A Data-driven Bidding Model for a Cluster of Price-responsive Consumers of Electricity

IEEE International Energy Conference, Leuven, Belgium

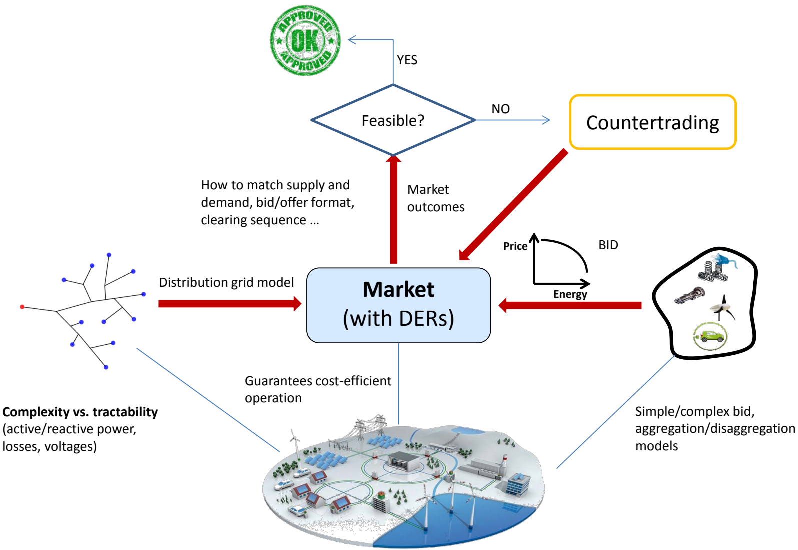
J. M. Morales [1] J. Saez-Gallego [1] M. Zugno H. Madsen [1]

[1] Technical University of Denmark

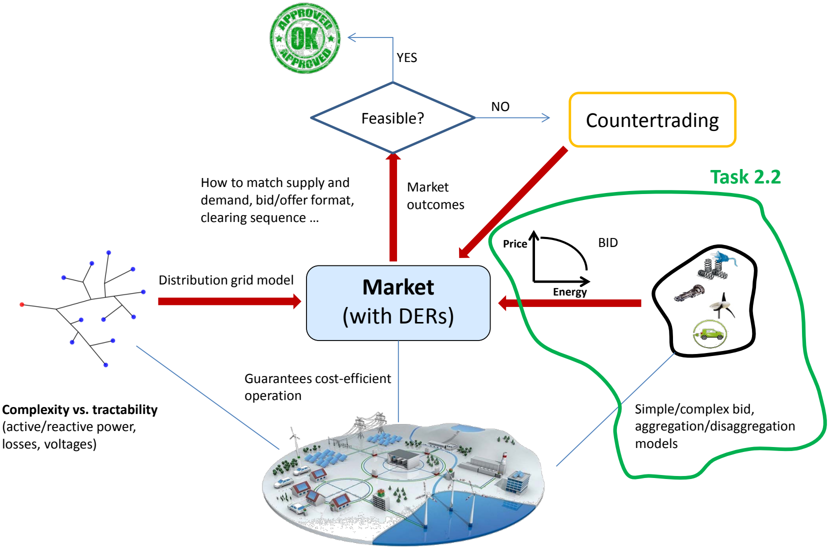
April 7^{th}, 2016

Work funded by the Centre for IT-Intelligent Energy Systems in cities (CITIES, 1035-00027B, http://smart-cities-centre.org/) and connected to the EU-H2020 project SmartNet "Smart TSO-DSO interaction schemes, market architectures and ICT Solutions for the integration of ancillary services from demand side management and distributed generation"

The Context: SmartNet



The Context: SmartNet



Outline

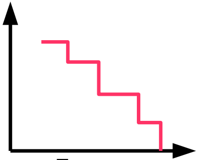
- Motivation
- Capturing the consumers' price-response in a market bid
- Defining the estimation problem
- Solving the estimation problem
- Case study: The Olympic Peninsula experiment
- Concluding remarks

