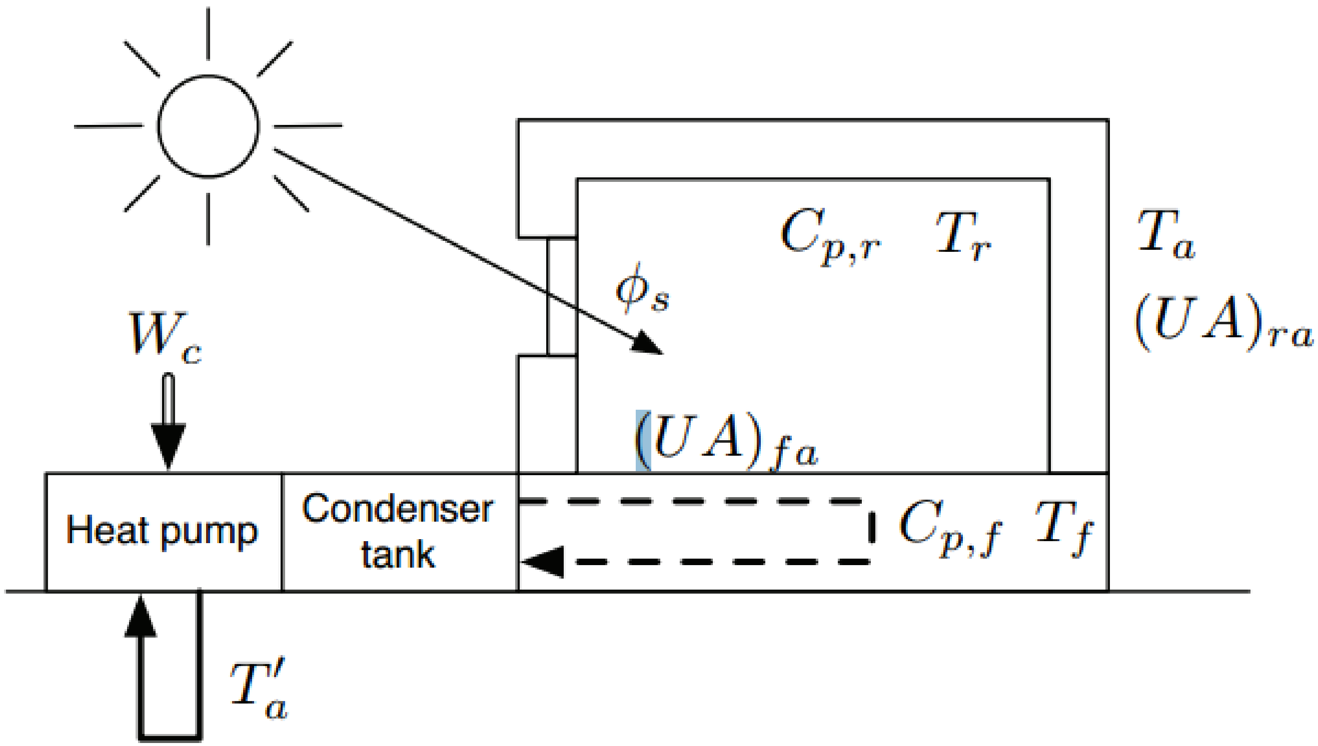

# Temperature Control

- Danfoss


- NEST (Google)

