Model Predictive Control
for
Smart Energy Systems

John Bagterp Jørgensen
Leo Emil Sokoler, Laura Standardi, Rasmus Halvgaard,
Tobias Gybel Hovgaard, Peter Juhler Dinesen, Gianluca Frison

Model Predictive Control Workshop
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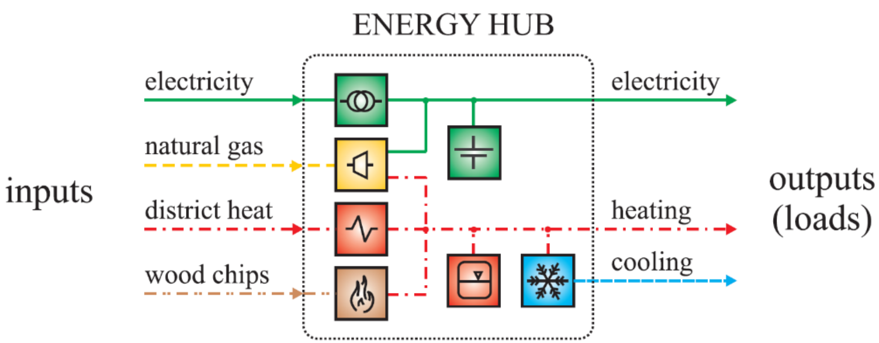
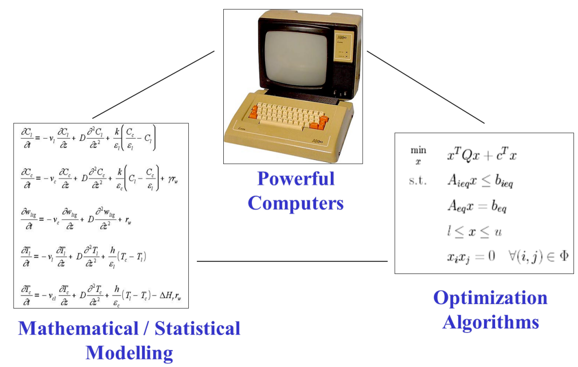
Technical University of Denmark

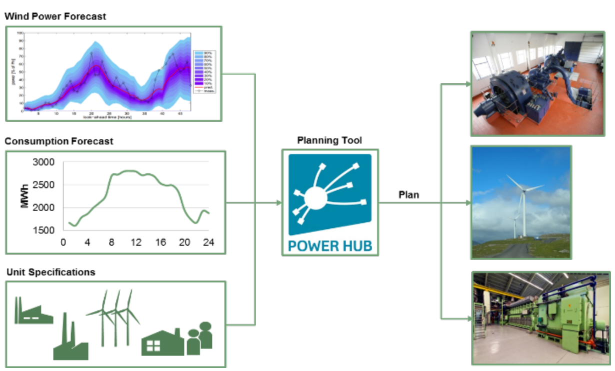
DTU Compute
Institut for Matematik og Computer Science

Main Objectives

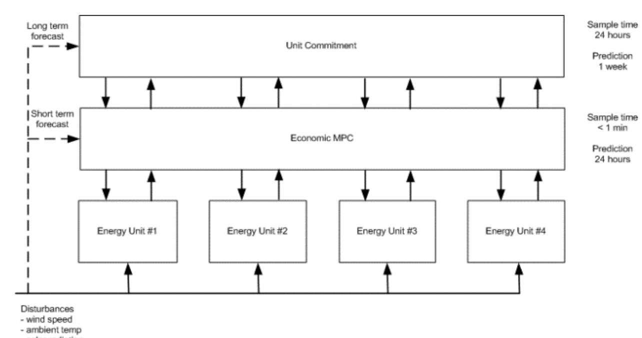
Develop and test optimization based predictive control systems for more efficient and flexible operation of integrated energy systems.







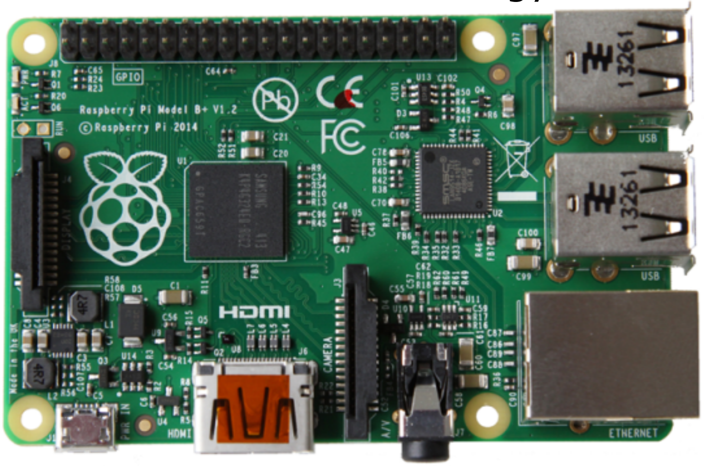
Hierarchical Control Structure

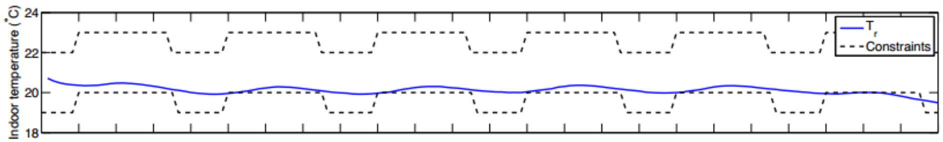


Control of Individual Energy Units

Raspberry Pi

Embedded MPC Algorithms for control of individual energy units





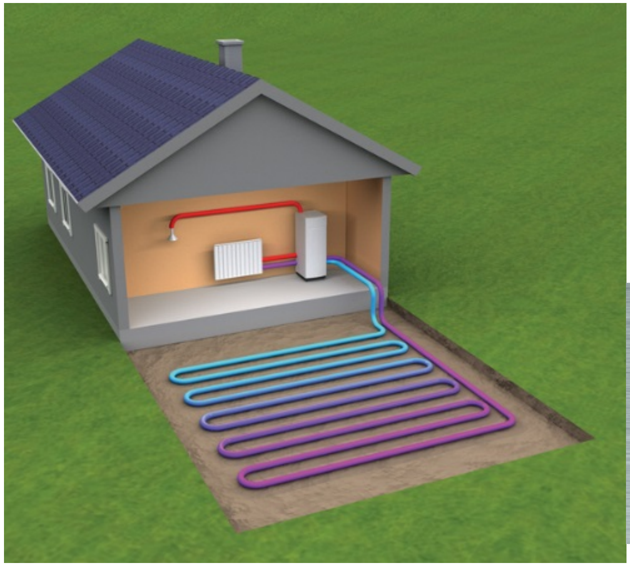
$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \sum_{k=0}^{N-1} l_k(x_k, u_k) + l_N(x_N) \tag{1a}$$

$$s.t. \quad x_{k+1} = A_k x_k + B_k u_k + b_k \quad k \in \mathcal{N} \tag{1b}$$

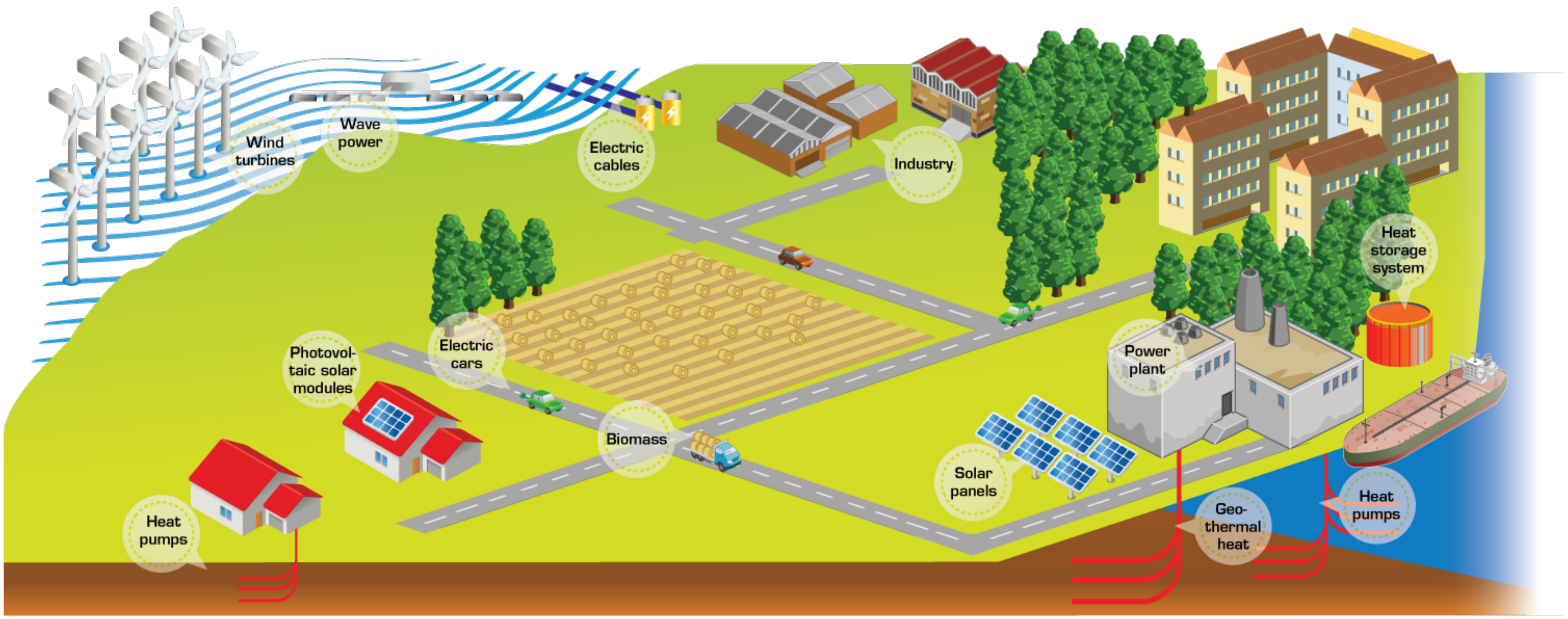
with $\mathcal{N} = \{0, 1, \ldots, N-1\}$ and stage costs defined by

$$l_k(x_k, u_k) = \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}' \begin{bmatrix} Q_k & M_k' \\ M_k & R_k \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \begin{bmatrix} q_k \\ s_k \end{bmatrix}' \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \rho_k \tag{2a}$$

$$l_N(x_N) = \frac{1}{2} x_N' P_N x_N + p_N' x_N + \gamma_N \tag{2b}$$

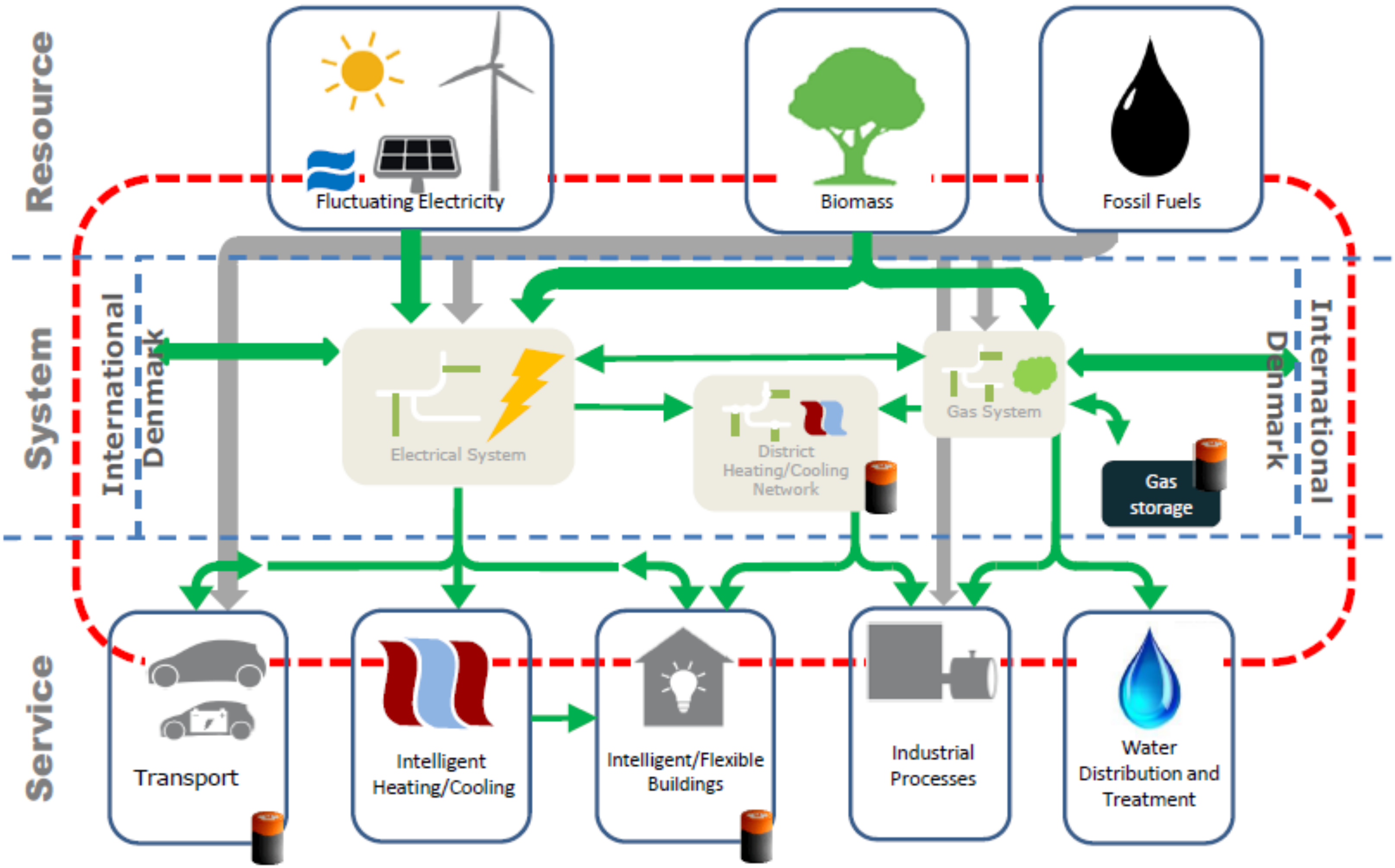


Smart Energy Systems

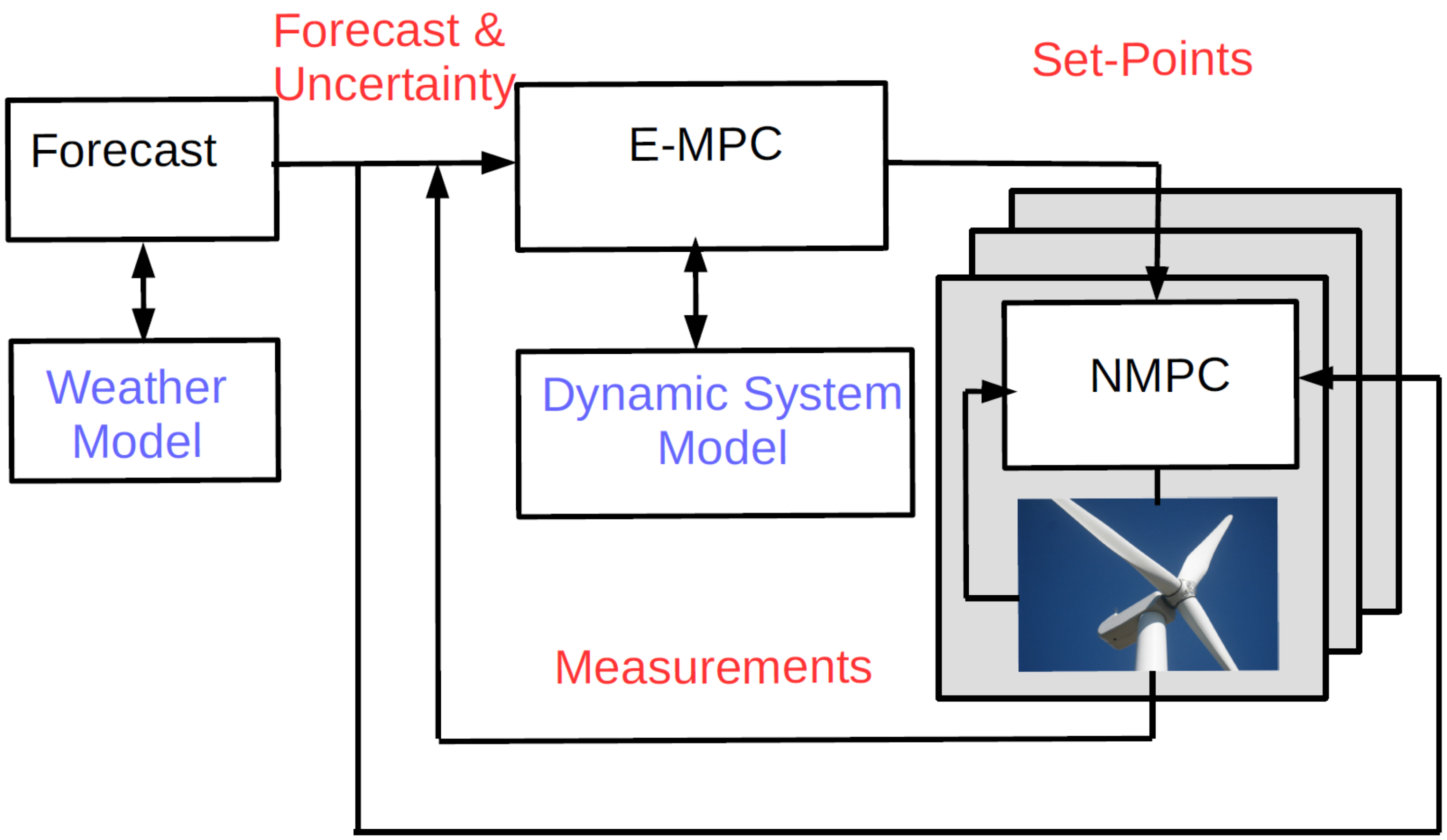


- **Thermal Storage**
  - Heating of floors etc
  - Heating of water accumulation tanks
  - Refrigeration Systems
- **Power / Heat Producers**
  - Wind Turbines
  - Photovoltaic Solar Modules
  - Solar Panels
  - CHP Plants
  - Fuel Cells