The Market Bid

The bid should capture the price-response of the aggregation of flexible consumers

Parameters θ of the *complex* bid:

- Step-wise marginal utility function ($a_{b,t}$)



- Maximum load pick-up and drop-off limits (r_t^u,r_t^d) (similar to the ramping limits of a conventional generating unit)
- Maximum and minimum power consumption ($\overline{P}_t$,$\underline{P}_t$)

The Market Bid ⇔ The Price-Response Model

Total consumption: $\underline{P}_t + \sum_{b \in \mathcal{B}} x_{b,t}$

$$\underset{x_{b,t}}{\text{Maximize}} \sum_{t \in \mathcal{T}} \left(\sum_{b \in \mathcal{B}} a_{b,t} x_{b,t} - p_t \sum_{b \in \mathcal{B}} x_{b,t} \right)$$

Subject to

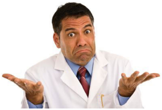
$$\underline{P}_t + \sum_{b \in \mathcal{B}} x_{b,t} - \underline{P}_{t-1} - \sum_{b \in \mathcal{B}} x_{b,t-1} \leq r_t^u \qquad t \in \mathcal{T}_{-1} \qquad (1a)$$

$$\underline{P}_{t-1} + \sum_{b \in \mathcal{B}} x_{b,t-1} - \underline{P}_t - \sum_{b \in \mathcal{B}} x_{b,t} \leq r_t^d \qquad t \in \mathcal{T}_{-1} \qquad (1b)$$

$$0 \leq x_{b,t} \leq \frac{\overline{P}_t - \underline{P}_t}{B} \qquad b \in \mathcal{B}, t \in \mathcal{T} \qquad (1c)$$

The Market Bid ⇔ The Price-Response Model

How do we determine the set of characteristic parameters θ that define the market bid?



The Estimation Problem

- We estimate the bid parameters θ from observational price-consumption data

Time	Price	Load
t_1	p_1	x_1^{meas}
t_2	p_2	x_2^{meas}
...	...	...

- **Inverse optimization**: the parameters of the bid are the parameters of an optimization problem
- We cast the inverse optimization problem as a **bilevel programming problem**

The Estimation Problem as a Bilevel Program

Upper-level problem

$$\underset{x,\theta}{\text{Minimize}} \ \|x - x^{meas}\|$$

$s.t.$ Constraints on bid parameters

Lower-level problem

$$\underset{x}{\text{Maximize}} \ \text{Utility}(a_b) \ - \text{Cost}$$

$s.t.$ Power bounds $(\overline{P}, \underline{P})$

Maximum pick-up rate (r^u)

Maximum drop-off rate (r^d)

$$\theta = \{a_b, r^u, r^d, \overline{P}, \underline{P}\}$$

Equivalent Single-level Optimization Problem

Parameter estimation

$$\underset{x,\theta}{\text{Minimize}} \sum_{t\in\mathcal{T}} w_t \left| \underline{P}_t + \sum_{b\in\mathcal{B}} x_{b,t} - x_t^{meas} \right|$$

subject to

$$a_{b,t} \geq a_{b+1,t} \qquad b \in \mathcal{B}, t \in \mathcal{T}$$

KKT conditions of lower-level problem

Equivalent Single-level Optimization Problem

Parameter estimation

$$\underset{x,\theta}{\text{Minimize}} \sum_{t\in\mathcal{T}} w_t \left| \underline{P}_t + \sum_{b\in\mathcal{B}} x_{b,t} - x_t^{meas} \right|$$

subject to

$$a_{b,t} \geq a_{b+1,t} \quad b \in \mathcal{B}, t \in \mathcal{T}$$

KKT conditions of lower-level problem

The weight w_t has a threefold purpose:

1. If the market bid is intended for a forward (e.g., day-ahead) market, then w_t could represent the cost of imbalances at time t
2. The most recent observations can be given larger weights
3. Zero weight for missing or wrong measurements

Including Auxiliary Information (Features Z)

Time	Price	Load	External Info.
t_1	p_1	x_1^{meas}	z_1
t_2	p_2	x_2^{meas}	z_2
...	...	...	...