# Schematics of House with Heat Pump



# Model of a ground-source heated building

*Energy conservation*

$$C_{pr}\dot{T}_r = Q_{fr} - Q_{ra} + (1-p)\phi_{sol}$$

$$C_{pf}\dot{T}_f = Q_{wf} - Q_{fr} + p\phi_{sol}$$

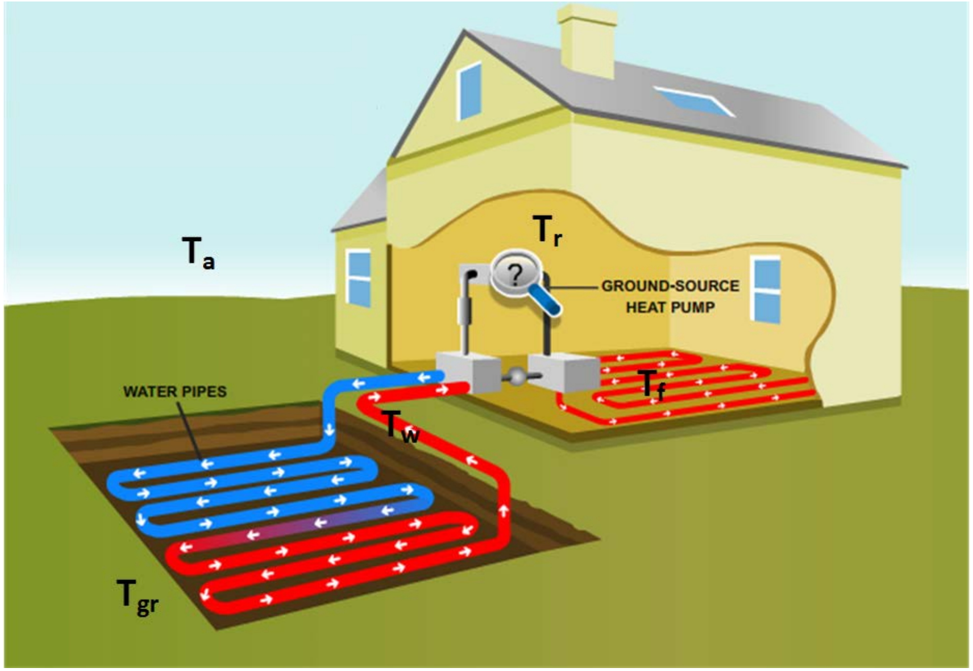
$$C_{pw}\dot{T}_w = Q_c - Q_{wf}$$

*Heat Transfer*

$$Q_{fr} = UA_{fr}(T_f - T_r)$$

$$Q_{ra} = UA_{wf}(T_w - T_f)$$

$$Q_{fr} = UA_{ra}(T_r - T_a)$$



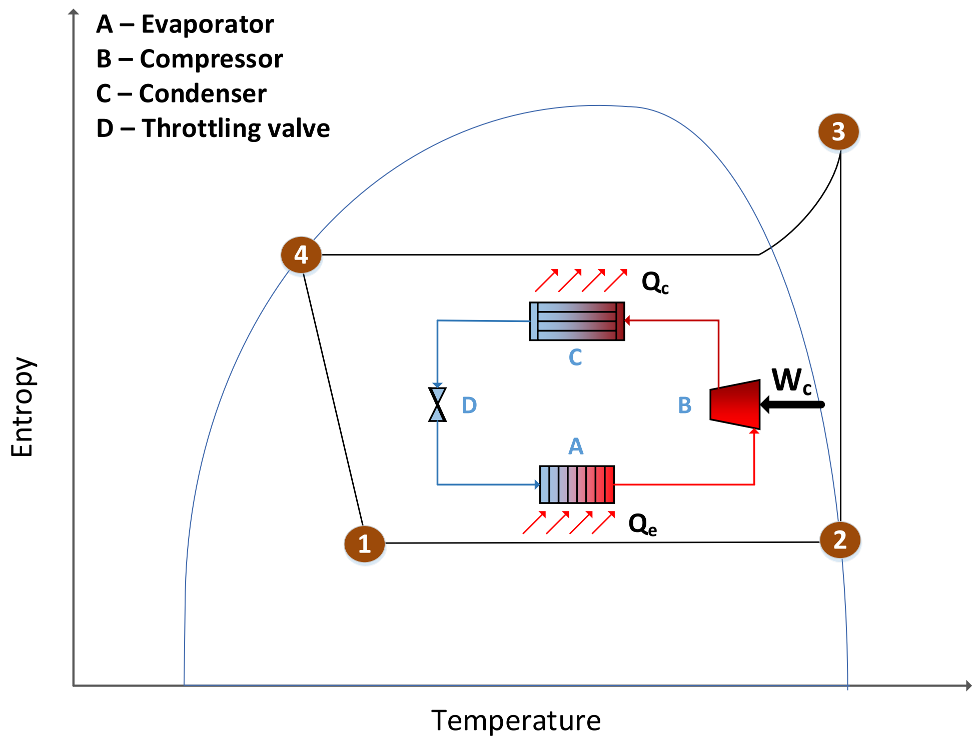
# Rigorous heat pump model

1 – 2: Isobar evaporation (ΔP = 0)
2 – 3: Isentropic compression (ΔS = 0)
3 – 4: Isobar condensation (ΔP = 0)
4 – 1: Isenthalpic expansion (ΔH = 0)