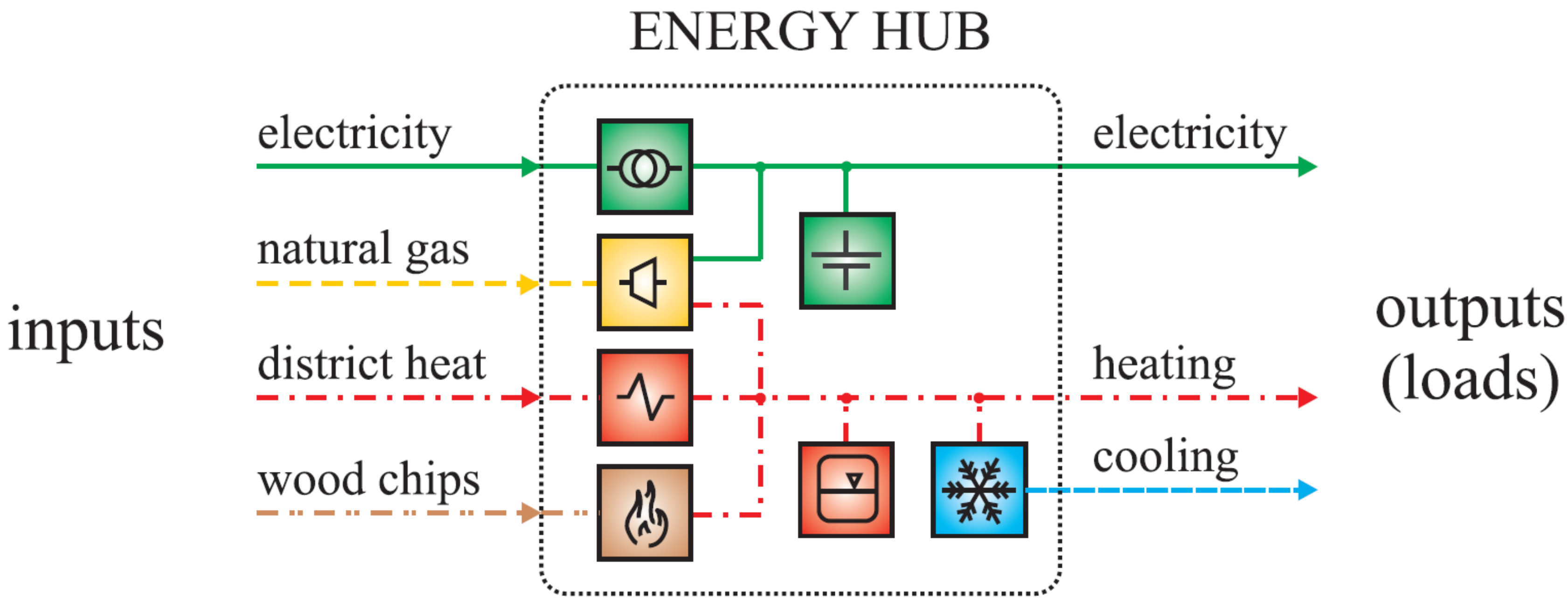
Connected Energy Systems



Forecast Based Hierarchical MPC



Electricity & Heating / Cooling

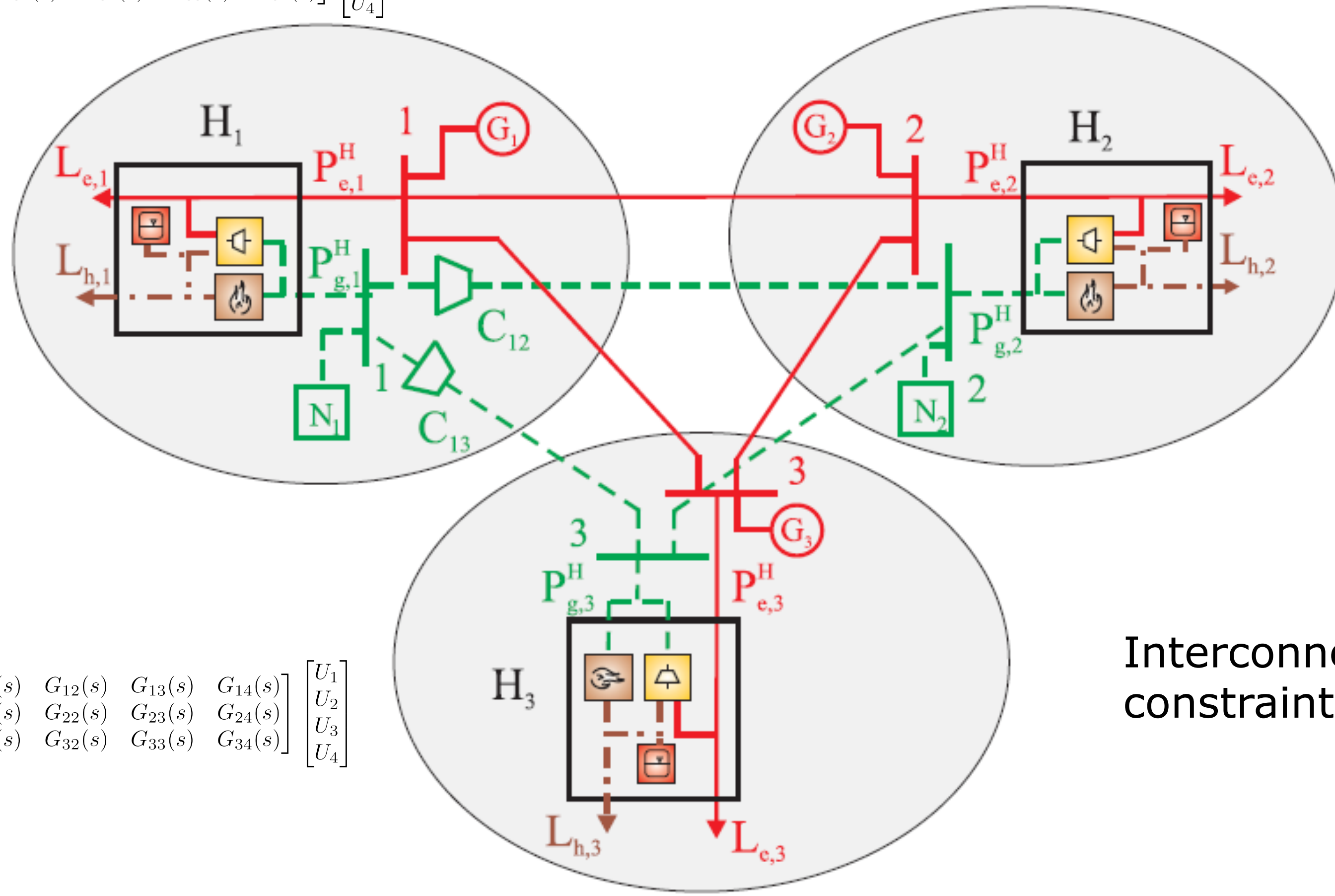


$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) & G_{34}(s) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

Multiple Connected Energy Hubs

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) & G_{34}(s) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

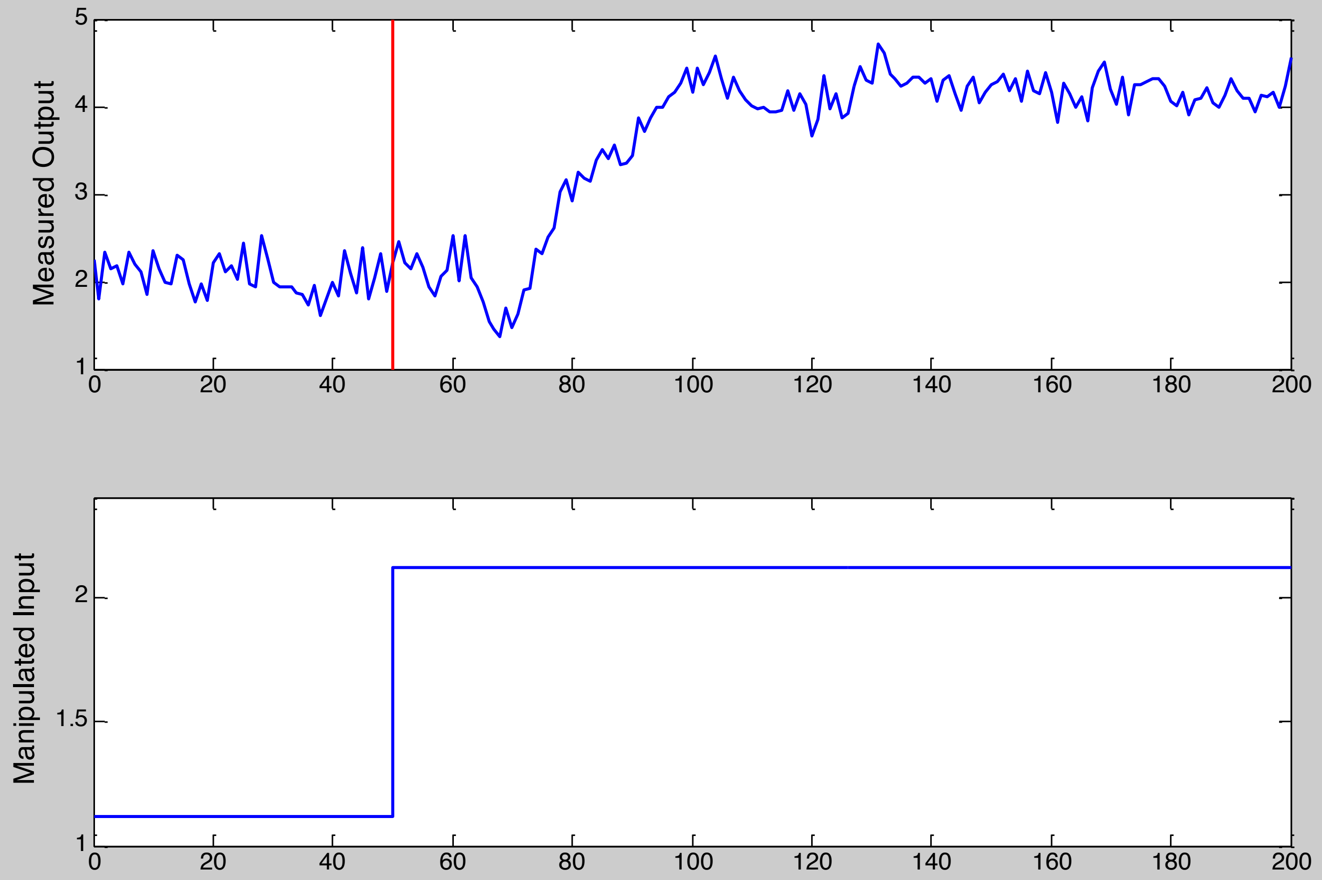
$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) & G_{34}(s) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$



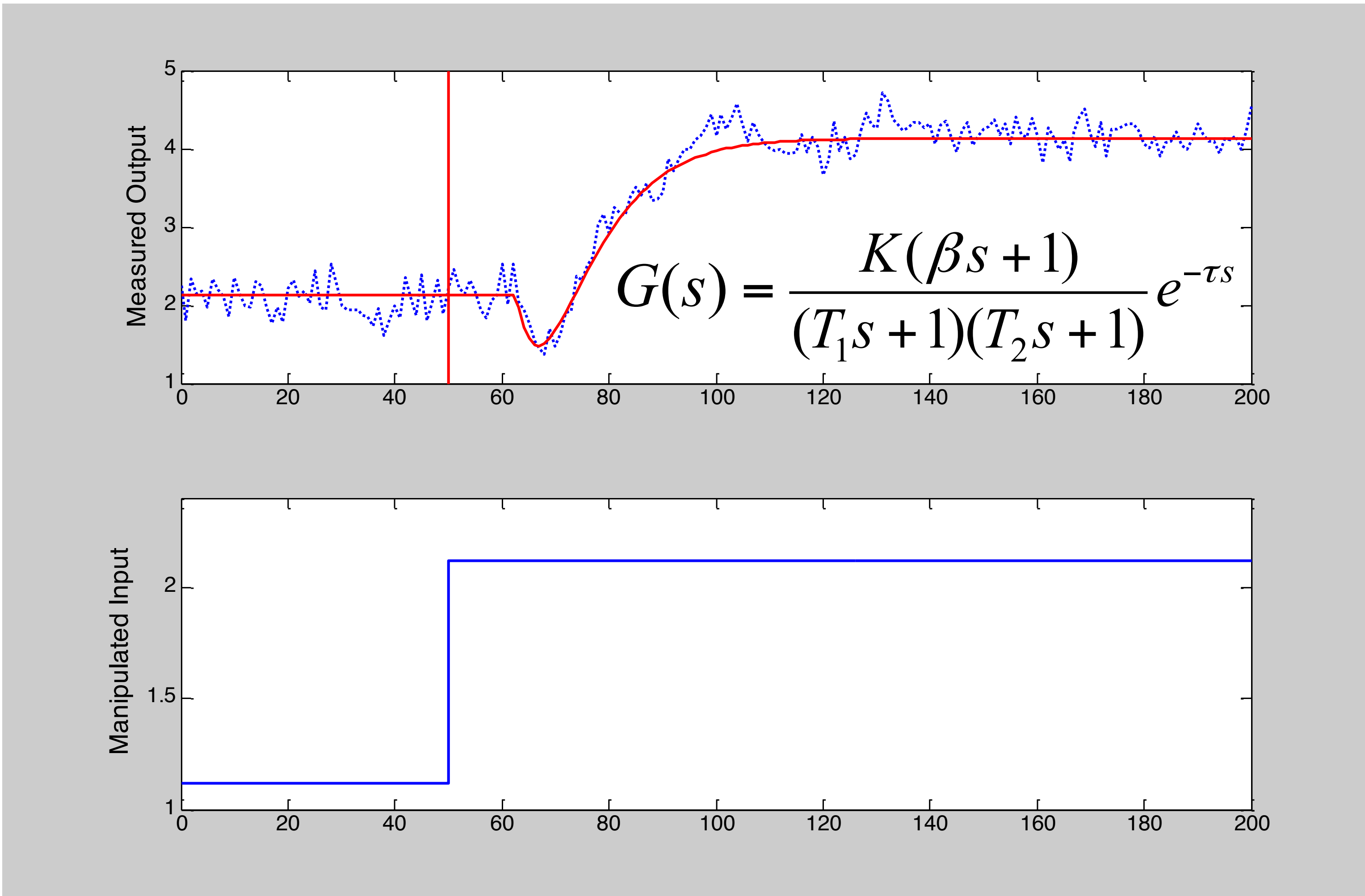
$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) & G_{34}(s) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

Interconnecting constraints

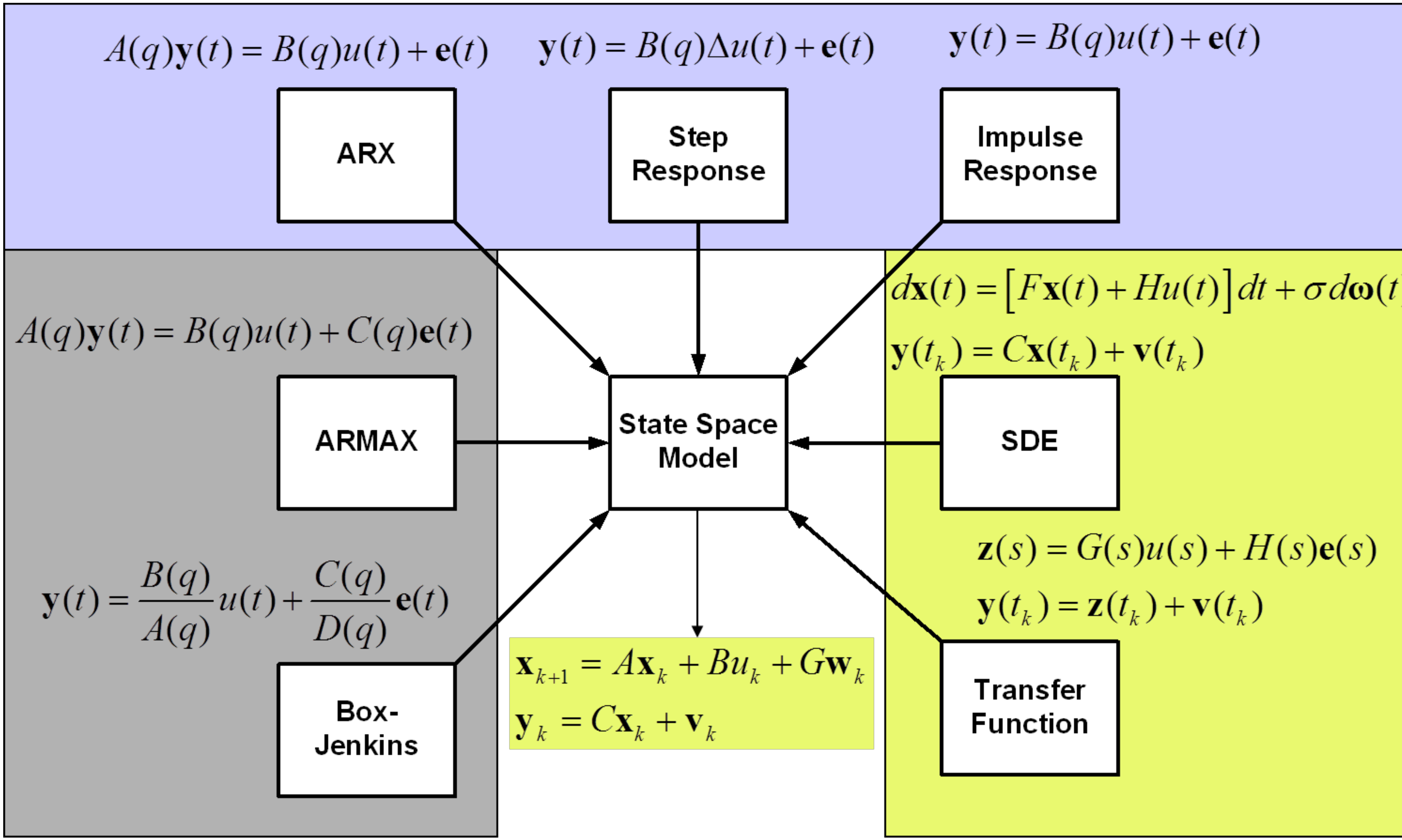
Step Response Experiments



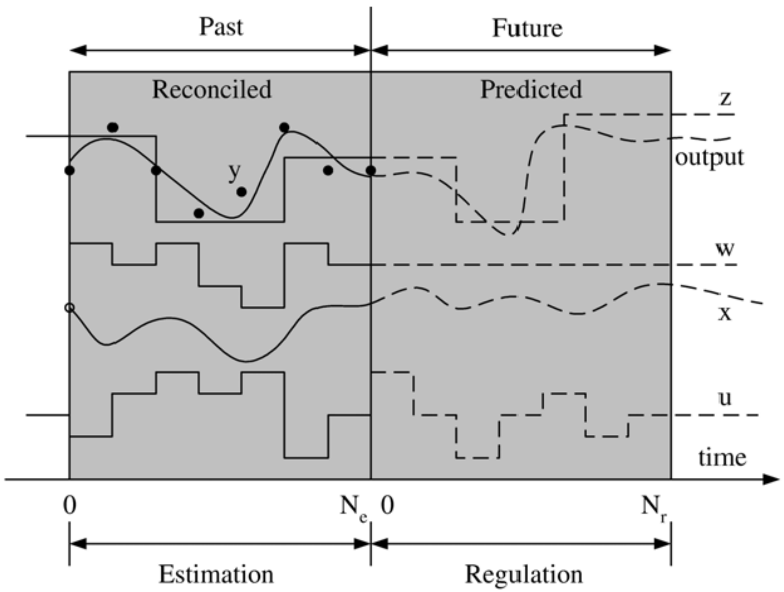
Step Response Experiments

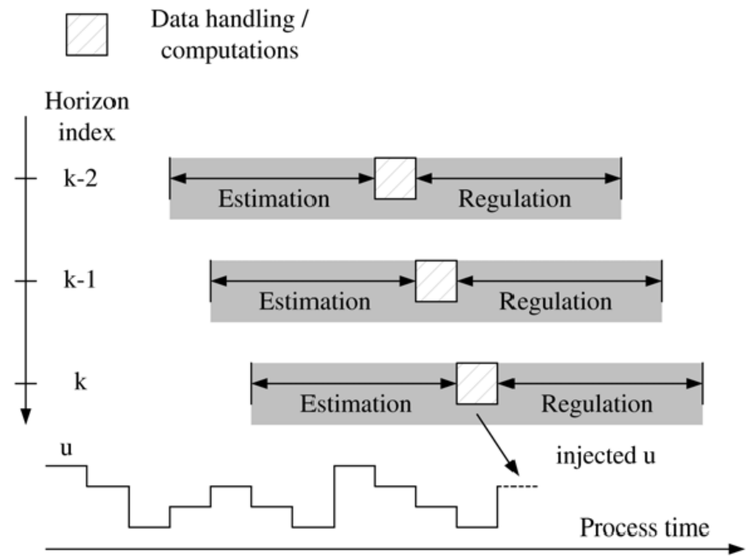


Identification of the Deterministic and Stochastic Model



Model Predictive Control



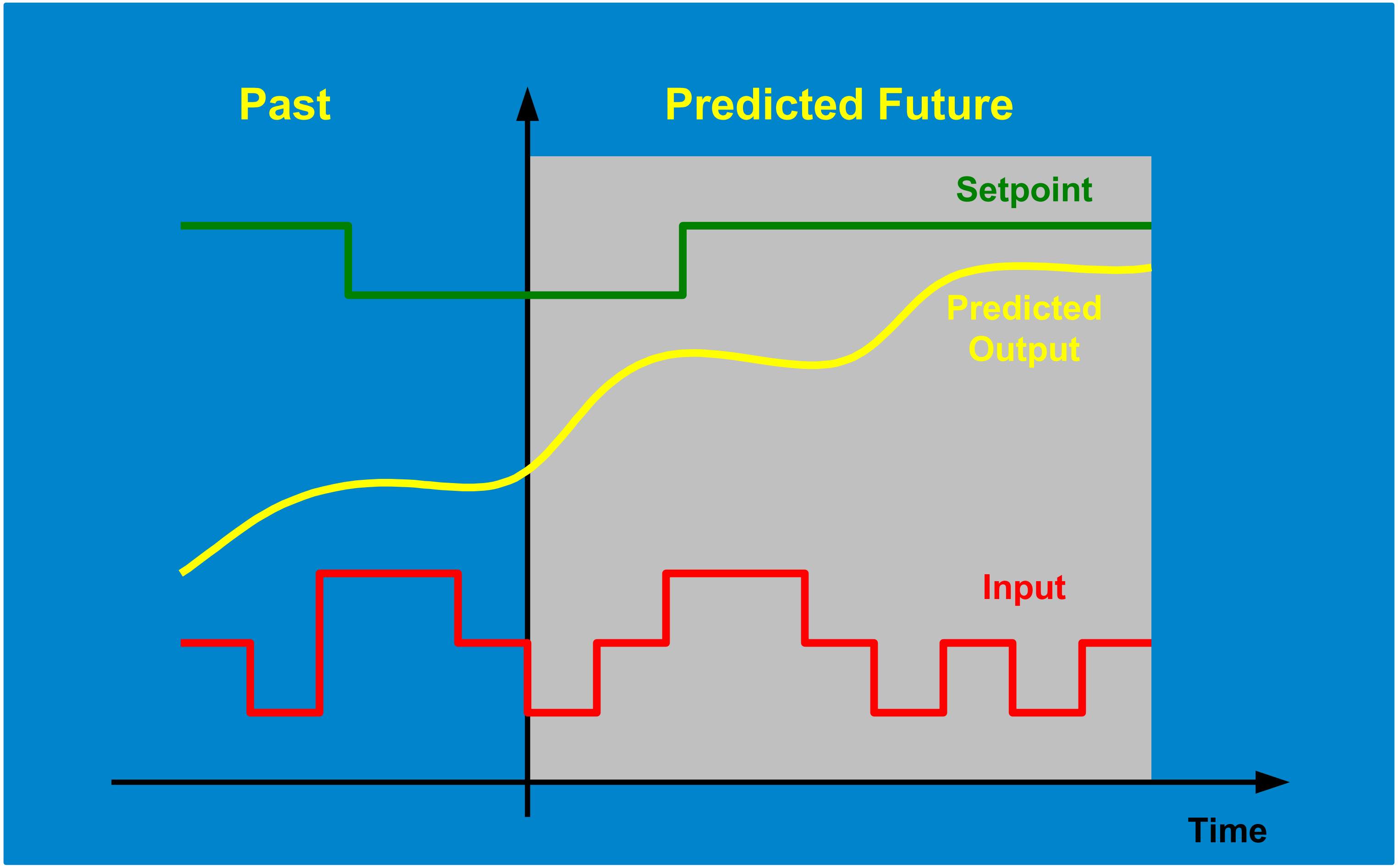


$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \phi(\{u_k, x_{k+1}\}_{k=0}^{N-1}; x_0, \theta)$$