Generalized framework for inverse optimization:

- x^{meas} needs not be optimal or even feasible for the lower-level problem
- Auxiliary information on features is leveraged (affine dependence)

Upper-level problem

$$\underset{x,\theta(Z)}{\text{Minimize}} \ \|x - x^{meas}\|$$

$$s.t. \quad \theta(Z) \in \Xi, \forall Z$$

Lower-level problem

$$\underset{x}{\text{Maximize}} \ \text{Utility} \ (a_b(Z)) \ - \text{Cost}$$

$$s.t. \quad \text{Power bounds} \ \left(\overline{P}(Z), \underline{P}(Z)\right)$$

$$\text{Maximum pick-up rate} \ (r^u(Z))$$

$$\text{Maximum drop-off rate} \ \left(r^d(Z)\right)$$

Two-step Procedure

Step 1: Solve a linear relaxation of the estimation problem (which is an MPEC)

Step 2: Recompute the parameters defining the utility function with the parameters defining the constraints of the lower-level problem fixed at the values estimated in Step 1

L-Penalty Method

We relax the complementarity conditions [Siddiqui and Gabriel, 2013]

$$\underset{x,\lambda}{\text{Minimize}}\ \mathrm{cx} \qquad \mathrm{Ax} - \mathrm{b} \geq 0 \perp \lambda \geq 0 \quad \Longrightarrow \quad \underset{x,\lambda}{\text{Minimize}}\ \mathrm{cx} + \mathrm{L}(\mathrm{Ax} - \mathrm{b} + \lambda) \qquad \mathrm{Ax} - \mathrm{b} \geq 0 \qquad \lambda \geq 0$$

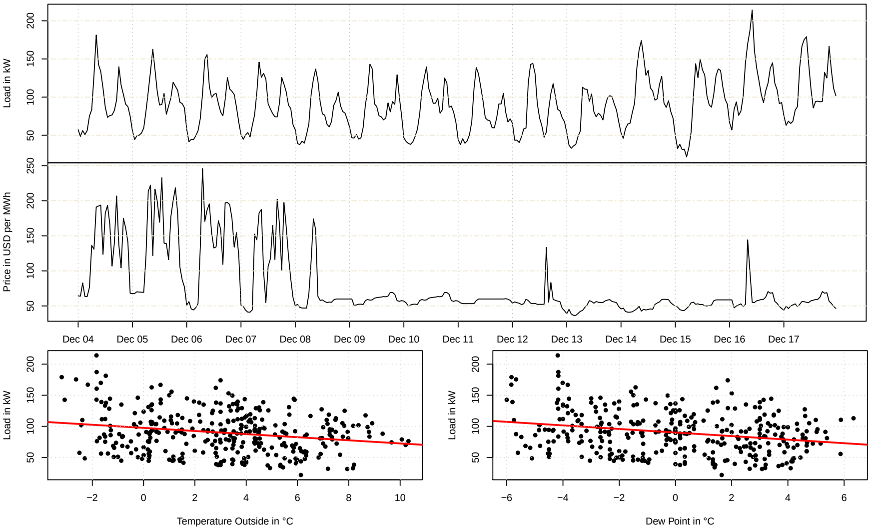
- Parameter L penalizes violations of the complementarity constraints
- Optimality is not guaranteed - practical usefulness proved
- Model validation to tune L

Case Study

- Data of price-responsive households from Olympic Peninsula project from May 2006 to March 2007
- Decisions made by the home-automation system based on occupancy modes, comfort settings, and price
- The price was sent out every 15 minutes to 27 households

Case Study

- Load, price, temperature and dew point during December



Benchmark models

ARX: Auto-Regressive model with eXogenous inputs [Dorini et al., 2013, Corradi et al., 2013]

$$x_t = \vartheta_x \boldsymbol{X}_{t-n} + \vartheta_z \boldsymbol{Z}_t + \epsilon_t,$$

with $\epsilon_t \sim \mathrm{N}(0,\sigma^2)$ and σ^2 is the variance.