$$W_c = \frac{Q_c}{\eta COP(T_w, T_{gr})}$$

$$COP = \frac{h_3(T_3, P_4) - h_4(T_w + \Delta T, P_4)}{h_3(T_3, P_4) - h_2(T_{gr} + \Delta T, P_2)}$$

**COP is a nonlinear function of $T_w$ and $T_{gr}$**




**Realistic Thermodynamics needed**

www.psetools.org

# Rigorous heat pump model

**Algorithm 1:** COP of a heat pump

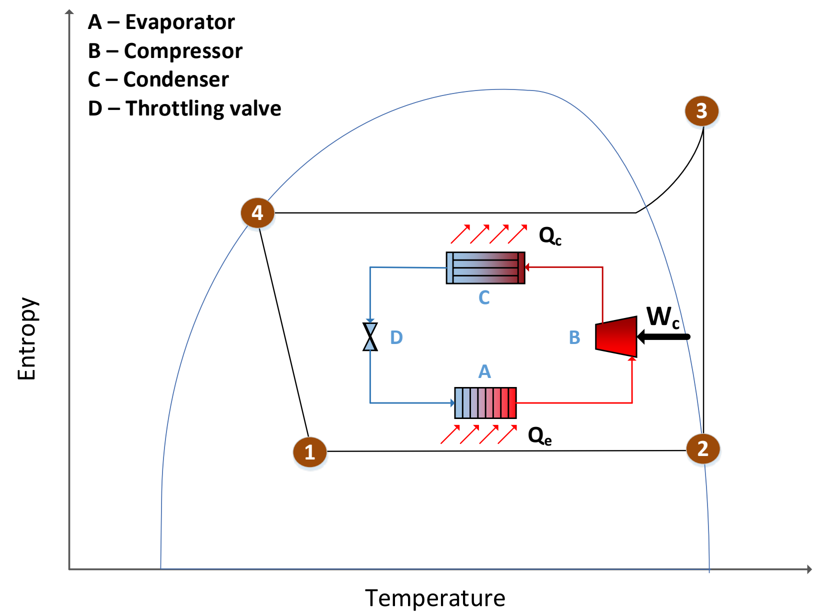
**Require:**

Ground temperature, $T_{gr}$
Water tank temperature, $T_w$
Heat exchanger temperature difference, $\Delta T$

**Compute using ThermoLib:**

Evaporator pressure, $P_2 = p_{sat}(T_2)$, where
$T_2 = T_{gr} - \Delta T$
Enthalpy out from evaporator, $h_2 = h(T_2, P_2)$
Entropy out from evaporator, $s_2 = s(T_2, P_2)$
Condenser pressure, $P_4 = p_{sat}(T_4)$, where
$T_4 = T_w + \Delta T$
Solve $s_2 - s_3 = 0$ for $T_3$, where $s_3 = s(T_3, P_4)$
Enthalpy out from compressor, $h_3 = h(T_3, P_4)$
Enthalpy out from condenser, $h_4 = h(T_4, P_4)$

$$COP = \frac{h_3(T_3, P_4) - h_4(T_w + \Delta T, P_4)}{h_3(T_3, P_4) - h_2(T_{gr} + \Delta T, P_2)}$$