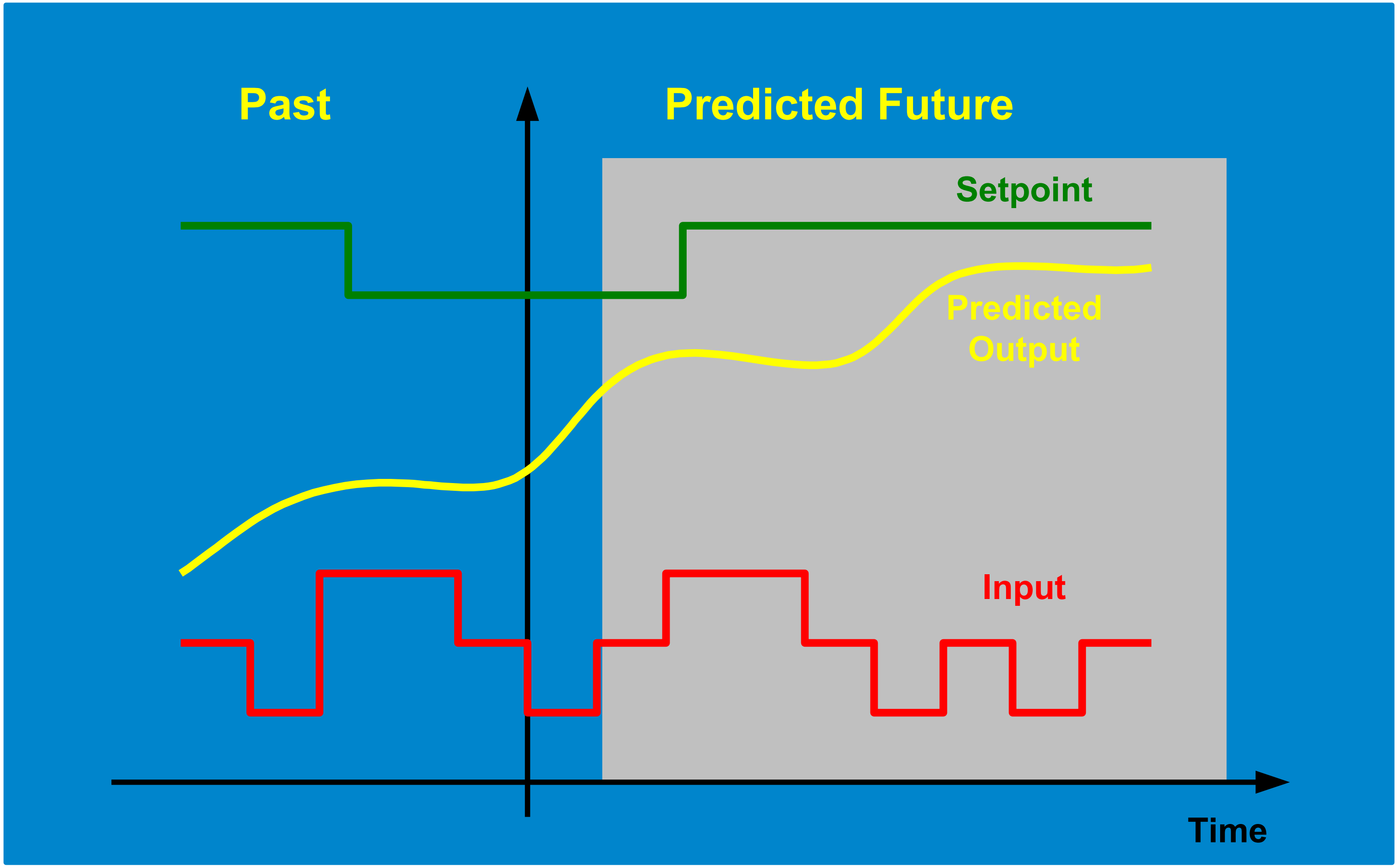
$$s.t. \quad x_{k+1} = F_k(x_k, u_k, \theta) \qquad k = 0, 1, \ldots, N-1$$