$\boldsymbol{Z}_t$: price, outside temperature, solar irradiance, wind speed, humidity, dew point, hour of the day and day of the week.

Simple Inv: Only marginal utilities are estimated (12 blocks). Based on Keshavarz et al. [2011], Chan et al. [2014].

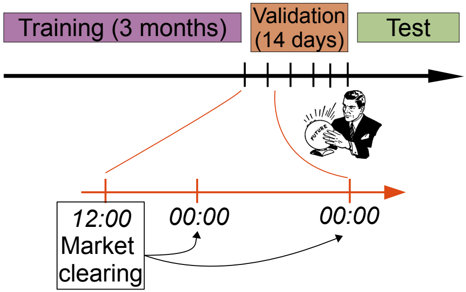
Inv Few: Our method with only outside temperature and hourly indicator variables as features. Includes exponential forgetting factor E through weights w_t.

Inv All: The same as **Inv Few**, but including all features.

Case Study

Rolling-horizon validation for model tuning (based on MAPE)

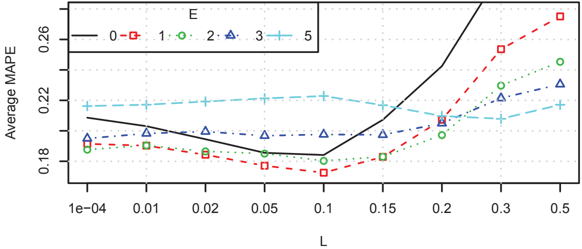
- Penalization parameter L
- Forgetting factor E



Case Study

Rolling-horizon validation for model tuning (based on MAPE)

- Penalization parameter *L*
- Forgetting factor *E*



Case Study

Rolling-horizon validation for model tuning (based on MAPE)

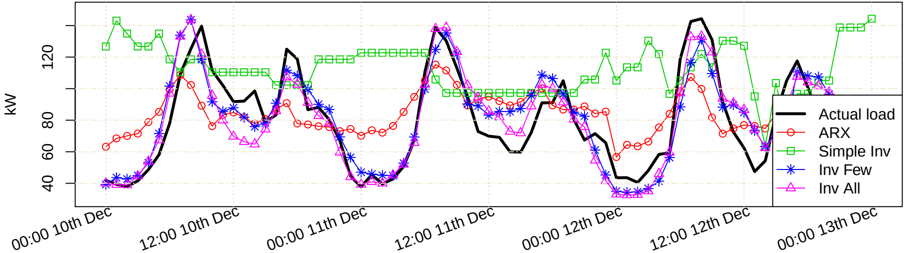
- Penalization parameter *L*
- Forgetting factor *E*

Month	L	E
September 2006	0.3	0
December 2006	0.1	1
March 2007	0.3	1

$L \uparrow \Rightarrow$ Price-responsiveness$\downarrow$

Case Study

Prediction capabilities of different benchmarked methods



	MAE	RMSE	MAPE
ARX	22.176	27.501	0.275
Simple Inv	44.437	54.576	0.586
Inv Few	17.318	23.026	0.189
Inv All	17.554	22.392	0.199

Case Study

	September			March		
	MAE	RMSE	MAPE	MAE	RMSE	MAPE
ARX	7.649	9.829	0.235	17.439	23.395	0.2509
Simple Inv	14.263	17.800	0.495	44.687	54.616	0.836
Inv Few	5.719	8.582	0.146	12.652	16.776	0.195
Inv All	5.815	8.494	0.151	14.797	19.119	0.239

The prediction performance of the proposed machinery is only slightly lower than that of the state-of-the-art prediction tool developed in Hosking et al. [2013] on the same dataset. However, our methodology produces a market bid!

Concluding Remarks

What we have done:

- We develop a novel approach to capture the **price-response** of the pool of flexible consumers in the form of a **market bid** using price-consumption data.
- We propose a **generalized inverse optimization framework** to estimate the market bid that best captures the price-response of the pool.
- We leverage auxiliary information on a **set of features** that may have predictive power on the consumption pattern of the cluster.
- We test our methodology using data from a **real-world experiment** and compare its performance with state-of-the-art prediction models on the same dataset.