**COP is a nonlinear function of $T_w$ and $T_{gr}$**

# Economic MPC

$$
\begin{aligned}
\min_{\hat{u}} \quad & \phi = \int_{t_0}^{t_f} (C(\hat{x}(t), \hat{u}(t), \hat{d}(t)) + V(\hat{y}(t)))dt, \\
\text{s.t.} \quad & \hat{x}(t_0) = \bar{x}_{k|k}, \quad k \geq 0 \\
& \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + E\hat{d}(t), \quad t \in [t_0, t_f] \\
& \hat{y}(t) = C\hat{x}(t), \quad t \in [t_0, t_f] \\
& u_{min}(t) \leq \hat{u}(t) \leq u_{max}(t), \quad t \in [t_0, t_f] \\
& \Delta u_{min}(t) \leq \Delta\hat{u}(t) \leq \Delta u_{max}(t), \quad t \in [t_0, t_f] \\
& y_{min}(t) - v(t) \leq \hat{y}(t), \quad t \in [t_0, t_f] \\
& y_{max}(t) + v(t) \geq \hat{y}(t), \quad t \in [t_0, t_f] \\
& v(t) \geq 0, \quad t \in [t_0, t_f]
\end{aligned}
$$

Energy cost function

Initial estimate from a Kalman filter

Linear house model

Bound constraints

Rate of change constraints

Soft thermal comfort constraints

$$\hat{x} = [T_r \quad T_f \quad T_w]^T \qquad \hat{u} = Q_c$$

$$\hat{y} = T_r$$

$$\hat{d} = [T_a \quad \phi_{sol}]^T$$

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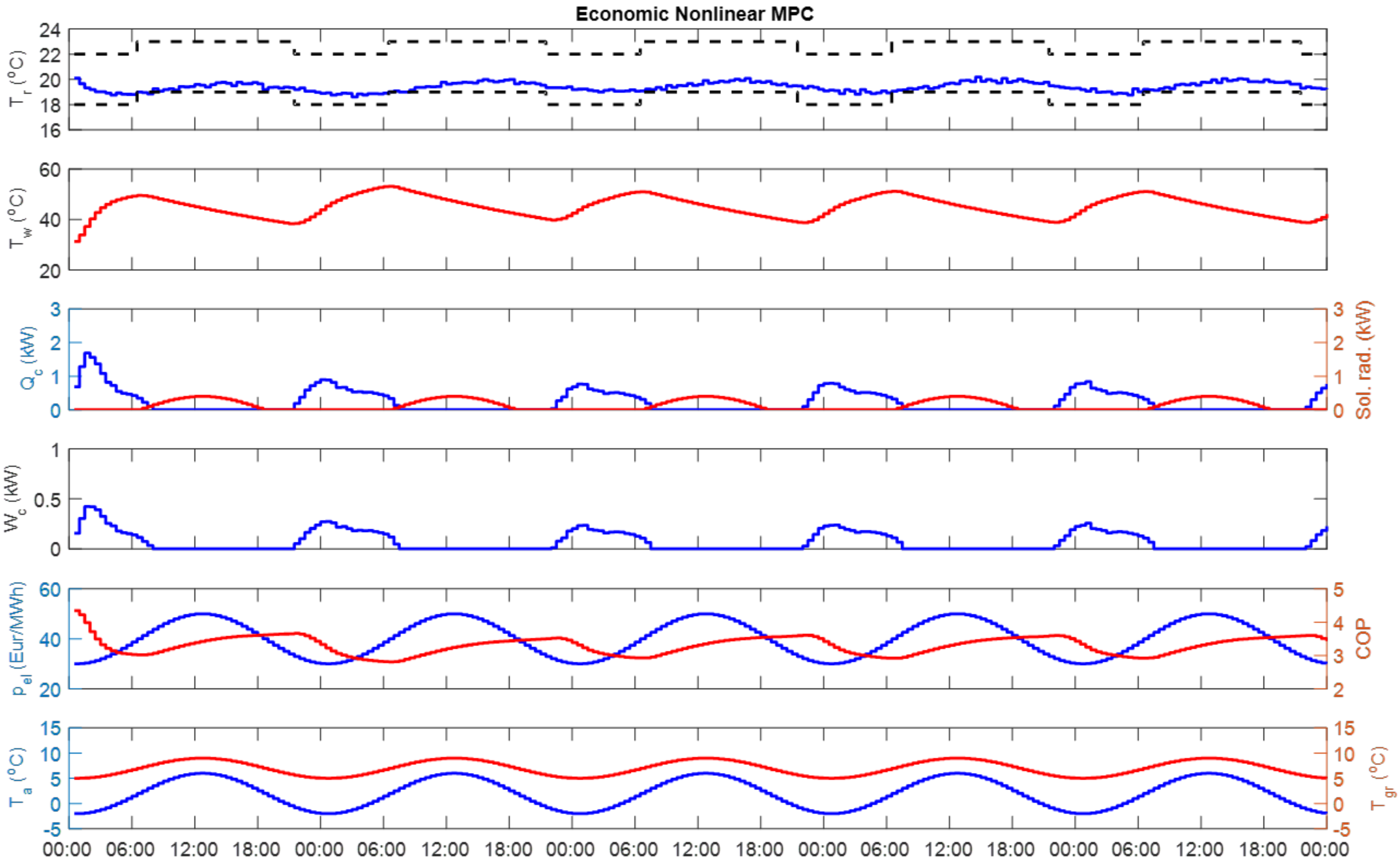
# Economic MPC