$$u_k \in \mathcal{U}$$

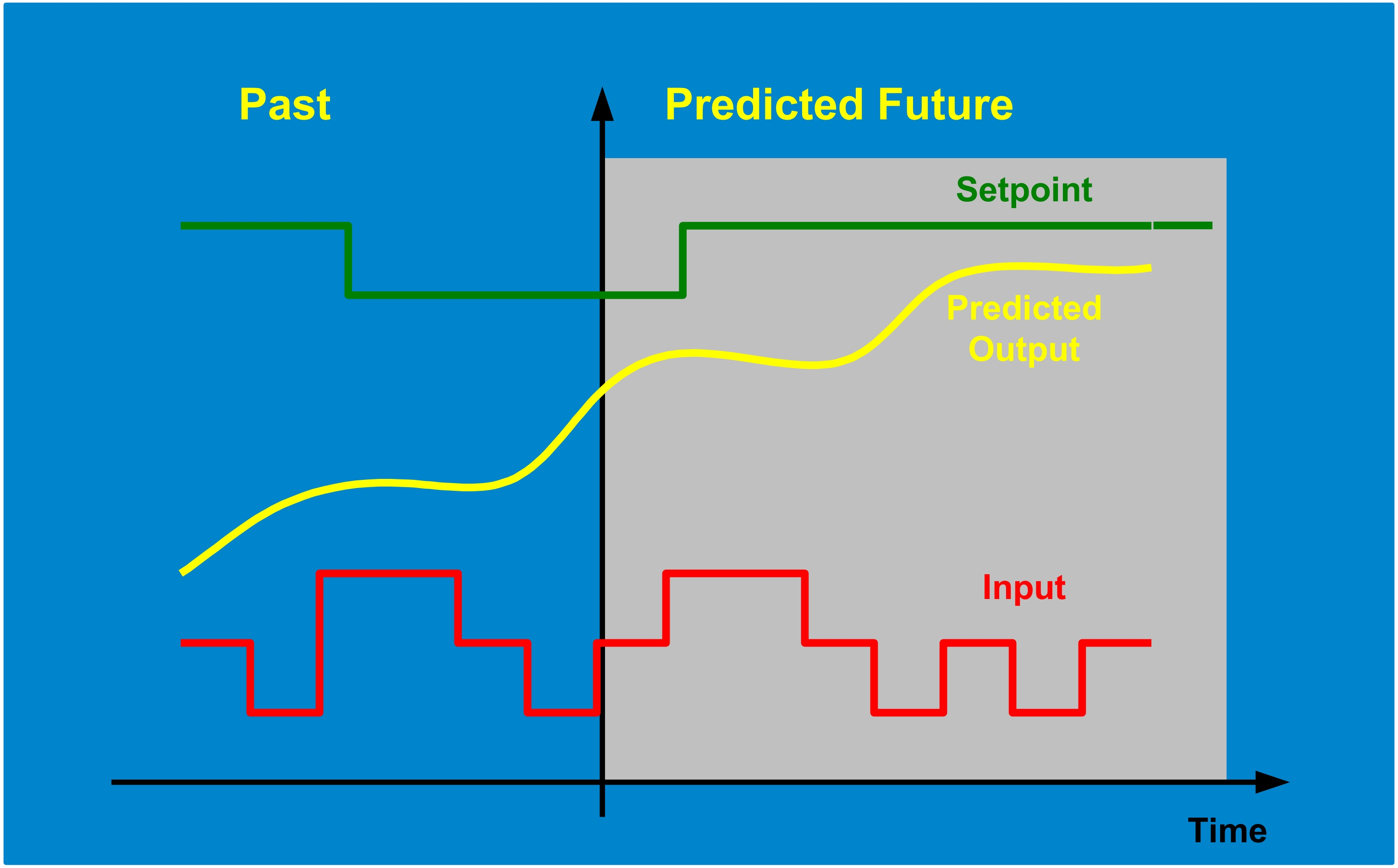
Moving Horizon Control



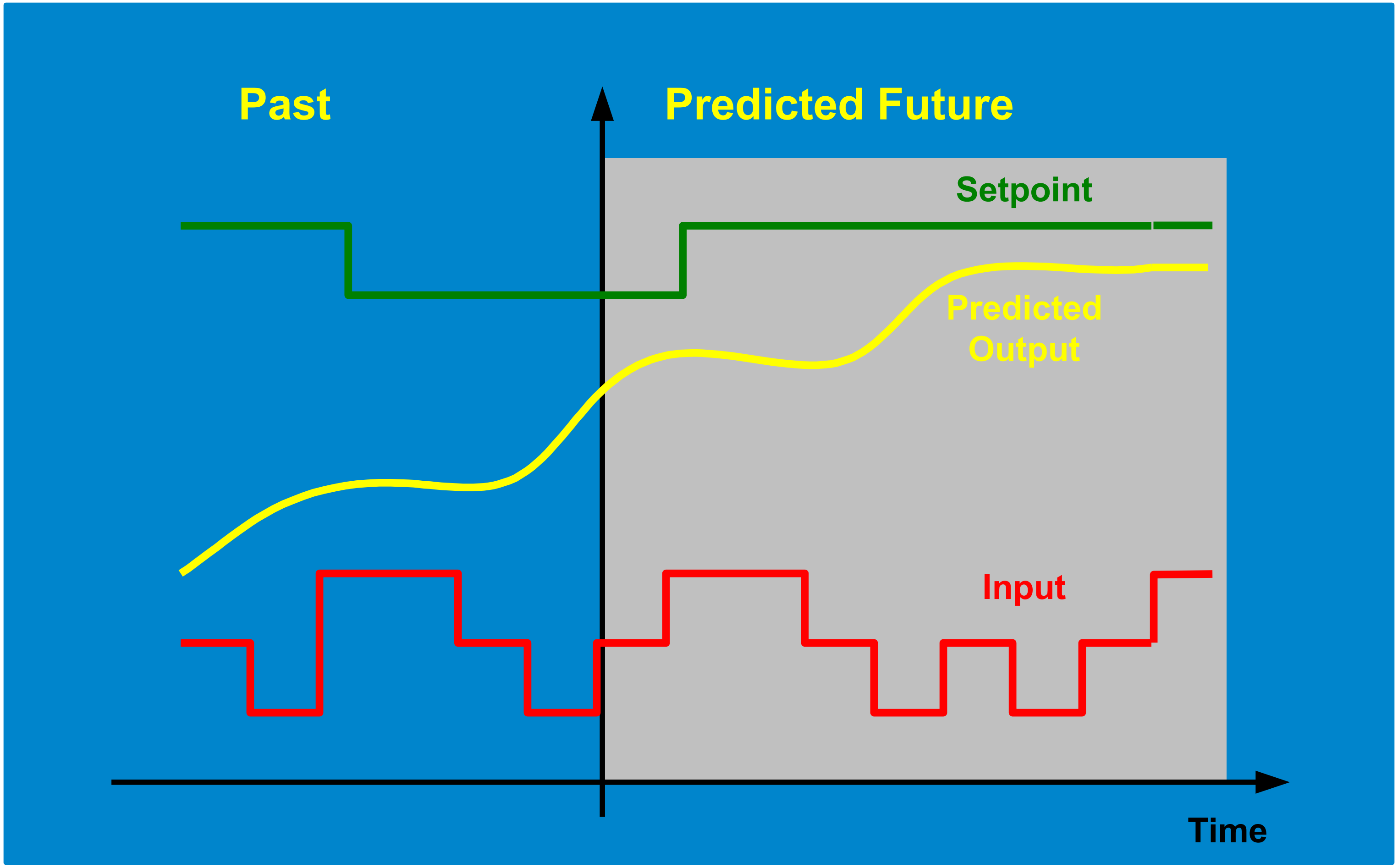
Moving Horizon Control



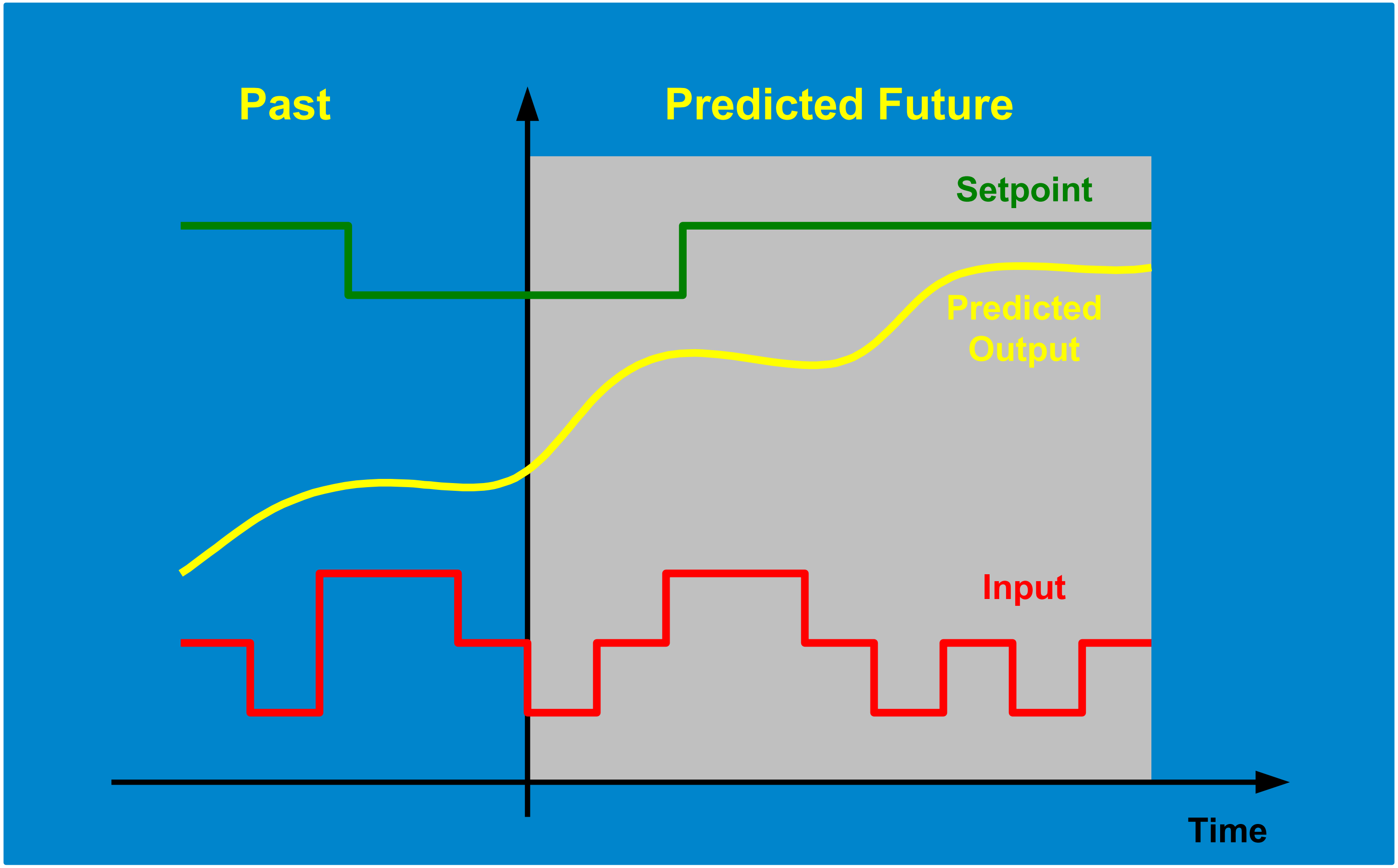
Moving Horizon Control



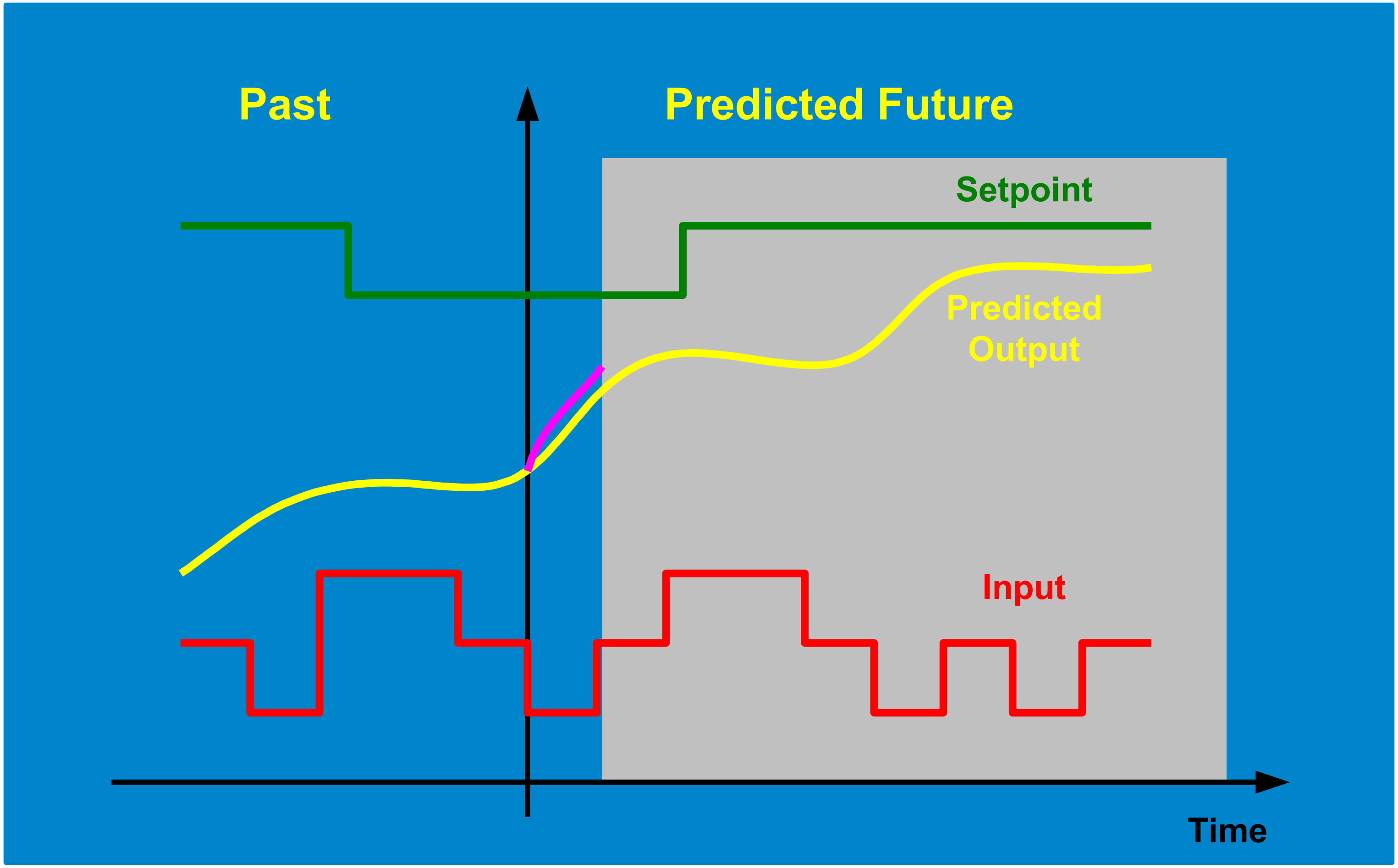
Moving Horizon Control



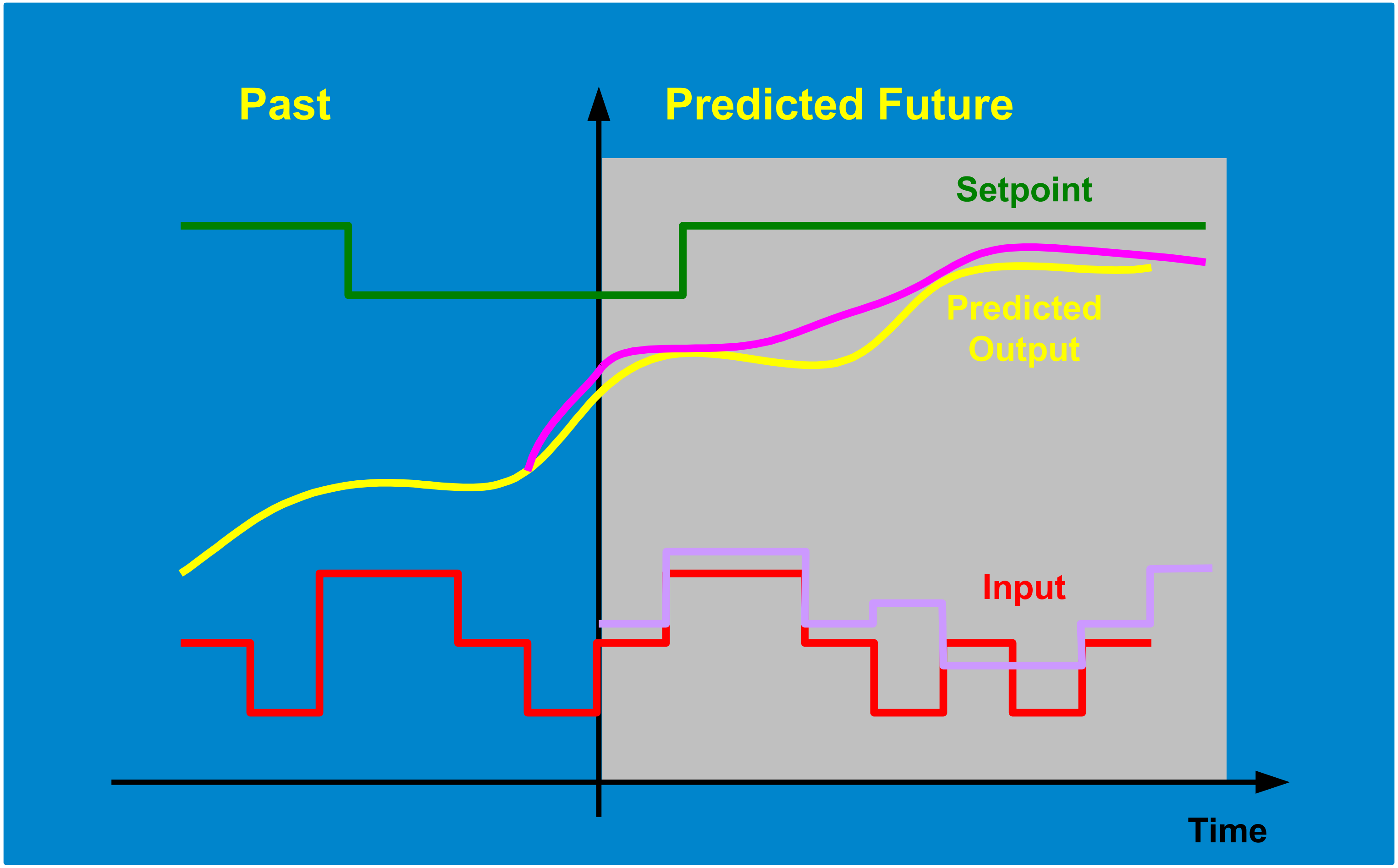
Moving Horizon Control



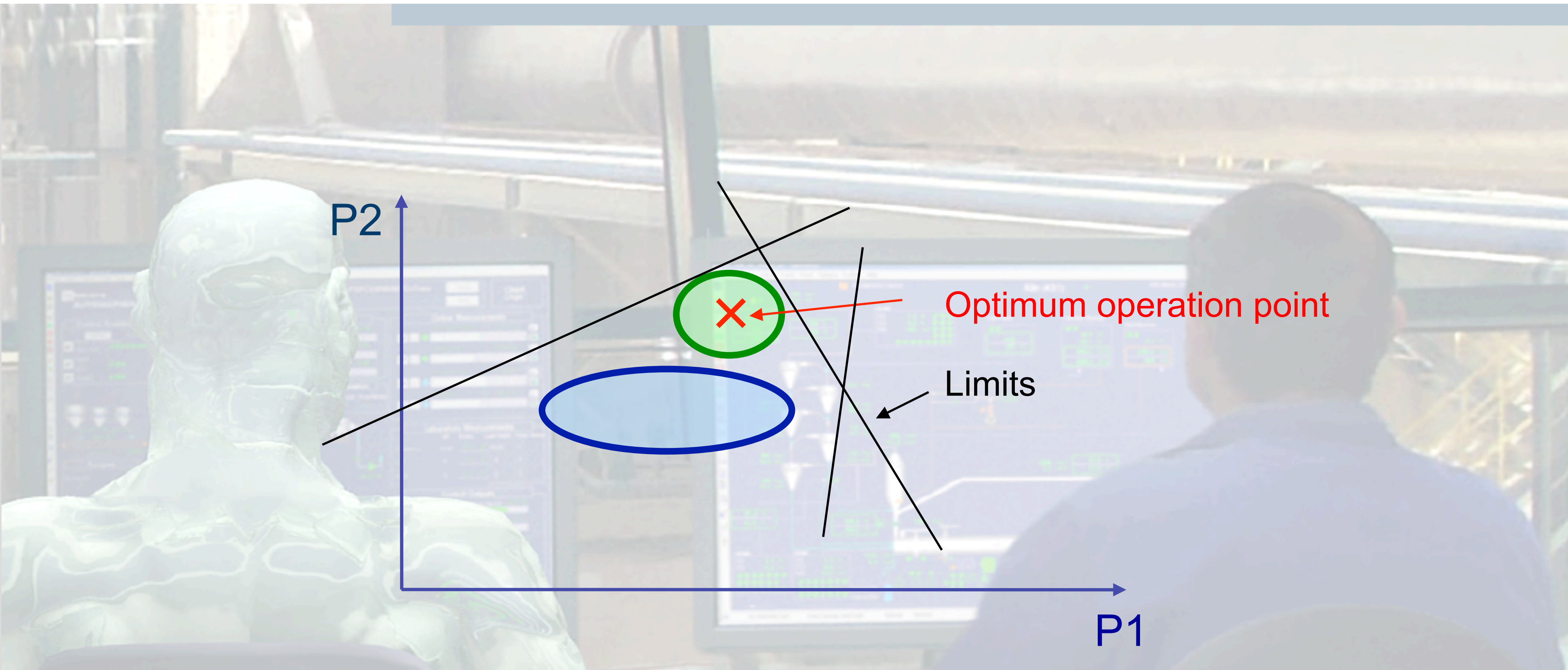
Moving Horizon Control



Moving Horizon Control

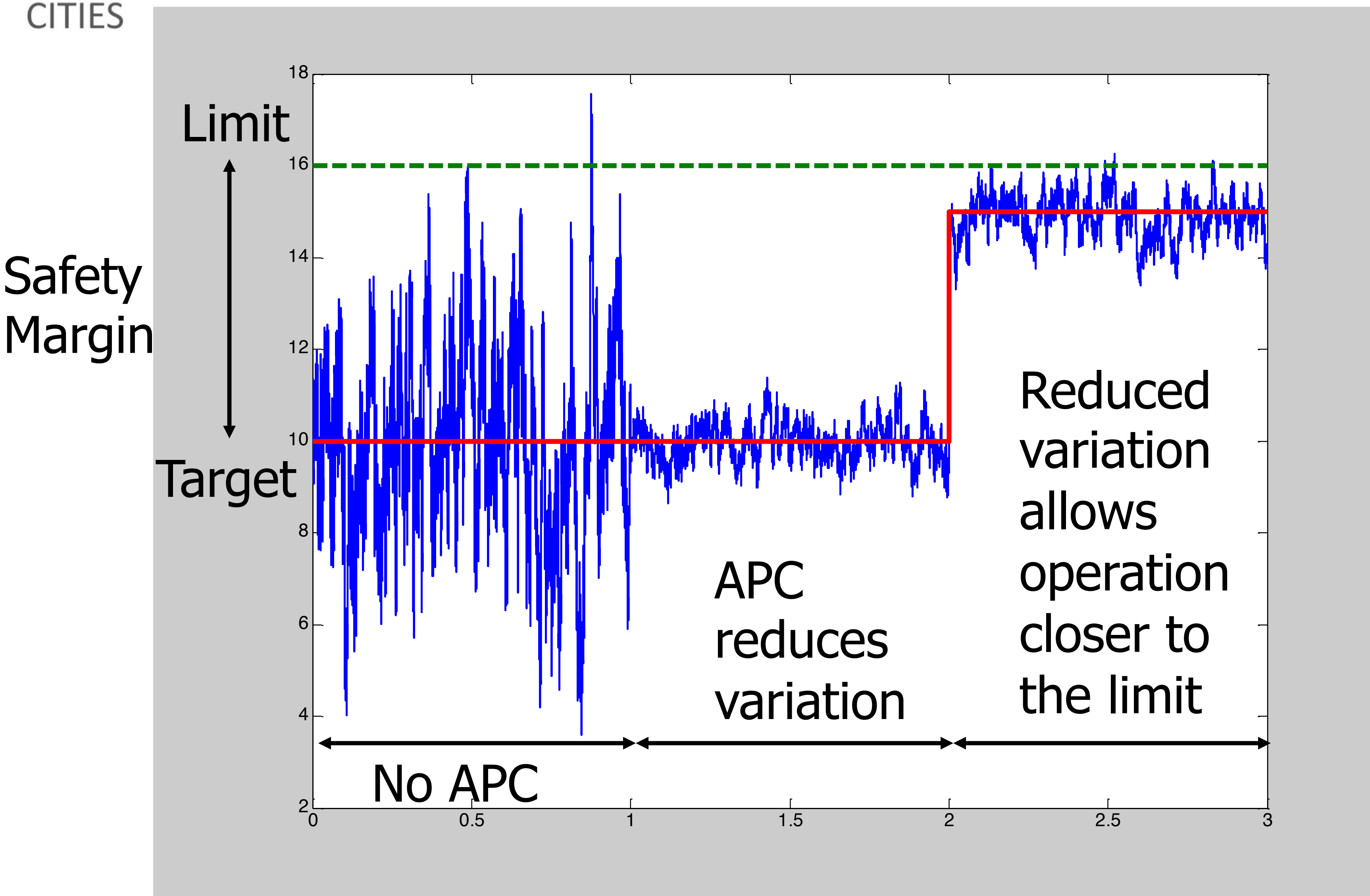


Optimal Operation is Close to Limits

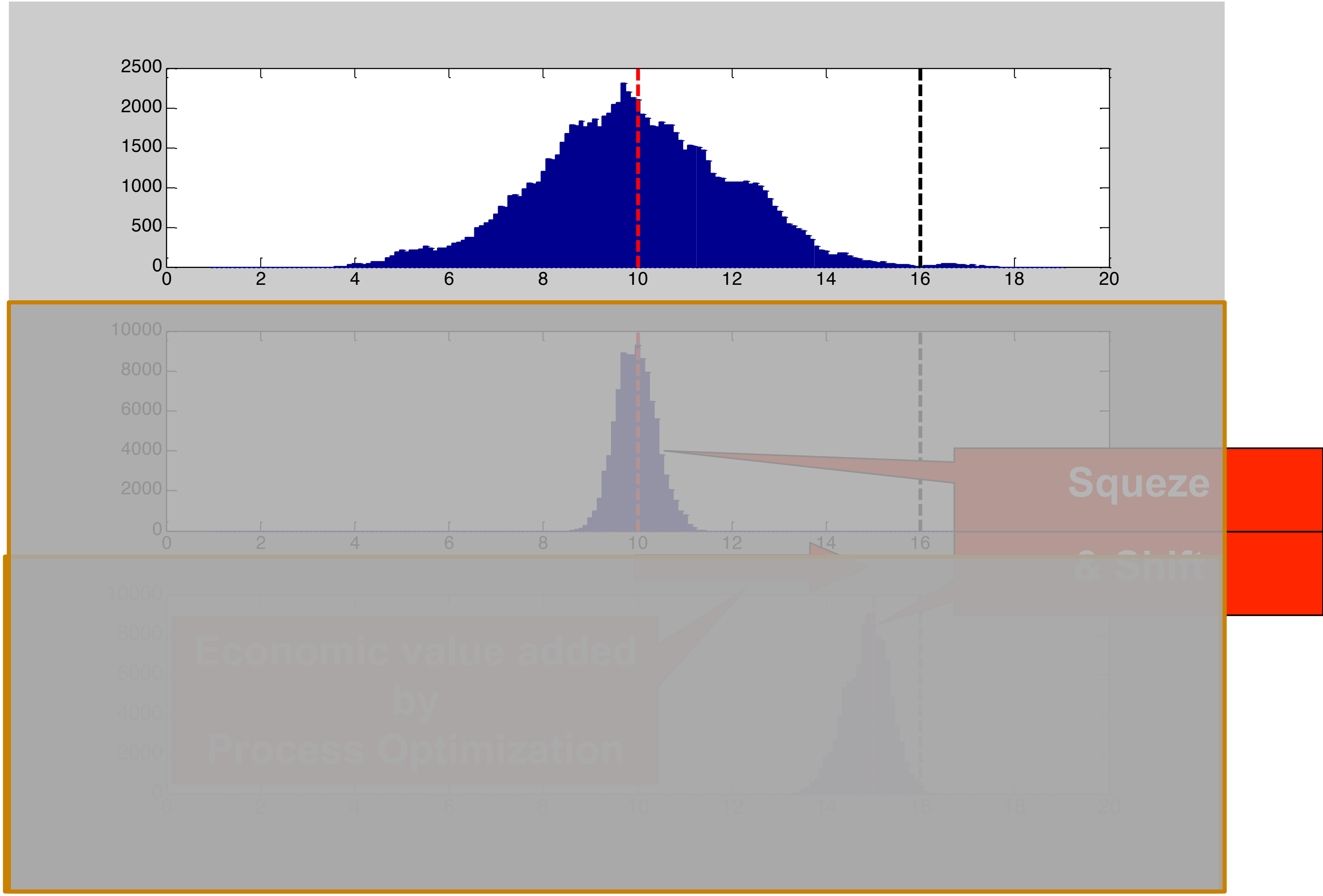


Optimum close to constraints requires process optimization and advanced process control

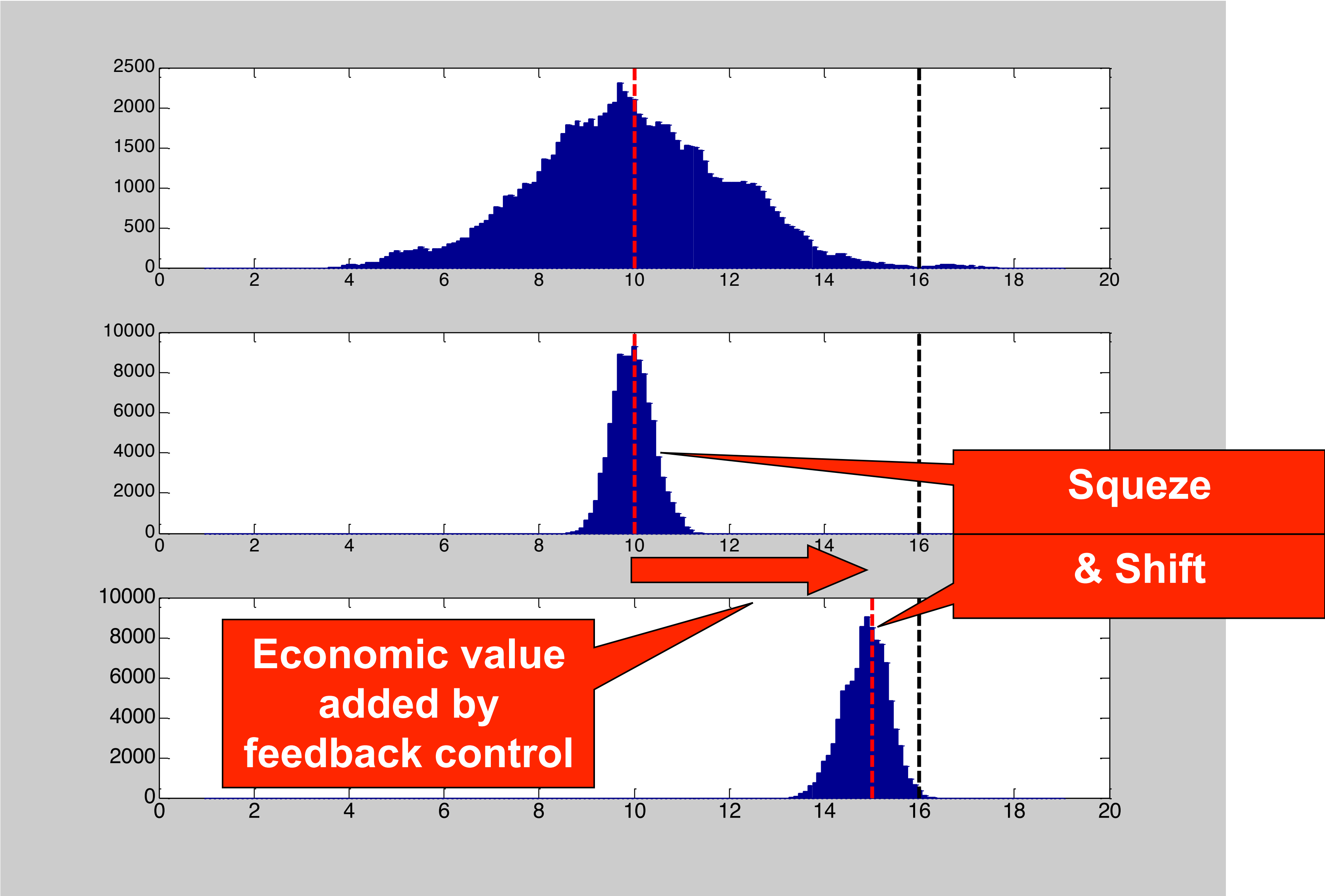
Economic Value of Process Optimization



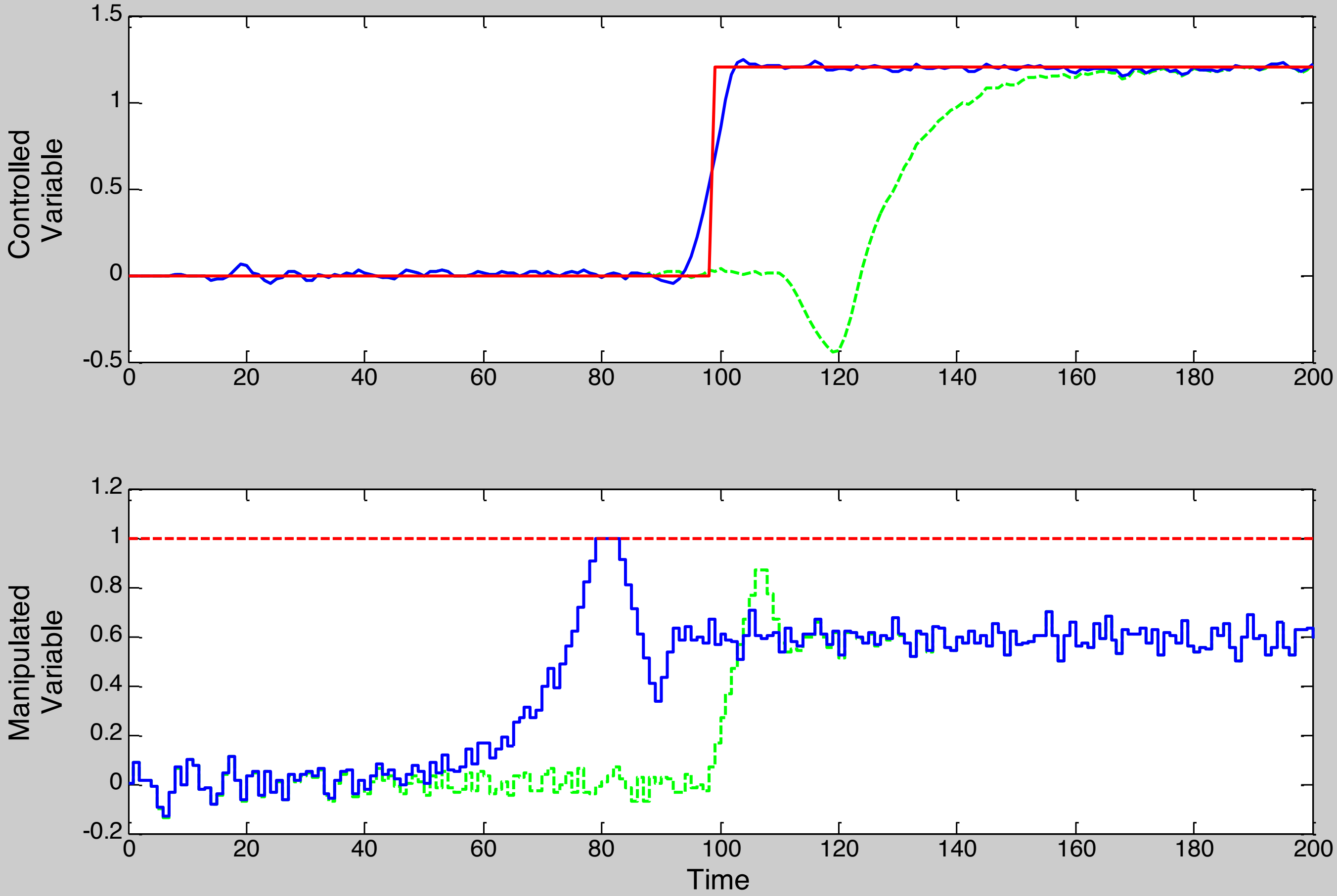
Economic Value of Proces Optimization



Economic Benefit of Process Control



Rapid Product Change



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Drivers of MPC

Powerful Computers

$$\frac{\partial C_l}{\partial t} = -v_l \frac{\partial C_l}{\partial z} + D \frac{\partial^2 C_l}{\partial z^2} + \frac{k}{\varepsilon_l}\left(\frac{C_c}{\varepsilon_l} - C_l\right)$$

$$\frac{\partial C_c}{\partial t} = -v_c \frac{\partial C_c}{\partial z} + D \frac{\partial^2 C_c}{\partial z^2} + \frac{k}{\varepsilon_c}\left(C_l - \frac{C_c}{\varepsilon_l}\right) + \gamma r_w$$

$$\frac{\partial w_{lig}}{\partial t} = -v_c \frac{\partial w_{lig}}{\partial z} + D \frac{\partial^2 w_{lig}}{\partial z^2} + r_w$$

$$\frac{\partial T_l}{\partial t} = -v_l \frac{\partial T_l}{\partial z} + D \frac{\partial^2 T_l}{\partial z^2} + \frac{h}{\varepsilon_l}\left(T_c - T_l\right)$$

$$\frac{\partial T_c}{\partial t} = -v_{cl} \frac{\partial T_c}{\partial z} + D \frac{\partial^2 T_c}{\partial z^2} + \frac{h}{\varepsilon_c}\left(T_l - T_c\right) - \Delta H_r r_w$$

