Economic Model Predictive Control for Energy Systems in Smart Homes

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Introduction

Elon Musk’s vision of an energy system

Source: https://www.tesla.com/solarroof

- How can we model this system?
- How can we control it in a smart way?
Our vision of a smart home energy system

An energy system based on renewable energy sources

- Solar panels
- Battery
- Heat pump
- EV
- Wind turbines
Goal for the smart home energy system

Heat pump model:
\[
\dot{x}_{hp} = A_{hp,c} x_{hp} + B_{hp,c} u_{hp} + E_{hp,c} d_{hp},
\]
\[
y_{hp} = C_{hp,c} x_{hp}.
\]

EV battery model:
\[
\dot{x}_{ev} = A_{ev,c} x_{ev} + B_{ev,c} u_{ev} + E_{ev,c} d_{ev},
\]
\[
y_{ev} = C_{ev,c} x_{ev}.
\]

Stationary battery model:
\[
\dot{x}_{bat} = A_{bat,c} x_{bat} + B_{bat,c} u_{bat} + E_{bat,c} d_{bat},
\]
\[
y_{bat} = C_{bat,c} x_{bat}.
\]

Smart home energy system model:
\[
\dot{x} = A_c x + B_c u + E_c d,
\]
\[
y = C_c x.
\]
Our idea of a smart home

1. Heat pump input power is regulated such that indoor temperature is kept between pre-specified intervals.
2. EV battery is charged such that a pre-defined driving pattern is possible.
3. Stationary battery is discharged to provide power for heat pump, to charge EV battery and to sell energy.
4. Stationary battery is charged by power from photo voltaic cells and purchasing power.
Economic MPC

Models of the devices in the energy system

Steps 1–4 are accomplished using an economic model predictive controller (EMPC) minimizing electricity costs

Linear state space models

Each device is modeled as a continuous-time linear state space model:

\[ \dot{x} = A_c x + B_c u + E_c d, \]
\[ y = C_c x. \]

Here \( x \) is the state-, \( u \) is the manipulated-, \( d \) is the disturbance- and \( y \) is the output-variable.

- The heat pump is modeled as a ground source based heat pump for a floor heating system with constant coefficient of performance [1]
- The batteries are modeled as simple integrators with transfer losses [2]

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Economic MPC
State space model of the smart home energy system

Coupling in the state space matrices

Variable definitions:

\[ x = (x_{hp}; x_{ev}; x_{bat}) \], \[ u = (u_{hp}; u_{ev}; u_{bat}^+; u_{bat}^-) \], \[ d = (d_{hp}; d_{ev}; d_{bat}) \]
\[ y = (y_{hp}; y_{ev}; y_{bat}) \]

State space matrices:

\[ A_c = \begin{pmatrix} A_{hp,c} & 0 & 0 \\ 0 & A_{ev,c} & 0 \\ 0 & 0 & A_{bat,c} \end{pmatrix}, \quad B_c = \begin{pmatrix} B_{hp,c} & 0 & 0 & 0 \\ 0 & B_{ev,c} & 0 & 0 \\ \ -c_{bat} & \ -c_{bat} & \ c_{bat} & \ -c_{bat} \end{pmatrix} \]
\[ E_c = \begin{pmatrix} E_{hp,c} & 0 & 0 \\ 0 & E_{ev,c} & 0 \\ 0 & 0 & c_s \cdot c_{bat} \end{pmatrix}, \quad C_c = \begin{pmatrix} C_{hp,c} & 0 & 0 \\ 0 & C_{ev,c} & 0 \\ 0 & 0 & C_{bat,c} \end{pmatrix} \]

Where \( d_{bat} = [\text{solar radiation power}] \),
\[ c_{bat} = \frac{[\text{charging efficiency}]}{[\text{battery capacity}]}, \quad c_s = [\text{#photo voltaic cells}] \cdot [\text{cell efficiency}]. \]
Economic MPC

State space model of the smart home energy system

Figure: Overview of the coupling in the smart home energy system.
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Optimization problem

Objective function: \[ \min_{u,v} \phi_1 = \sum_{k=0}^{N-1} c_{u,k} u_k + \rho_v v_{k+1} \]

Discretized state space model: \[ \begin{align*}
    x_{k+1} &= A x_k + B u_k + E d_k & k \in \mathcal{N} \\
    y_k &= C x_k & k \in \mathcal{N}^+ 
\end{align*} \]

Input constraints: \[ \begin{align*}
    u_{\min,k} &\leq u_k \leq u_{\max,k} & k \in \mathcal{N} \\
    \Delta u_{\min,k} &\leq \Delta u_k \leq \Delta u_{\max,k} & k \in \mathcal{N} 
\end{align*} \]

Soft output constraints: \[ \begin{align*}
    y_{\min,k} &\leq y_k + v_k & k \in \mathcal{N}^+ \\
    y_{\max,k} &\geq y_k - v_k & k \in \mathcal{N}^+ 
\end{align*} \]