| Special nonlinear economic MPC (NMPC) | Linear economic MPC (LMPC) | Linear economic MPC constant COP (LMPC w./ COP = 4.5) |
|---|---|---|

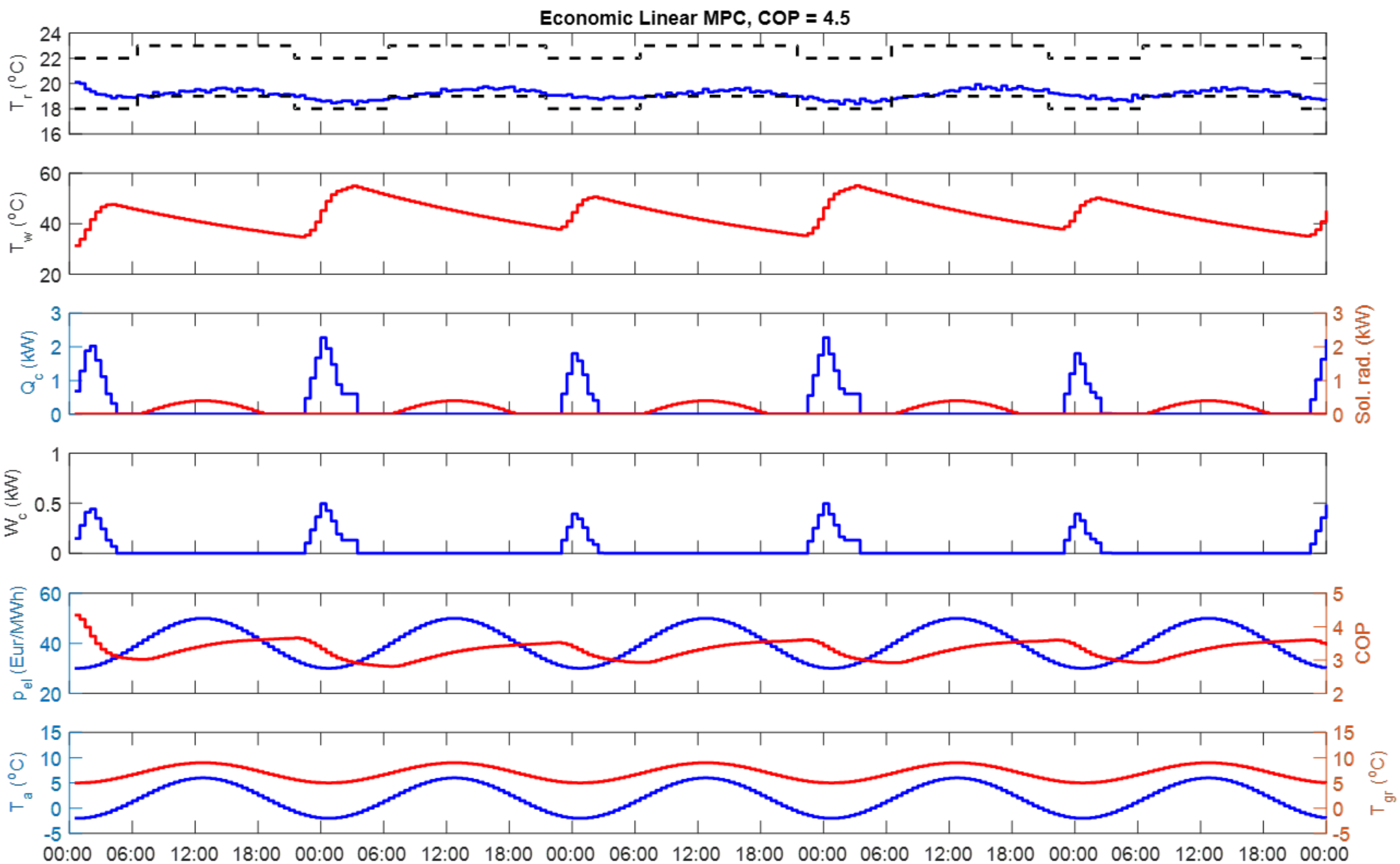
Energy cost function: $C\left(\hat{x}, \hat{u}, \hat{d}\right) = p_{el}(t)W_c(t)$