Mathematical / Statistical Modelling

$$\begin{aligned} \min_{x} \quad & x^T Q x + c^T x \\ \text{s.t.} \quad & A_{ieq} x \leq b_{ieq} \\ & A_{eq} x = b_{eq} \\ & l \leq x \leq u \\ & x_i x_j = 0 \quad \forall (i,j) \in \Phi \end{aligned}$$

Optimization Algorithms

Economic MPC
Mathematical Optimization

The portfolio power generation problem can be stated as

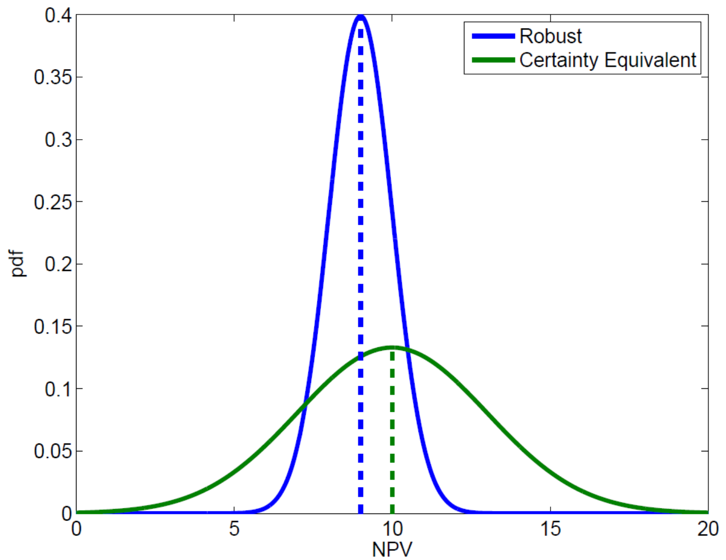
$$\min_{\{u_k\}_{k=0}^{N-1}} \quad \phi = \sum_{k=0}^{N-1} c' u_k$$

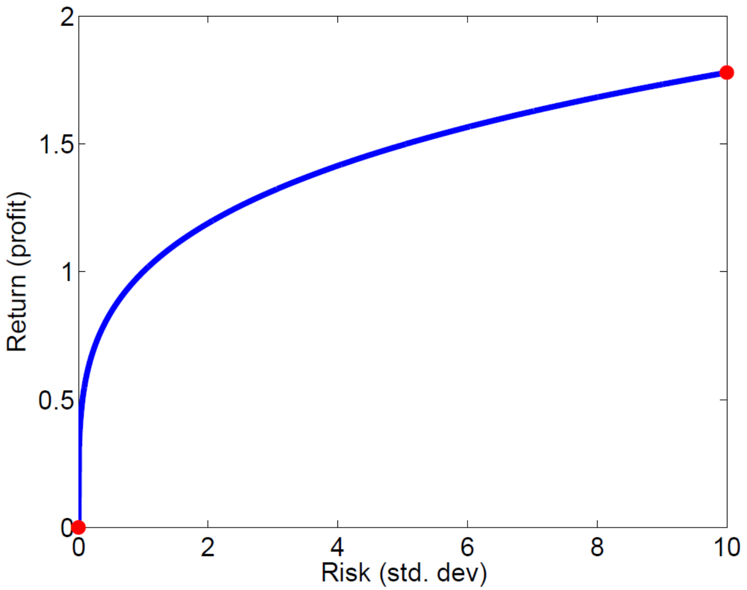
$$\begin{aligned}
s.t. \quad & x_{k+1} = A x_k + B u_k + E d_k && k = 0, 1, \dots, N-1 \\
& y_k = C x_k && k = 1, 2, \dots, N \\
& u_{\min} \leq u_k \leq u_{\max} && k = 0, 1, \dots, N-1 \\
& \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} && k = 0, 1, \dots, N-1 \\
& y_k \geq r_k && k = 1, 2, \dots, N
\end{aligned}$$

The parameters for this problem are

- Initial state, x_0, and previous decision, u_{-1}
- Predicted loads on non-controllable generators (e.g. wind speed on wind turbines): $\{d_k\}_{k=0}^{N-1}$
- Predicted power demand: $\{r_k\}_{k=1}^{N}$

Economic MPC for Uncertain Systems





$$\max_{x \in \mathbb{R}^n} \quad \psi(x) = \alpha \overbrace{E_\theta \left\{ R(x,\theta) \right\}}^{\text{Mean profit}} - (1-\alpha) \overbrace{V_\theta \left\{ R(x,\theta) \right\}}^{\text{Profit variance}}$$

Bi-Criterion Economic MPC

$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \phi(\{u_k, x_{k+1}\}_{k=0}^{N-1}; x_0, \theta)$$

$$s.t. \quad x_{k+1} = F_k(x_k, u_k, \theta) \qquad k = 0, 1, \ldots, N-1$$

$$u_k \in \mathcal{U}$$

Least Squares Objective

$$\phi_{\text{reg}} = \frac{1}{2}\left(\sum_{k=0}^{N-1} \|x_k(\theta) - \bar{x}_k\|_Q^2 + \|u_k(\theta) - \bar{u}_k\|_R^2\right) + \frac{1}{2}\|x_N(\theta) - \bar{x}_N\|_P^2$$

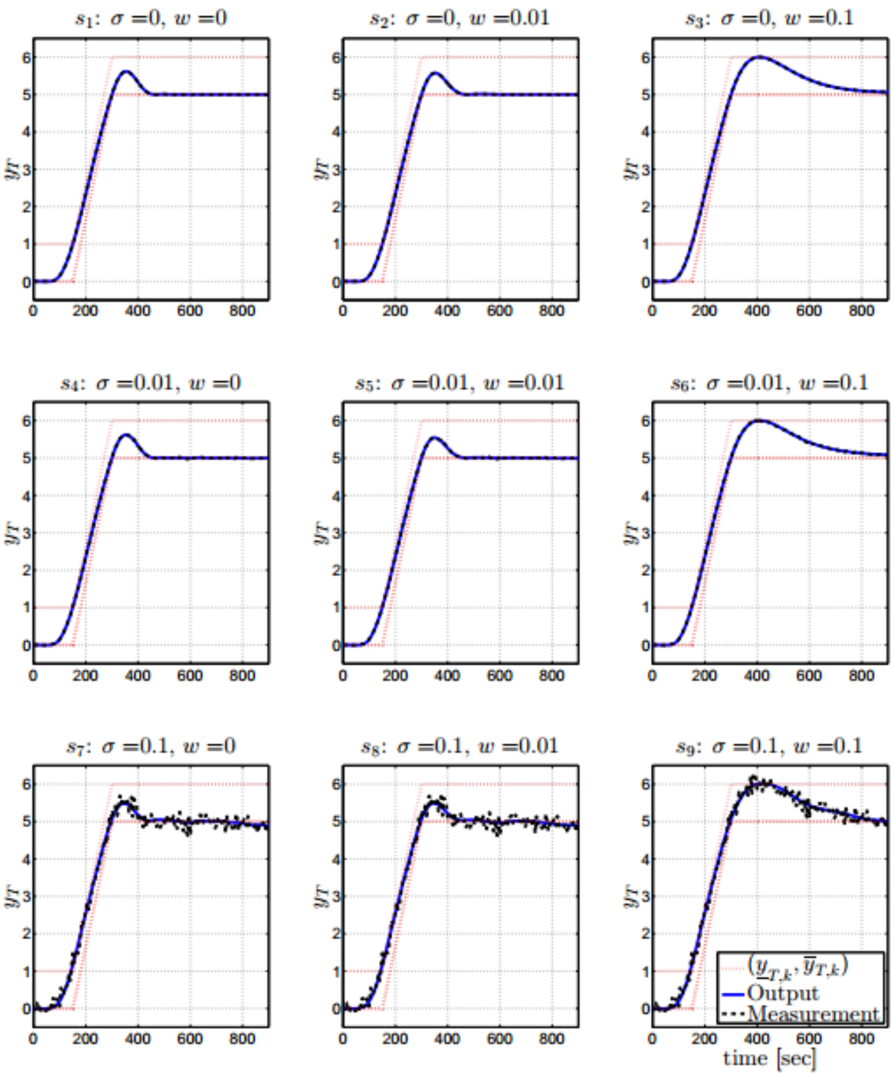
Economic Objective - cost, profit, ...

$$\phi_{\text{eco}} = \sum_{k=0}^{N-1} l_k(x_k, u_k, \theta) + l_N(x_N, \theta)$$

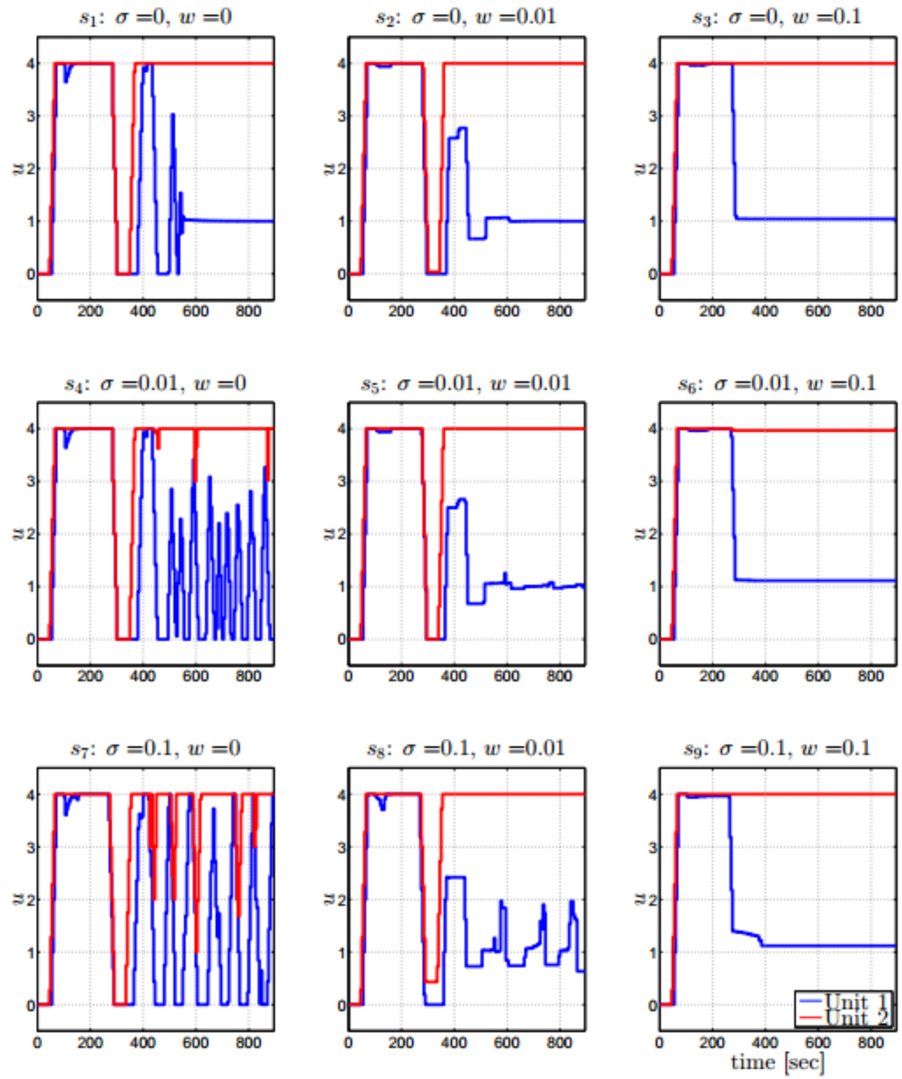
Bi-criterion (cost and variance)

$$\phi = \phi(x, u, \theta) = \alpha\phi_{\text{eco}} + (1-\alpha)\phi_{\text{reg}} \qquad \alpha \in [0,\ 1]$$

Risk Mitigation (Regularization)

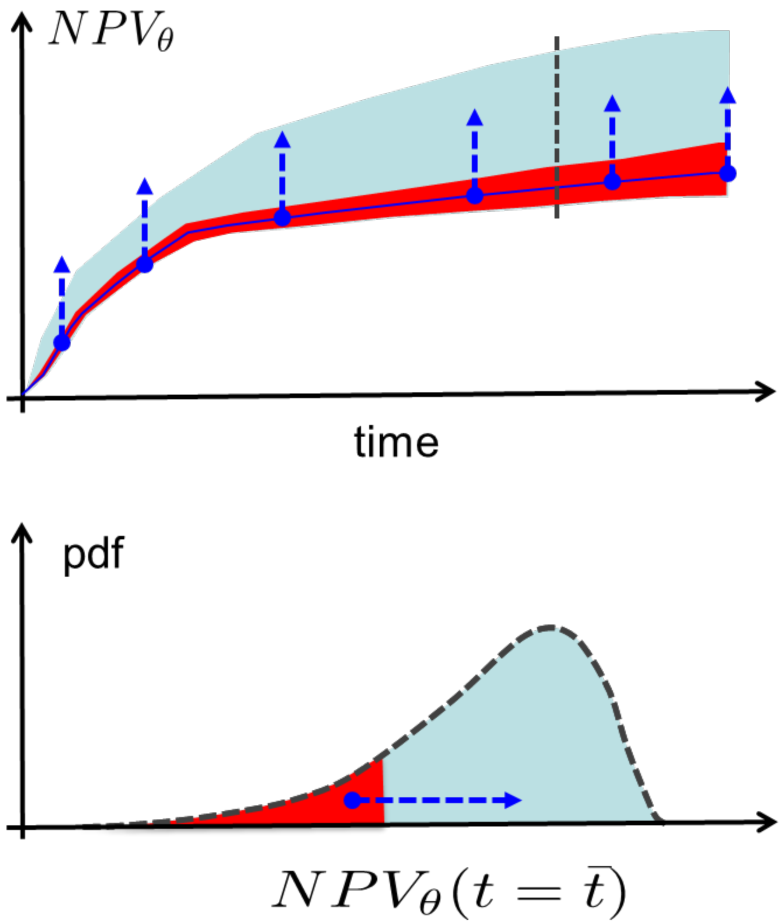


(a) Total power output, y_T.



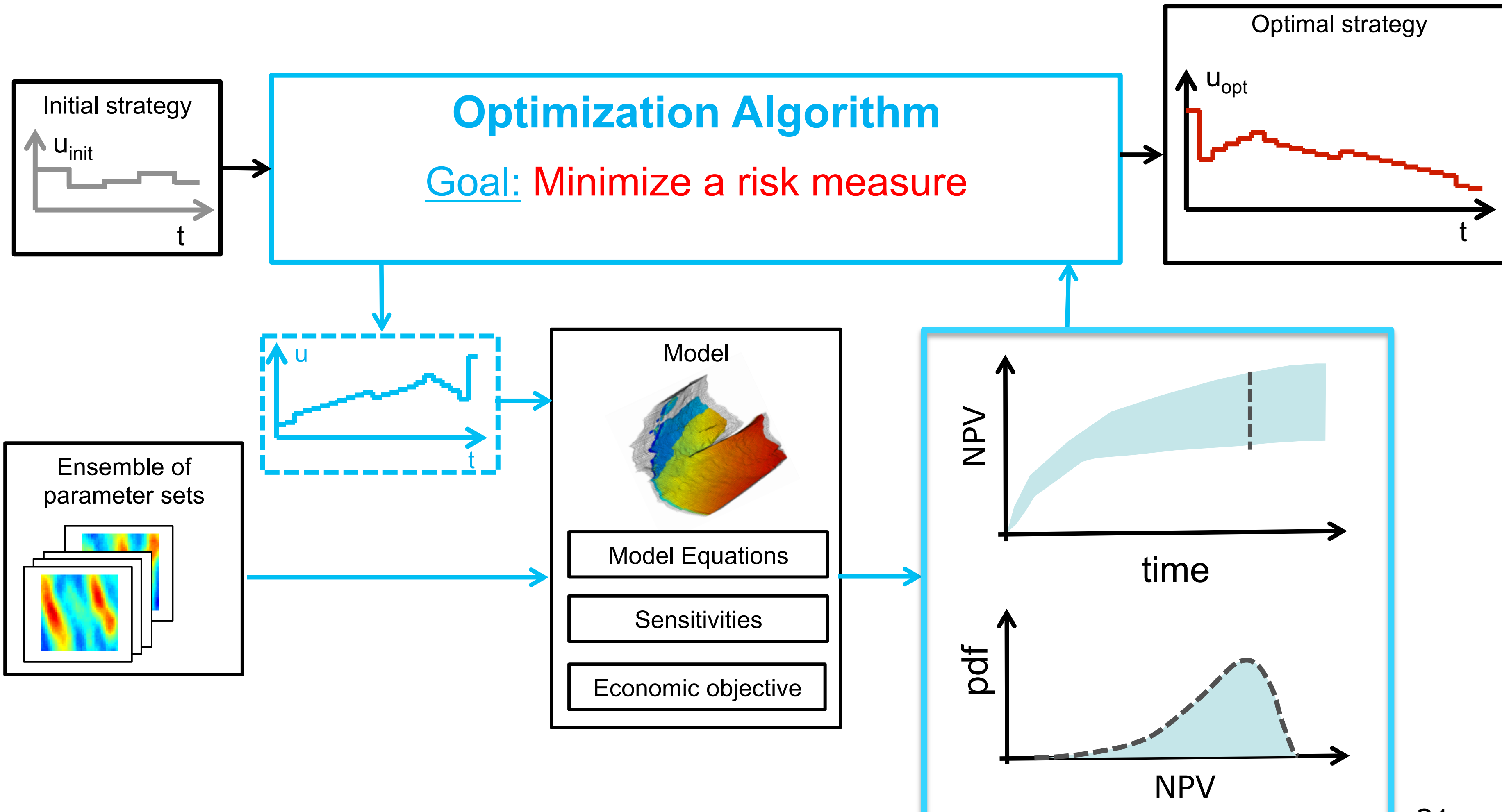
(b) Inputs (power setpoints), u_1 and u_2, for each power plant.