Slack variable constraint: \[ 0 \leq v_k & \quad k \in \mathcal{N}^+ \]

With \( \mathcal{N} = \{0, \ldots, N - 1\} \) and \( \mathcal{N}^+ = \{1, \ldots, N\} \), where \( N \) is the prediction horizon, \( c_{u,k} \) are electricity prices and \( \rho_v \) is a penalty parameter.
Economic MPC
Challenge for the heat pump

Choice of the penalty parameter

- When the penalty parameter $\rho_v$ is chosen too small the controller will not turn on the heat pump
- When $\rho_v$ is chosen too large the controller will avoid violating the lower temperature constraint leading to higher electricity costs

Solution approach

We introduce multi-level soft constraints such that temperature violations are tolerated to some degree:

Figure: Illustration of multi-level soft constraints. Violations below the borderline are penalized harder than above.
Economic MPC

Enhance heat pump performance

Figure: Heat pump performance for different penalty parameters. SSC = standard soft constraints, MLSC = multi-level soft constraints.
Economic MPC

Challenge for short prediction horizons

**Challenge**

When using a short prediction horizon there is a chance that the controller does not turn the device on, which can lead to higher electricity costs later.

**Solution: Cost-to-go term**

The solution is to account for the value of stored energy in the end of the prediction horizon:

\[ \phi_2 = \phi_1 + \text{[cost-to-go term]} \]

![Figure: Battery performance in presence of a constant discharge of 0.1 kW. \( \phi_1 \) = standard objective function, \( \phi_2 \) = objective function with cost-to-go term.](image)

Figure: Battery performance in presence of a constant discharge of 0.1 kW. \( \phi_1 \) = standard objective function, \( \phi_2 \) = objective function with cost-to-go term.
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Performance of the smart home energy system

![Graphs showing performance of heat pump, EV, Battery, and Solar]

Figure: Illustration of EMPC performance.
Contributions

We formulated:

- A simple state space model of a smart home energy system

We controlled the smart home energy system by an economic MPC, where we used

- Multi-level soft constraints for more intuitive tuning
- An objective function with a cost-to-go term to account for the value of stored energy in the end of the prediction horizon

Source: https://www.tesla.com/solarroof
Thank you for your attention!

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Heat pump model

Additional slides
Heat pump model

Energy balances:

\[ C_{p,r} \dot{T}_r = R_{fr} - R_{ra} + (1 - p)\phi_s, \]
\[ C_{p,f} \dot{T}_f = R_{wf} - R_{fr} + p\phi_s, \]
\[ C_{p,w} \dot{T}_w = \eta W_c - R_{wf}, \]

Heat transfer rates:

\[ R_{wf} = (UA)_{wf}(T_w - T_f), \]
\[ R_{fr} = (UA)_{fr}(T_f - T_r), \]
\[ R_{ra} = (UA)_{ra}(T_r - T_a), \]
## Heat pump model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$</td>
<td>°C</td>
<td>Room air temperature</td>
</tr>
<tr>
<td>$T_f$</td>
<td>°C</td>
<td>Floor temperature</td>
</tr>
<tr>
<td>$T_w$</td>
<td>°C</td>
<td>Water temperature in the floor heating pipes</td>
</tr>
<tr>
<td>$T_a$</td>
<td>°C</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>$T'_a$</td>
<td>°C</td>
<td>Ground temperature</td>
</tr>
<tr>
<td>$W_c$</td>
<td>W</td>
<td>Heat pump compressor input power</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>W</td>
<td>Solar radiation power: Solar radiation (W/m$^2$) times effective window area (2.9 m$^2$)</td>
</tr>
</tbody>
</table>
Battery model

\[ \dot{\zeta} = \frac{1}{Q_n} \left( \eta^+ P_c^+ - \eta^- P_c^- \right). \]

| Description                        |  
|------------------------------------|---
| \( \zeta \)                        | State of Charge (SOC)  
| \( P_c^+ \)                        | Charge power  
| \( P_c^- \)                        | Discharge power  
| \( \eta^+ \)                       | Charging efficiency  
| \( \eta^- \)                       | Discharging efficiency  
| \( Q_n \)                          | Nominal battery capacity |
Multi-level soft constraints formulation

\[ \phi = \sum_{k=0}^{N-1} (c'_u u_k + \rho^1_v v^1_{k+1} + \rho^2_v v^2_{k+1}) \]

\[ y_{\text{min},k} \leq y_k + v^1_k + v^2_k, \]

\[ y_{\text{max},k} \geq y_k - v^1_k, \]

\[ 0 \leq v^1_k \leq v^1_{\text{max}}, \]

\[ 0 \leq v^2_k. \]
Additional slides

Value of stored energy at the end of the prediction horizon

Formulation:

\[ \phi_2 = \phi_1 - (\bar{c}_N x_N - \bar{c}_0 x_0), \]

where \( \bar{c}_0 \) and \( \bar{c}_N \) is the value of stored energy at the start and the end of the prediction horizon.