| | | |
|---|---|---|
| $$W_c = \frac{Q_c}{\eta COP(T_w, T_{gr})}$$ | $$W_c = \frac{Q_c}{\eta COP}$$ | $$W_c = \frac{Q_c}{\eta COP}$$ |
| COP is calculated using estimates of $T_w$ and forecasts of $T_{gr}$ | COP is recalculated at each iteration step | COP is constant (COP = 4.5) |

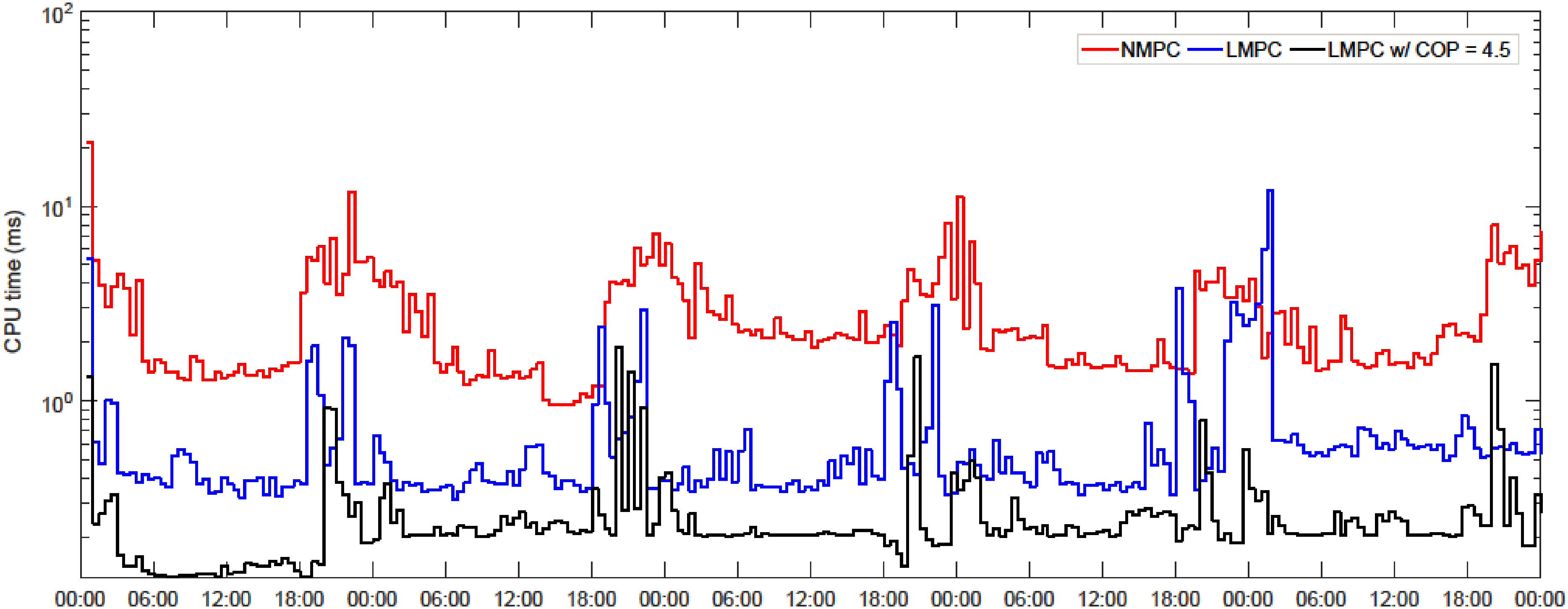
# Economic Nonlinear MPC



# Economic Linear MPC – Constant COP

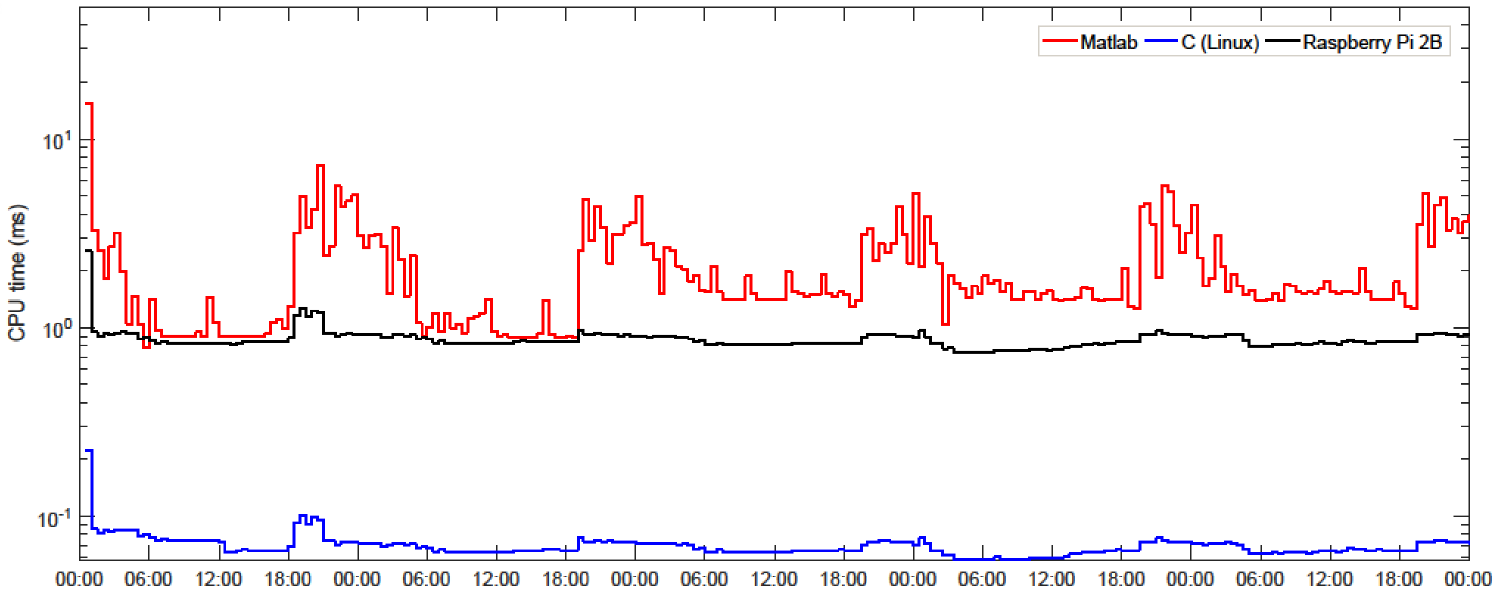


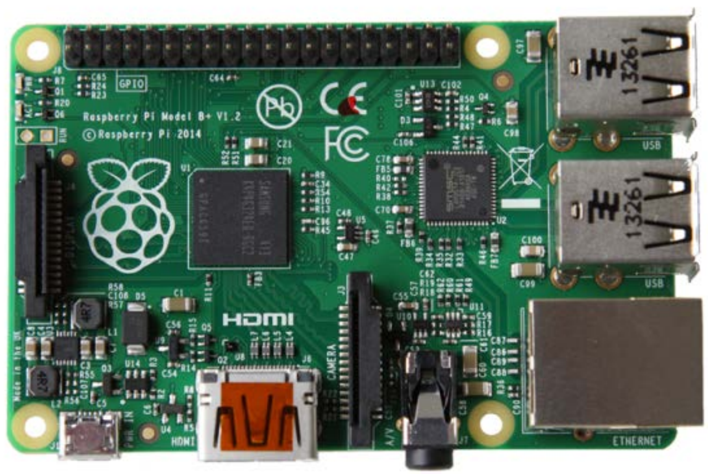
# Computational performance: NMPC vs LMPC