Concluding Remarks

J. Saez-Gallego, J. M. Morales, M. Zugno, and H. Madsen (2016). A Data-Driven Bidding Model for a Cluster of Price-Responsive Consumers of Electricity. *IEEE Transactions on Power Systems*. DOI: 10.1109/TPWRS.2016.2530843.

Thanks for your attention!

Website: `https://sites.google.com/site/jnmmgo/`

T. C. Y. Chan, T. Craig, T. Lee, and M. B. Sharpe. Generalized inverse multiobjective optimization with application to cancer therapy. *Operations Research*, 62(3):680–695, 2014.

Antonio J Conejo, Miguel Carrión, and Juan M Morales. *Decision making under uncertainty in electricity markets*, volume 1. Springer, 2010.

O. Corradi, H. Ochsenfeld, H. Madsen, and P. Pinson. Controlling electricity consumption by forecasting its response to varying prices. *Power Systems, IEEE Transactions on*, 28(1): 421–429, Feb 2013. ISSN 0885-8950. doi: 10.1109/TPWRS.2012.2197027.

C. De Jonghe, B. F. Hobbs, and R. Belmans. Optimal generation mix with short-term demand response and wind penetration. *IEEE Transactions on Power Systems*, 27(2):830–839, May 2012. ISSN 0885-8950. doi: 10.1109/TPWRS.2011.2174257.

G. Dorini, P. Pinson, and H. Madsen. Chance-constrained optimization of demand response to price signals. *Smart Grid, IEEE Transactions on*, 4(4):2072–2080, Dec 2013. ISSN 1949-3053. doi: 10.1109/TSG.2013.2258412.

J. R. M. Hosking et al. Short-term forecasting of the daily load curve for residential electricity usage in the smart grid. *Applied Stochastic Models in Business and Industry*, 29(6):604–620, 2013.

T. G. Hovgaard, S. Boyd, L. F. Larsen, and J. B. Jørgensen. Nonconvex model predictive control for commercial refrigeration. *International Journal of Control*, 86(8):1349–1366, 2013.

E. B. Iversen, J. M. Morales, and H. Madsen. Optimal charging of an electric vehicle using a markov decision process. *Applied Energy*, 123:1–12, 2014.

A. Keshavarz, Y. Wang, and S. Boyd. Imputing a convex objective function. In *IEEE International Symposium on Intelligent Control (ISIC)*, pages 613–619. IEEE, 2011.

M. Parvania, M. Fotuhi-Firuzabad, and M. Shahidehpour. Optimal demand response aggregation in wholesale electricity markets. *IEEE Transactions on Smart Grid*, 4(4):1957–1965, 2013.

F. A. Qureshi, T. T. Gorecki, and C. Jones. Model predictive control for market-based demand response participation. In *19th World Congress of the International Federation of Automatic Control*, number EPFL-CONF-197950, 2014.

S. Siddiqui and S. A. Gabriel. An SOS1-based approach for solving MPECs with a natural gas market application. *Networks and Spatial Economics*, 13(2):205–227, 2013.

M. Zugno, J. M. Morales, P. Pinson, and H. Madsen. A bilevel model for electricity retailers' participation in a demand response market environment. *Energy Economics*, 36:182–197, 2013.