Conditional Value at Risk

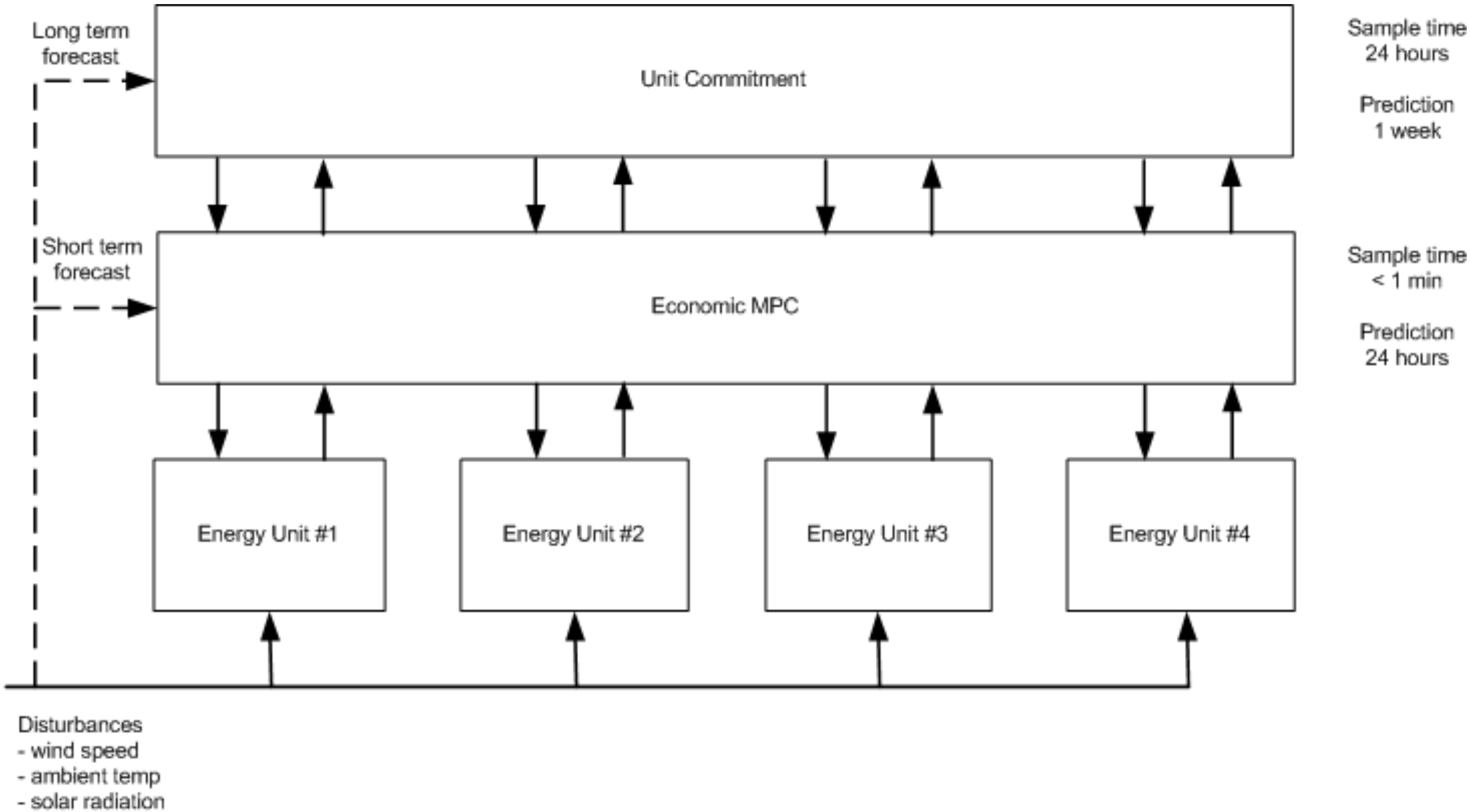


- Risk level
  - How many of the worst outcomes will you consider e.g. 5%
- Short-term / long-term considerations
- Use this measure to select the manipulated variables

Conditional Value at Risk (CVaR)



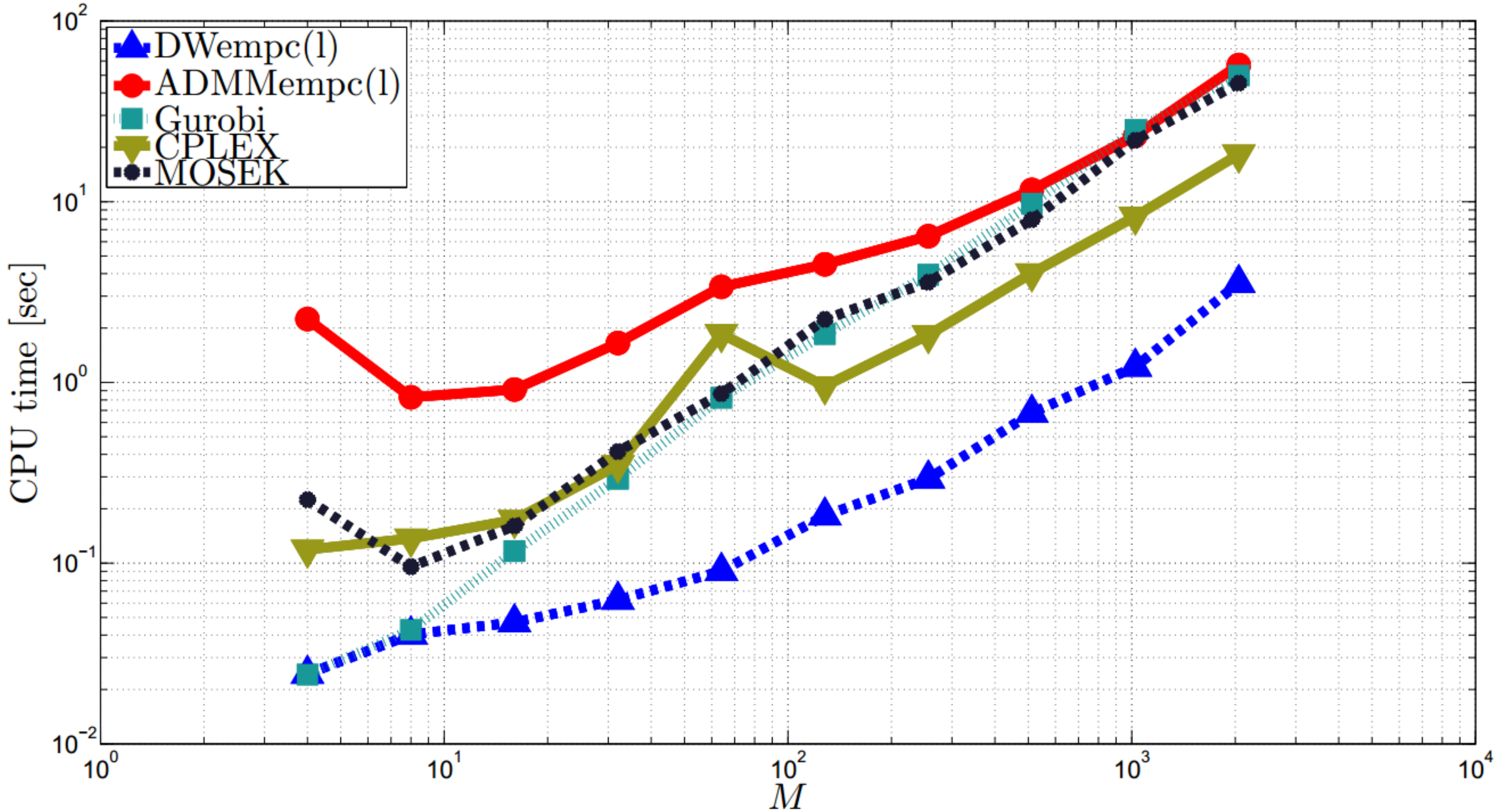
Hierarchical Control Structure



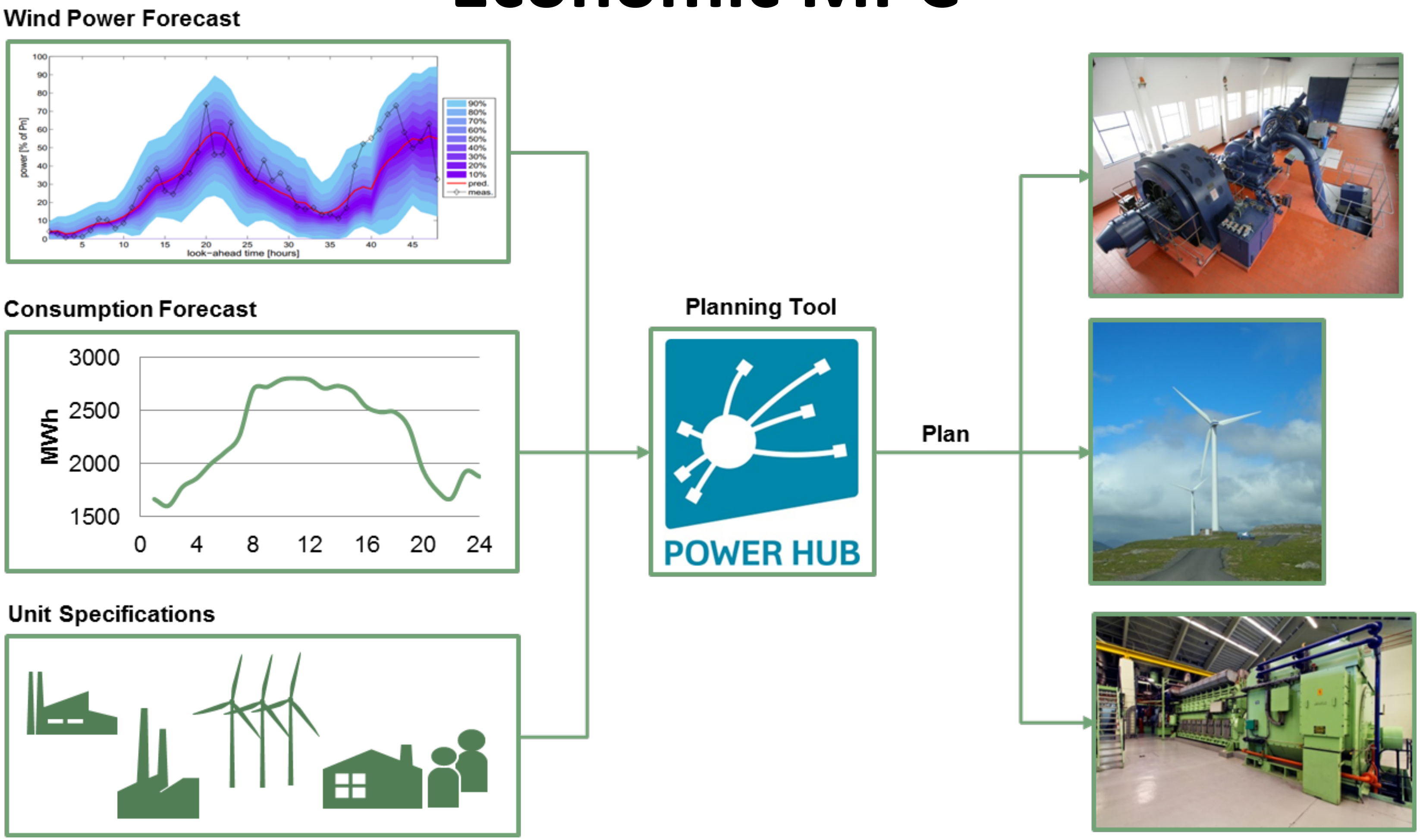
Fast Solver for Direct Control of an Entire City

A Dantzig-Wolfe Decomposition Algorithm for
Linear Economic Model Predictive Control of Dynamically Decoupled Subsystems

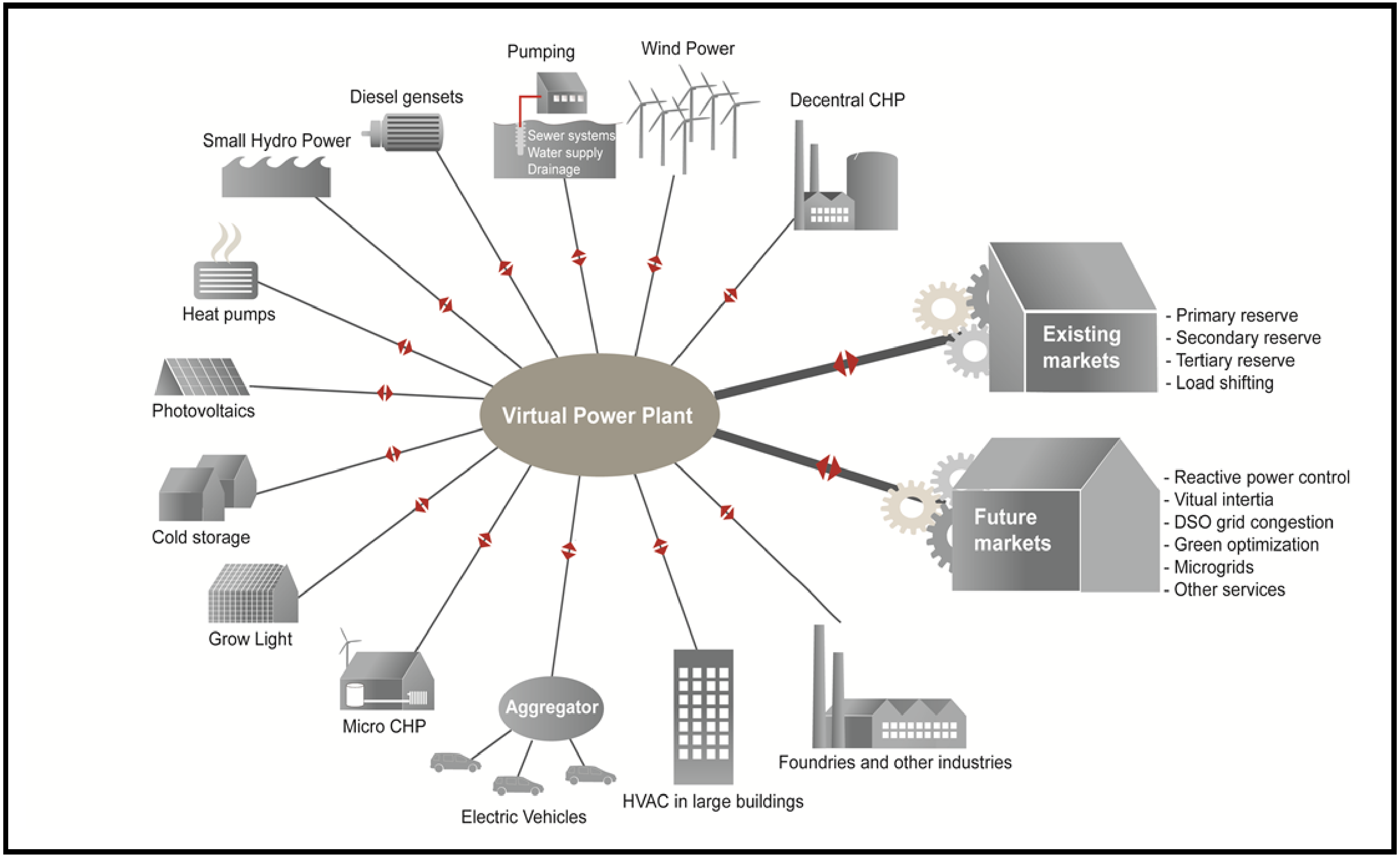
L.E. Sokoler[a,b], L. Standardi[a], K. Edlund[b], N.K. Poulsen[a], H. Madsen[a], J.B. Jørgensen[*,a]



Control of Smart Energy Systems = Economic MPC

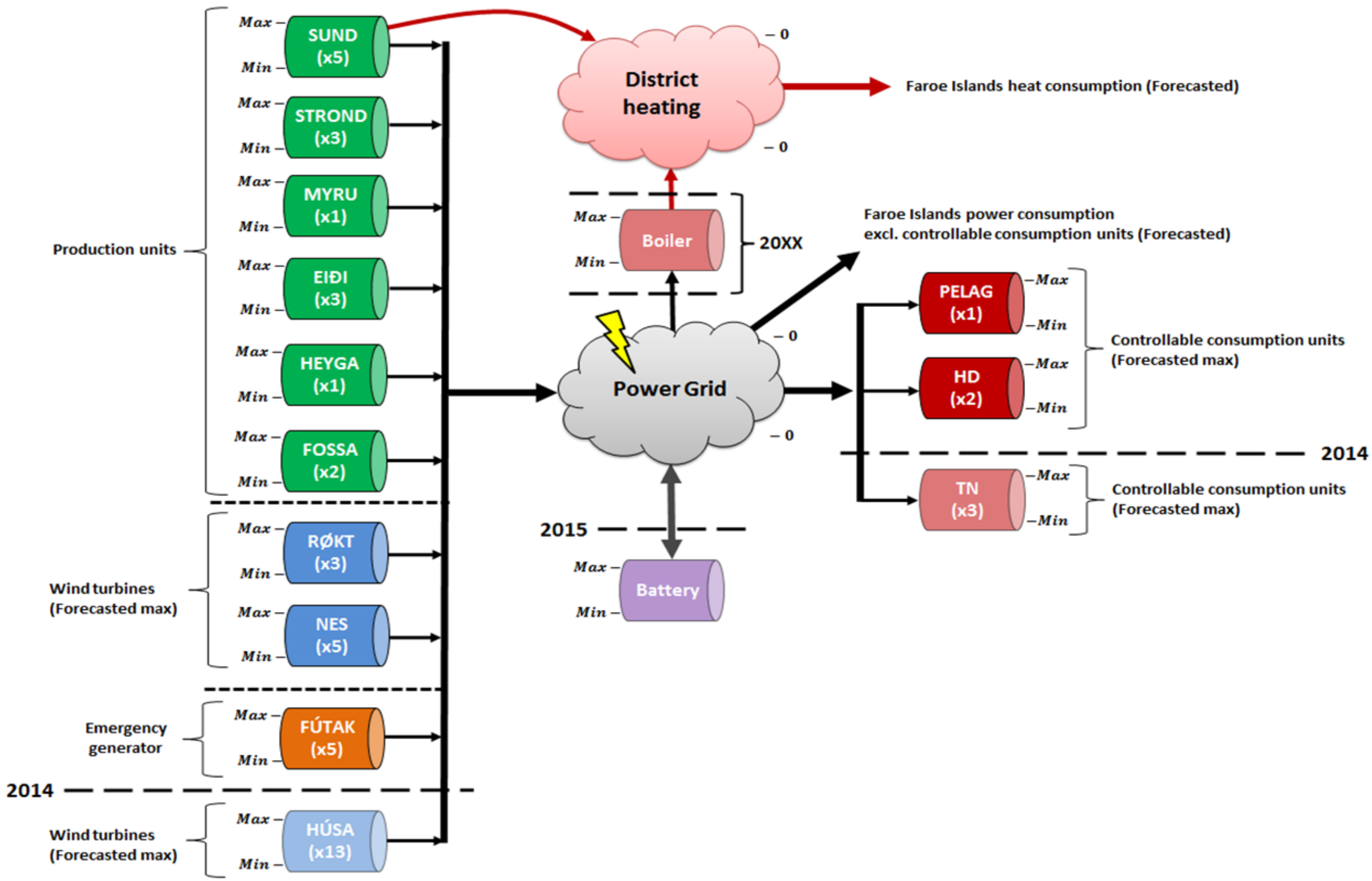


Virtual Power Plants – Controlling the Power Demand

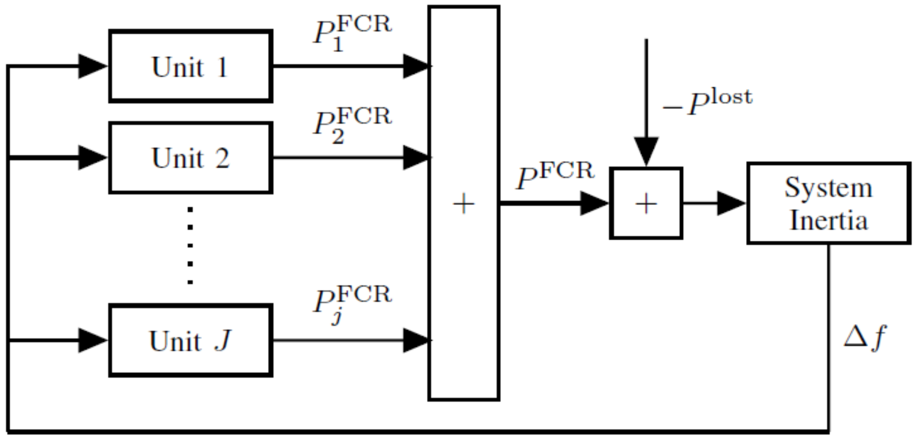


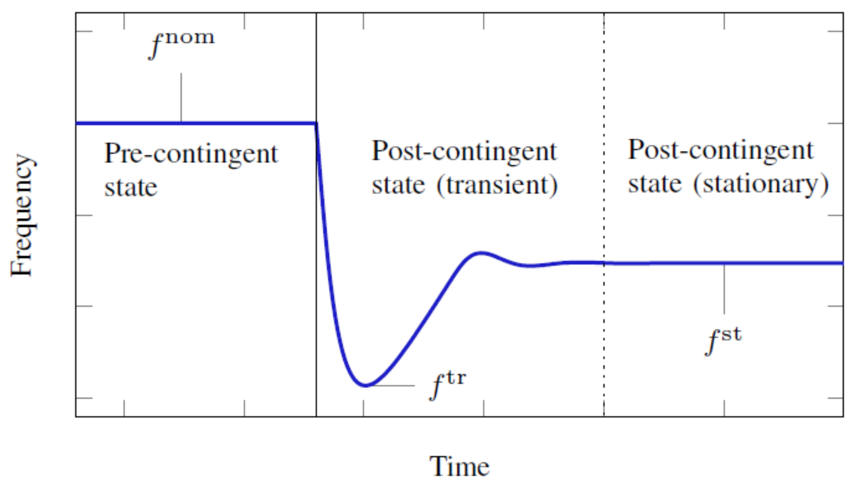
Faroe Islands

The Faroe Island Power System

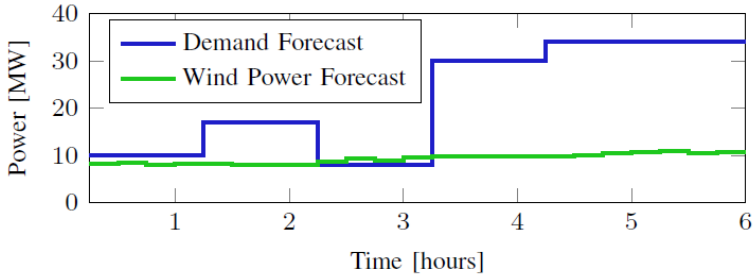


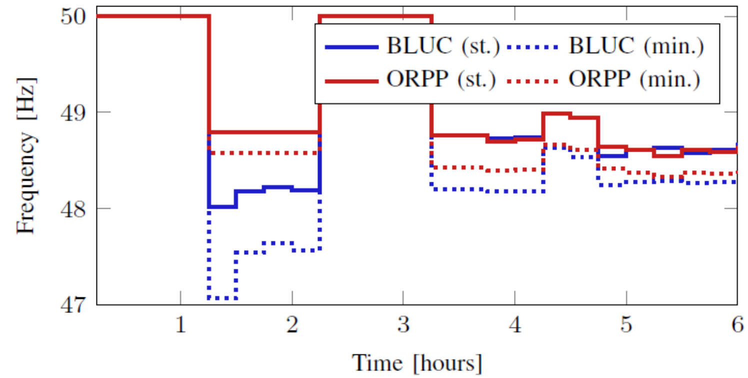
Frequency Dynamics at a Contingency Event