NMPC and LMPC have comparable computation time

# Computational performance: Matlab vs Linux-C



| Simulation platform | Cold start (ms) | Average warm start (ms) |
|---|---|---|
| Raspberry Pi 2B | 2.5 | 0.86 |
| Workstation (C) | 0.22 | 0.069 |
| Workstation (Matlab) | 15.51 | 2.07 |

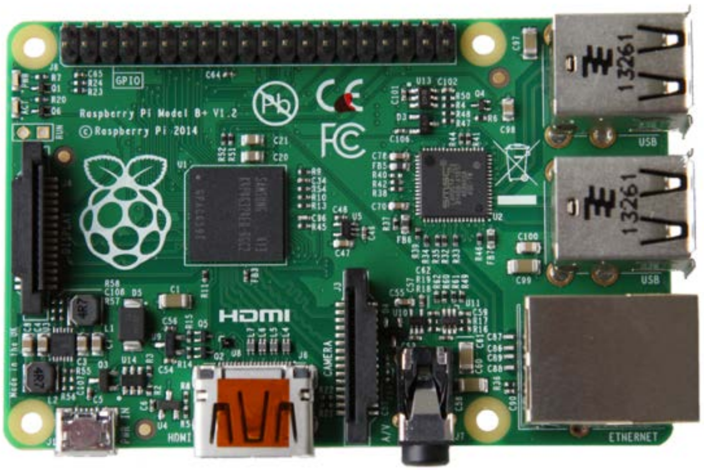
# MPC & Process Optimization Toolbox

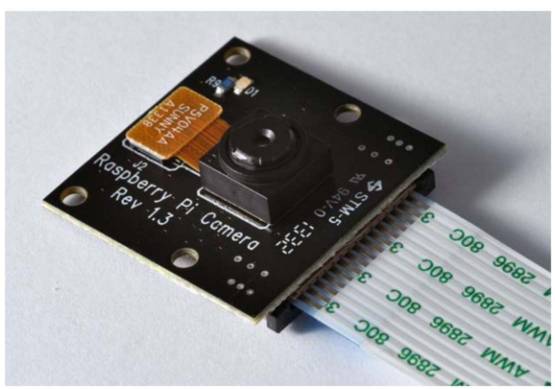
**Features**

- Simplicity - easy to
  - Commission
  - Tune
  - Maintain
- Customizable and adaptable to
  - Process dynamics
  - Process modifications
  - Operational strategies
- Includes frontier technologies in
  - Mathematical Optimization
  - Process Control
  - Software Engineering
  - Mathematical/Statistical Modeling and Simulation

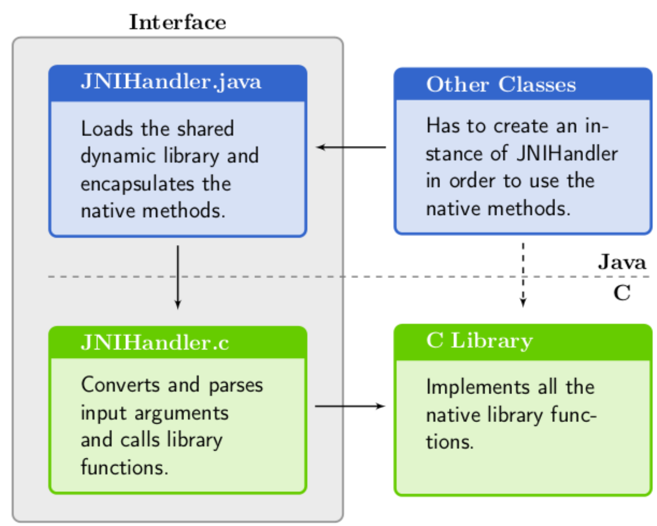
# Software Implementation

## Rasperry PI  C/C++




## Smart Phone  C/Java/Matlab

# Thank You – Q & A

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**DTU Compute**

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