Background Material

- A number of works address the load scheduling problem under real-time/dynamic pricing:
  - ✓ Often follow the principles of Model Predictive Control
  - ✓ *Rational-behavior models*: the price-response of the pool seeks to minimize the electricity cost
  - ✓ A variety of different types of power loads, e.g., a refrigeration system [Hovgaard et al., 2013], an electric vehicle [Iversen et al., 2014], or the HVAC system of a building [Qureshi et al., 2014, Zugno et al., 2013]
- *Statistical models*: the price-response of the pool is inferred from observed data [Corradi et al., 2013, Hosking et al., 2013]
  - ✓ Also econometric models that rely on the concept of price elasticities [De Jonghe et al., 2012]
- Bidding models for large consumers and for retailers that supply an inelastic and uncertain demand [Conejo et al., 2010, Ch. 8 and 9]
- Bids often boil down to offering load reduction or to buying the price-based predicted consumption of the pool [Parvania et al., 2013, Qureshi et al., 2014]

Equivalent Single-level Optimization Problem

Parameter estimation

$$\underset{x,\theta}{\text{Minimize}} \sum_{t \in \mathcal{T}} w_t \left(e_t^+ + e_t^-\right)$$

subject to

$$\underline{P}_t + \sum_{b \in \mathcal{B}} x_{b,t} - x_t^{meas} = e_t^+ - e_t^- \qquad t \in \mathcal{T}$$

$$e_t^+, e_t^- \geq 0 \qquad t \in \mathcal{T}$$

$$a_{b,t} \geq a_{b+1,t} \qquad b \in \mathcal{B}, t \in \mathcal{T}$$

KKT conditions of lower-level problem

Including Auxiliary Information (Features Z)

We assume that the bid parameters are affine functions of the features, e.g.,

$$\underline{P}_t(Z) = \underline{P} + \sum_{i \in \mathcal{I}} \alpha_i^{\underline{P}} Z_{i,t}, \quad t \in \mathcal{T}$$

Step 1: Solution to the relaxed estimation problem

$$\underset{\substack{x_t,\theta_t,e_t^+,e_t^-,\alpha_i^u,\alpha_i^d \\ \alpha_i^{\overline{P}},\alpha_i^{\underline{P}},\psi_t^{\overline{P}},\psi_t^{\underline{P}},\lambda_t^u,\lambda_t^d \\ \overline{\phi}_{i,t},\underline{\phi}_{i,t},\overline{\varphi}_{i,t},\underline{\varphi}_{i,t},\overline{\eta}_{i,t},\underline{\eta}_{i,t}}}{\text{Minimize}} \quad \overbrace{\sum_{t\in\mathcal{T}} w_t(e_t^+ + e_t^-)}^{\text{Estimation error}} +$$

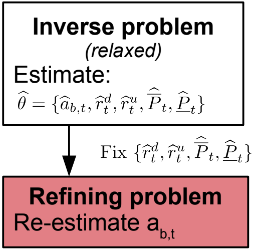
$$+ \overbrace{L\left(\sum_{\substack{b\in\mathcal{B}\\ t\in\mathcal{T}}} w_t\left(\psi_{b,t}^{\overline{P}} + \psi_{b,t}^{\underline{P}} + \frac{\overline{P}_t - \underline{P}_t}{B}\right) + \sum_{t\in\mathcal{T}_{-1}} w_t\left(\lambda_t^u + \lambda_t^d + r_t^u + r_t^d\right)\right)}^{\text{Penalization of complementarity conditions}}$$

subject to:

1. Upper-level constraints: linear reformulation of absolute value, constraints on bid parameters and the α's
2. Lower-level constraints (price-response model): stationarity, and primal and dual feasibility

Step 2: Refining the marginal utilities $a_{b,t}$

- Reformulate the inverse problem using primal-dual formulation [Chan et al., 2014, Keshavarz et al., 2011]
- In the lower-level, fix the parameters appearing in the constraints at the values estimated in Step 1
- Replace the estimated load (x) by the measured one (x^{meas})



Minimize $w\epsilon =$ Weigthed Duality Gap

subject to

Primal Ojective = Dual Objective $+ \epsilon$

Primal Constraints

Dual Constraints

Benchmark models

ARX: Auto-Regressive model with eXogenous inputs [Dorini et al., 2013, Corradi et al., 2013]

$$x_t = \boldsymbol{\vartheta}_x \boldsymbol{X}_{t-n} + \boldsymbol{\vartheta}_z \boldsymbol{Z}_t + \epsilon_t,$$

with $\epsilon_t \sim \mathrm{N}(0,\sigma^2)$ and σ^2 is the variance.