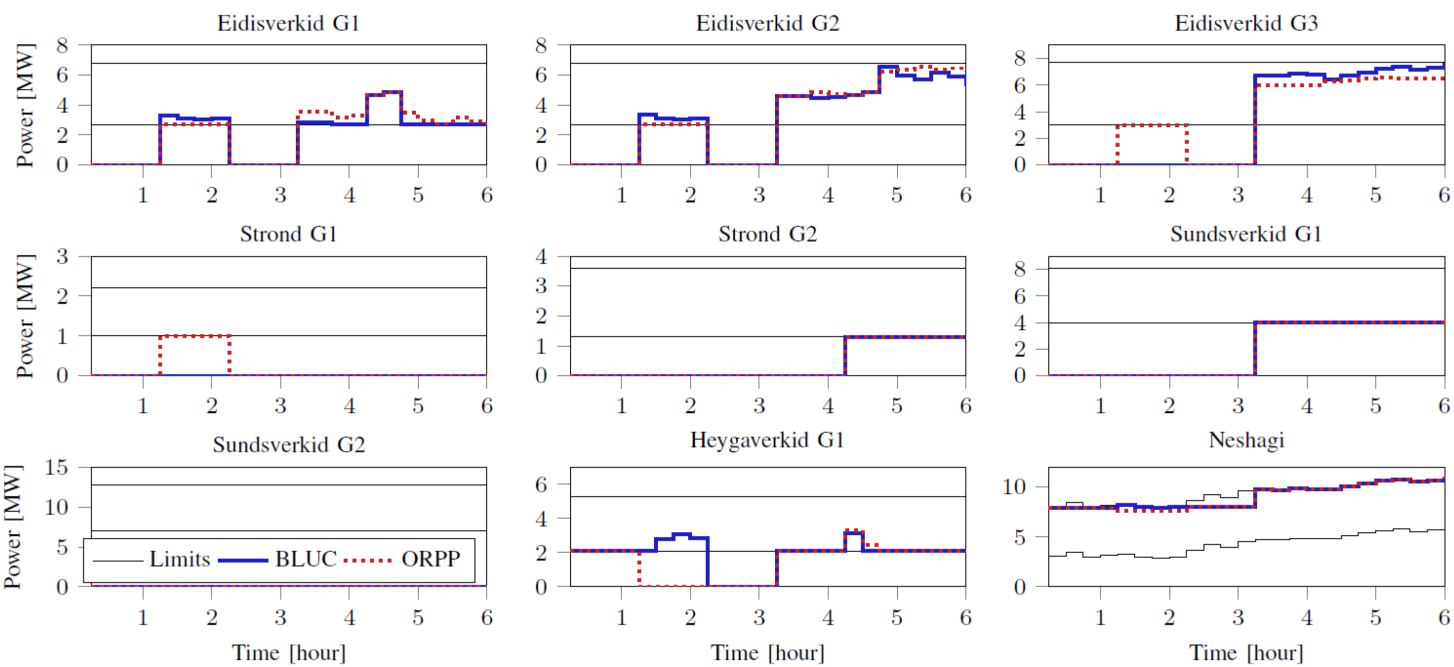




Unit Commitment for Island Operation





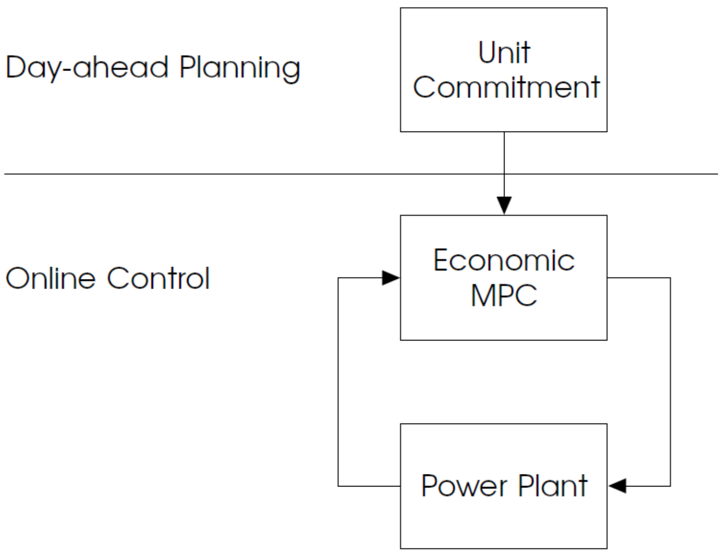


The Idea of Coupling the Unit Commitment and Economic MPC

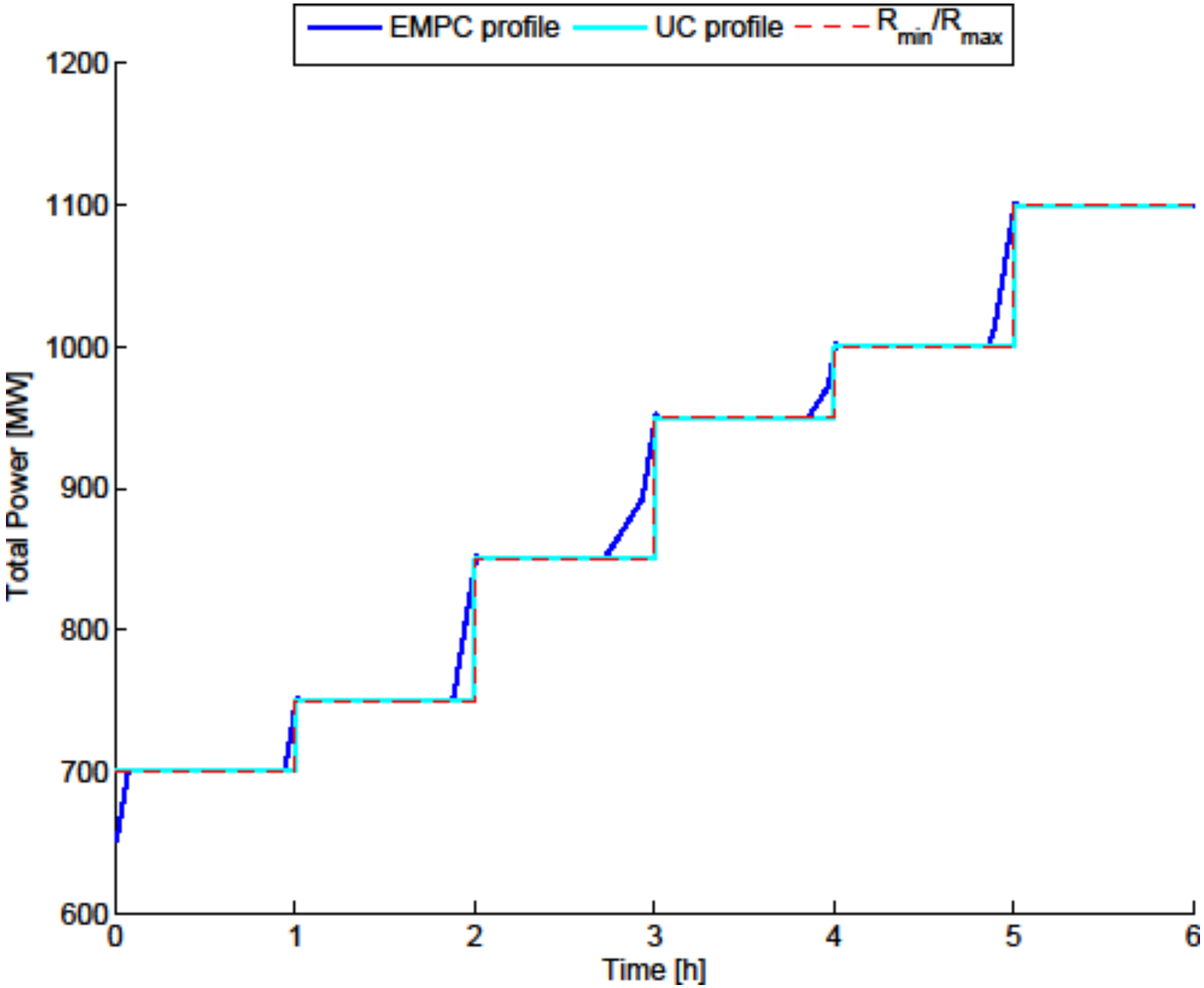
- Day-ahead planning:
  - Prepare power production schedule the next 24 hours based on available forecast on tomorrow demand load and power production from renewable sources

- Online Control:
  - Online framework adapts during the day the predefined schedule based on new and more reliable forecast on power production from renewable sources

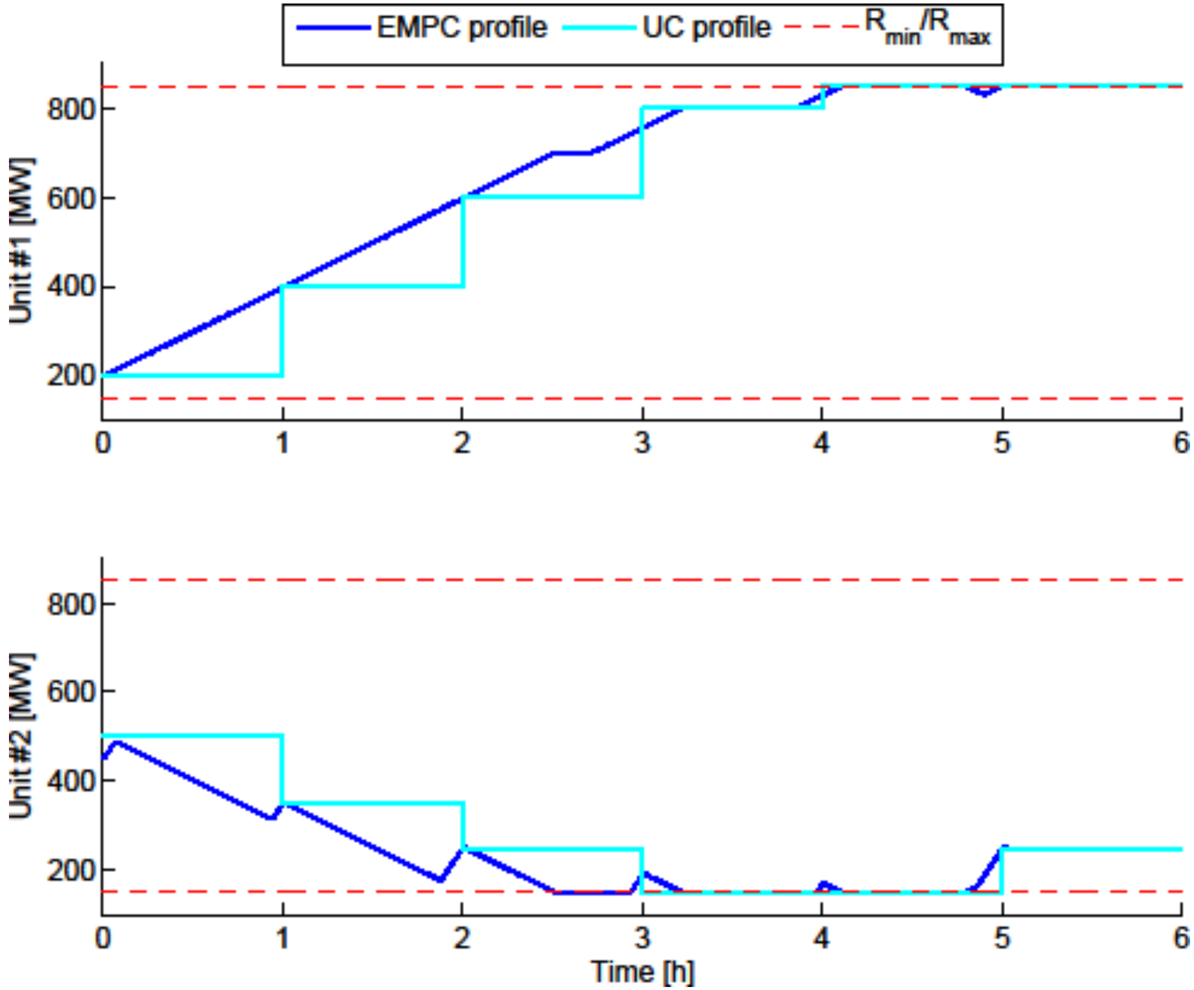
- The key ideas as we will see is that the Unit Commitment problem should be solved as often as possible or coupled with the Economic MPC. To this requires better numerical methods than available today



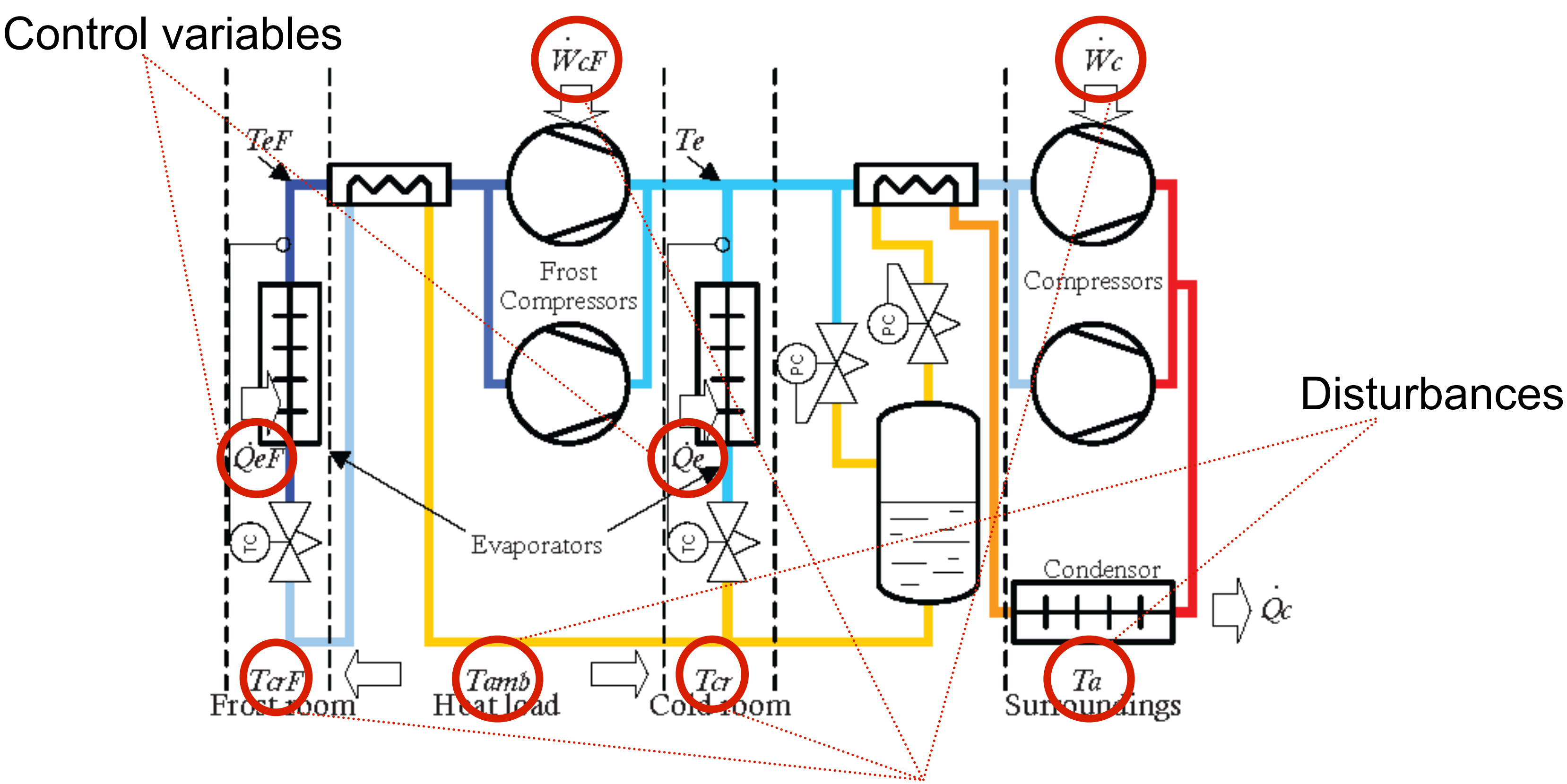
Unit Commitment & Economic MPC



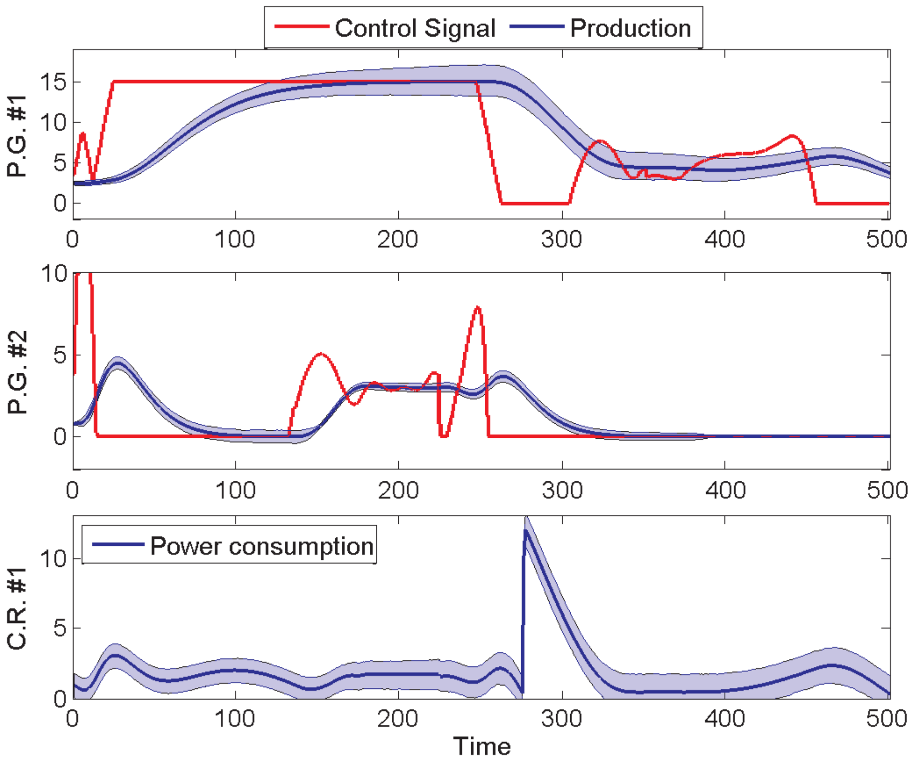
Unit Commitment & Economic MPC

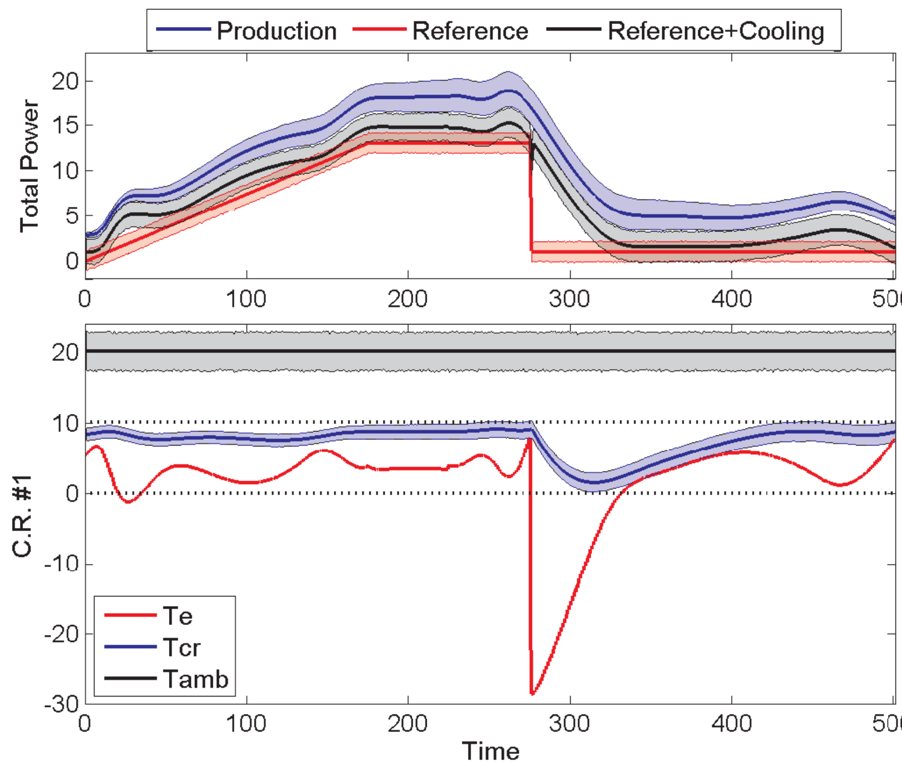


Supermarket Refrigeration



Supermarket Refrigeration





Case Study

Energy Efficient Refrigeration and Flexible Power Consumption in a Smart Grid

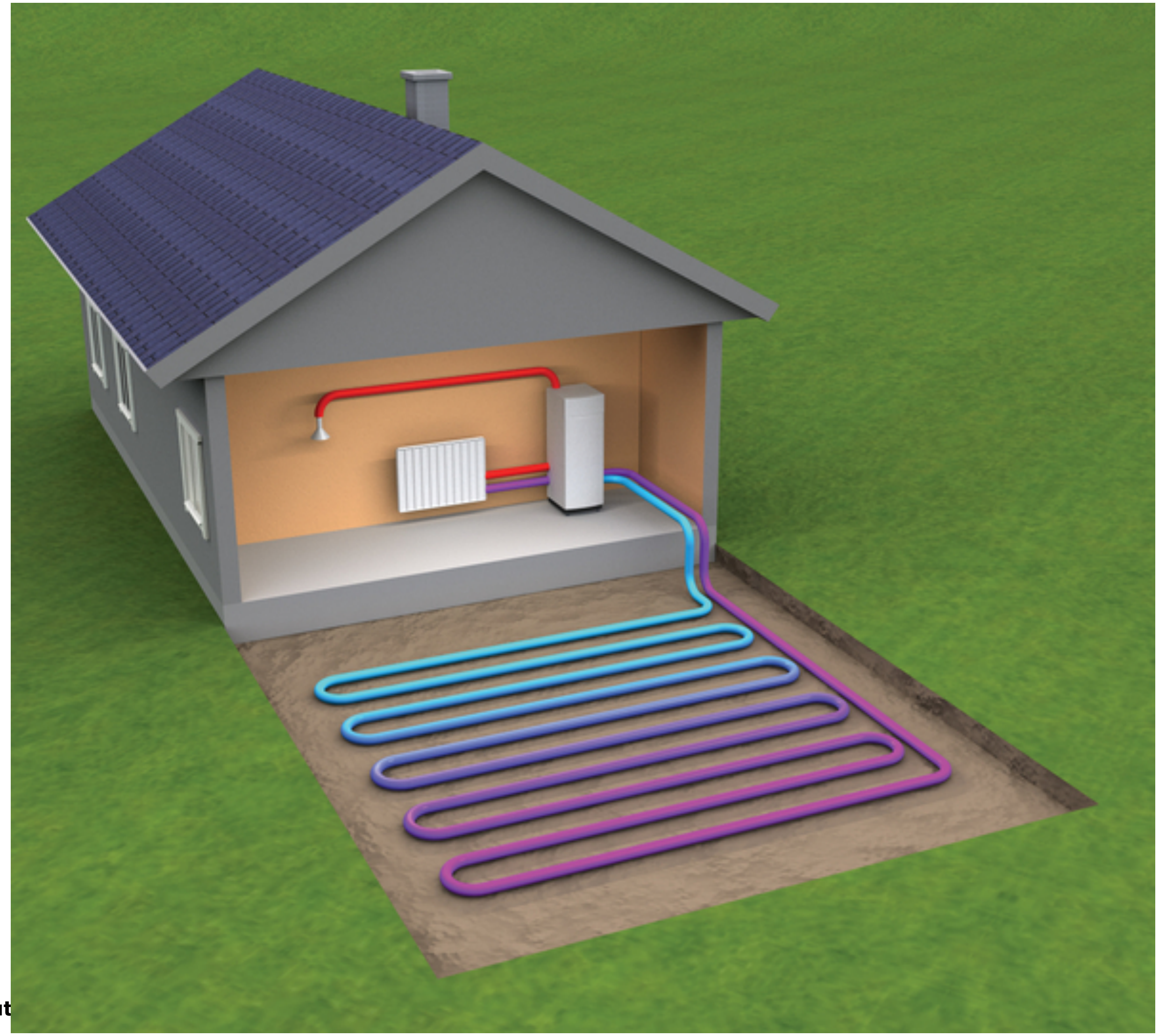
Tobias Gybel Hovgaard, Rasmus Halvgaard, Lars F. S. Larsen and John Bagterp Jørgensen

Risø International Energy Conference 2011   Proceedings   Page 164

Economic Model Predictive Control for Building Climate Control in a Smart Grid

Rasmus Halvgaard, Niels Kjølstad Poulsen, Henrik Madsen and John Bagterp Jørgensen

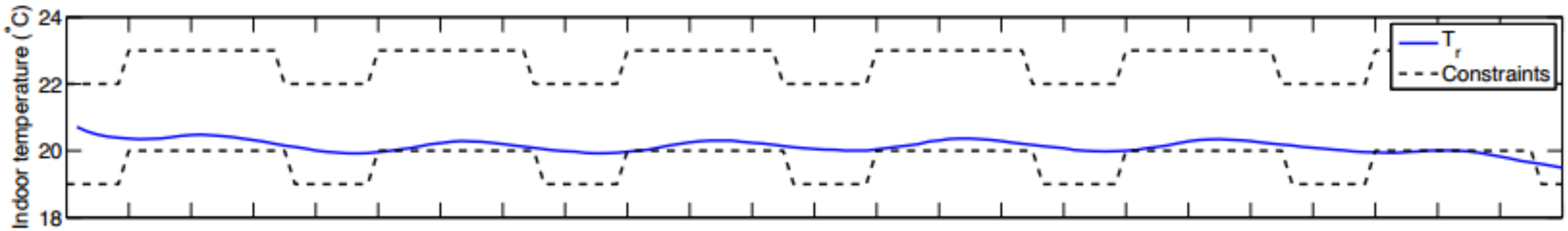
House with Heat Pump



Temperature Control

- Danfoss

- NEST (Google)



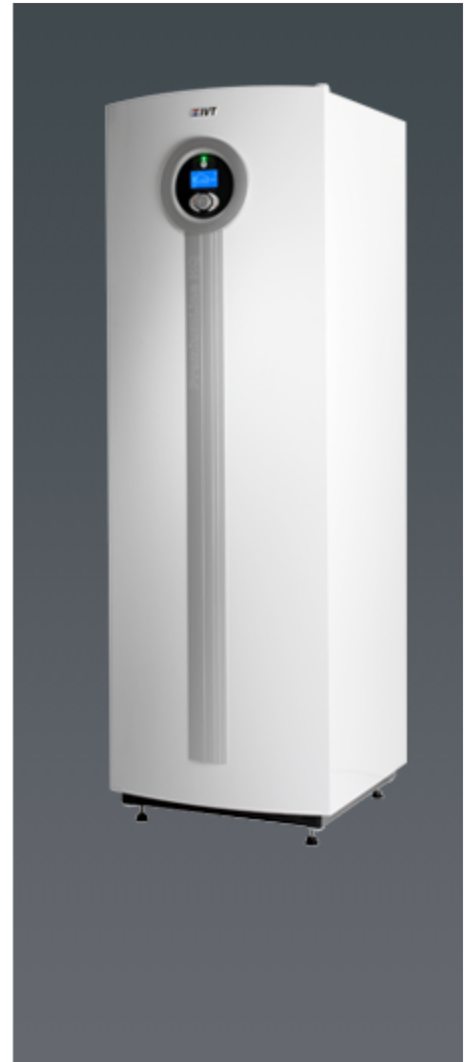
Smart Grid Ready Heat Pump

IVT PremiumLine HQ - En smartare värmepump

- SG READY — Framtidssäkrad med Smartgrid.
- AWS II – Patentsökt funktion som anpassar varmvattenproduktionen efter behovet.
- Lågenergiteknik som kan spara över 2000 kr extra per år.
- Vår tystaste bergvärmepump någonsin.
- Kvalitets- och miljömärkt med Svanen.
- 10 års garanti på kompressorn ingår. För vi chansar aldrig.

Klicka på symbolerna för att läsa mer.

Med IVT PremiumLine HQ introducerar vi nästa generation av smarta värmepumpar. Styrsystemet är förberett med Smartgrid. Det innebär att värmepumpen kan kopplas direkt mot den nordiska elbörsen, och själv anpassar så att den jobbar hårdast när elpriset är lägst. Den här tekniken sparar både pengar åt dig samtidigt som den bidrar till en jämnare och mer hållbar energianvändning. För att kunna utnyttja funktionen behöver värmesystemet kompletteras med IVT Anywhere samt ett abonnemang som kostar 39 kronor per år – en investering som snabbt sparas in.



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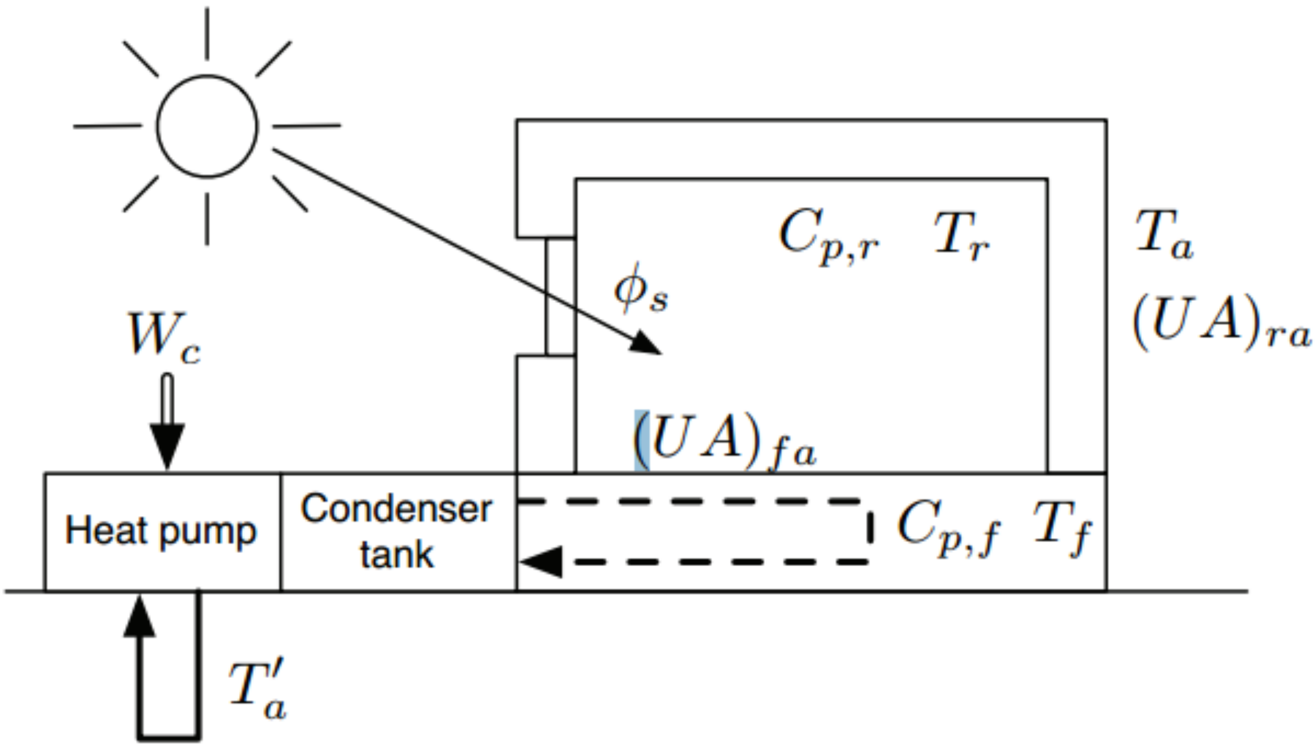
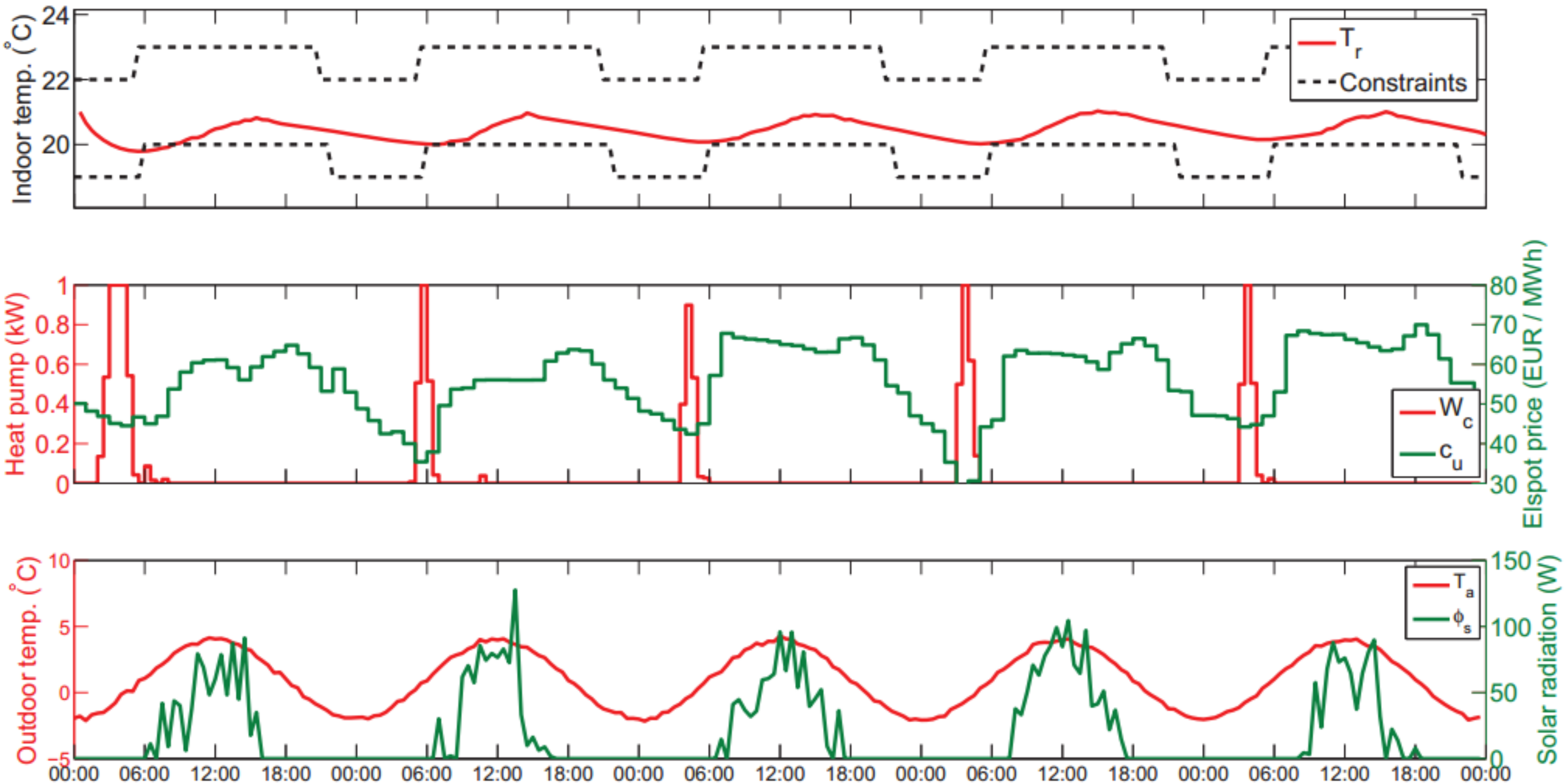
Schematics of House with Heat Pump



Economic MPC for Building Control



MPC & Process Optimization Toolbox

Features

- Simplicity - easy to
  - Commission
  - Tune
  - Maintain
- Customizable and adaptable to
  - Process dynamics
  - Process modifications
  - Operational strategies
- Includes frontier technologies in
  - Mathematical Optimization
  - Process Control
  - Software Engineering
  - Mathematical/Statistical Modeling and Simulation

The Extended LQ Problem

$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \quad \phi = \sum_{k=0}^{N-1} l_k(x_k, u_k) + l_N(x_N)$$

$$s.t. \qquad x_{k+1} = A_k x_k + B_k u_k + b_k \quad k \in \mathcal{N}$$

with $\mathcal{N} = \{0, 1, \ldots, N-1\}$ and stage costs defined by

$$l_k(x_k, u_k) = \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}' \begin{bmatrix} Q_k & M_k' \\ M_k & R_k \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \begin{bmatrix} q_k \\ s_k \end{bmatrix}' \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \rho_k$$

$$l_N(x_N) = \frac{1}{2} x_N' P_N x_N + p_N' x_N + \gamma_N$$

KKT System for the Extended LQ Problem

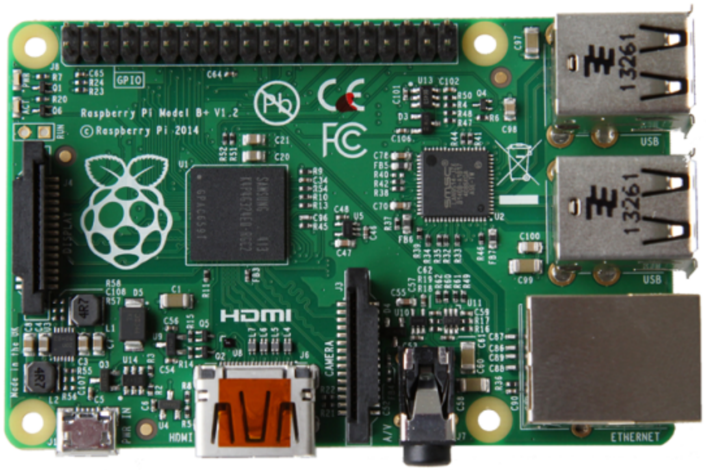
$$
\left[\begin{array}{cccccc|ccc}
R_0 & & & & & & B_0' & & \\
 & Q_1 & M_1' & & & & -I & A_1' & \\
 & M_1 & R_1 & & & & & B_1' & \\
 & & & Q_2 & M_2' & & & -I & A_2' \\
 & & & M_2 & R_2 & & & & B_2' \\
 & & & & & P_3 & & & -I \\
\hline
B_0 & -I & & & & & & & \\
 & A_1 & B_1 & -I & & & & & \\
 & & & A_2 & B_2 & -I & & &
\end{array}\right]
\left[\begin{array}{c}
u_0 \\ x_1 \\ u_1 \\ x_2 \\ u_2 \\ x_3 \\ \hline \pi_1 \\ \pi_2 \\ \pi_3
\end{array}\right]
= -
\left[\begin{array}{c}
\tilde{s}_0 \\ q_1 \\ s_1 \\ q_2 \\ s_2 \\ p_3 \\ \hline \tilde{b}_0 \\ b_1 \\ b_2
\end{array}\right]
$$

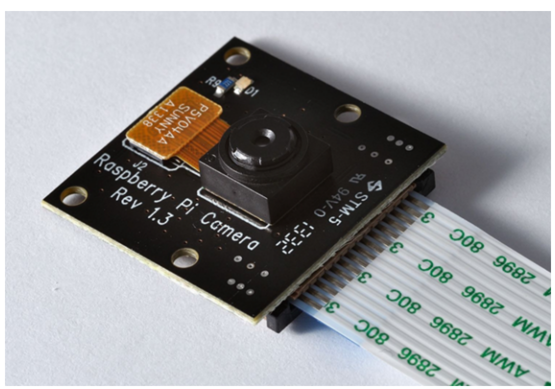
KKT System for the Extended LQ Problem

$$
\left[\begin{array}{cc|ccc|ccc|c}
R_0 & B_0' & & & & & & & \\
B_0 & 0 & -I & & & & & & \\
\hline
 & -I & Q_1 & M_1' & A_1' & & & & \\
 & & M_1 & R_1 & B_1' & & & & \\
 & & A_1 & B_1 & 0 & -I & & & \\
\hline
 & & & & -I & Q_2 & M_2' & A_2' & \\
 & & & & & M_2 & R_2 & B_2' & \\
 & & & & & A_2 & B_2 & 0 & -I \\
\hline
 & & & & & & & -I & P_3
\end{array}\right]
\left[\begin{array}{c}
u_0 \\ \pi_1 \\ \hline x_1 \\ u_1 \\ \pi_2 \\ \hline x_2 \\ u_2 \\ \pi_3 \\ \hline x_3
\end{array}\right]
= -
\left[\begin{array}{c}
\tilde{s}_0 \\ \tilde{b}_0 \\ \hline q_1 \\ s_1 \\ b_1 \\ \hline q_2 \\ s_2 \\ b_2 \\ \hline p_3
\end{array}\right]
$$

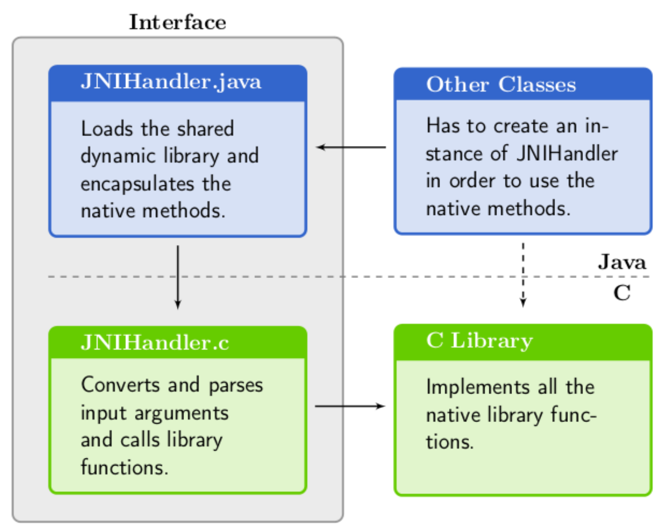
Software Implementation

Raspberry PI C/C++





Smart Phone C/Java/Matlab



Thank You – Q & A

John Bagterp Jørgensen

Technical University of Denmark

E-mail: jbjo@dtu.dk

DTU Compute

Department of Applied Mathematics and Computer Science