$\boldsymbol{Z}_t$: outside temperature, solar irradiance, wind speed, humidity, dew point (up to 36 hours in the past), plus binary indicators for the hour of the day and the day of the week.

Simple Inv: Only the marginal utilities are estimated (12 blocks) as in Step 2, the rest of bid parameters to historical maximum/minimum values observed in the last seven days. Inspired from Keshavarz et al. [2011], Chan et al. [2014].

Benchmark models

Inv Few: Our inverse optimization scheme only with the outside temperature and hourly indicator variables as features.

$$w_t = gap_t \left(\frac{t}{T}\right)^E, t \in \mathcal{T}$$

$E \geq 0$, forgetting factor.

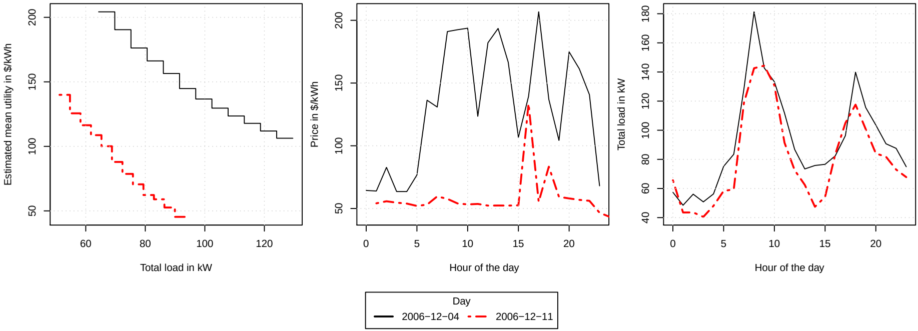
T: total number of periods.

gap indicates whether the observation was correctly measured ($gap = 1$) or not ($gap = 0$).

Inv All: The same as **Inv Few**, but including all features and regularization.

Case Study

Estimated marginal utility for the pool of price-responsive consumers



Consider the complementary slackness condition $(\mathrm{Ax}-\mathrm{b})\lambda = 0$. It can be equivalently reformulated as [Siddiqui and Gabriel, 2013]:

$$y_1 = 0.5\left((\mathrm{Ax}-\mathrm{b}) + \lambda\right) \quad (2a)$$

$$y_2 = 0.5\left((\mathrm{Ax}-\mathrm{b}) - \lambda\right) \quad (2b)$$

$$y_1^2 - y_2^2 = (\mathrm{Ax}-\mathrm{b})\lambda = 0 \quad (2c)$$

Noting that $\mathrm{Ax} - \mathrm{b} \geq 0$ and $\lambda \geq 0$:

$$y_1 = 0.5\left((\mathrm{Ax} - \mathrm{b}) + \lambda\right) \quad (3a)$$
$$y_2 = 0.5\left((\mathrm{Ax} - \mathrm{b}) - \lambda\right) \quad (3b)$$
$$y_1 = |y_2| \quad (3c)$$

We replace the absolute value by the sum of the two positive variables y_{2t}^{+} and y_{2t}^{-}, i.e.,

$$y_1 = 0.5\left((\mathrm{Ax} - \mathrm{b}) + \lambda\right) \quad (4a)$$
$$y_2^{+} - y_2^{-} = 0.5\left((\mathrm{Ax} - \mathrm{b}) - \lambda\right) \quad (4b)$$
$$y_1 = y_2^{+} + y_2^{-} \quad (4c)$$
$$y_2^{+}, y_2^{-} \geq 0 \quad (4d)$$

and penalize this sum in the objective function by adding the term $L(y_2^{+} + y_2^{-})$.

We can make a few substitutions and finally, obtain

$$y_2^+ = 0.5\,(\mathrm{Ax} - \mathrm{b}) \tag{5a}$$
$$y_2^- = 0.5\lambda \tag{5b}$$

which is equivalent to penalizing $(\mathrm{Ax} - \mathrm{b} + \lambda)$ in the objective function.