Economic Model Predictive Control for Energy Systems in Smart Homes

Hjørdis Amanda Schlüter, Dimitri Boiroux, Niels Kjølstad Poulsen, Henrik Madsen, John Bagterp Jørgensen

DTU Compute
Department of Applied Mathematics and Computer Science

Elon Musk's vision of an energy system

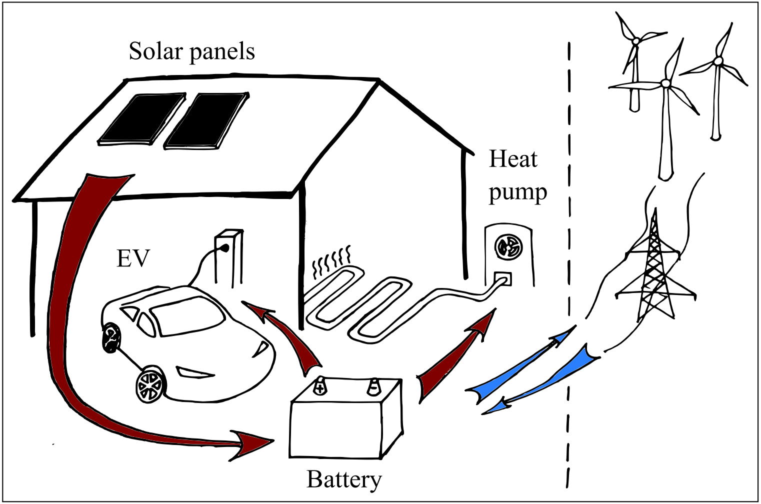
Source: https://www.tesla.com/solarroof

How can we model this system?

How can we control it in a smart way?

Our vision of a smart home energy system

An energy system based on renewable energy sources



Goal for the smart home energy system

Heat pump model:

$$\dot{x}_{hp} = A_{hp,c}x_{hp} + B_{hp,c}u_{hp} + E_{hp,c}d_{hp},$$
$$y_{hp} = C_{hp,c}x_{hp}.$$

EV battery model:

$$\dot{x}_{ev} = A_{ev,c}x_{ev} + B_{ev,c}u_{ev} + E_{ev,c}d_{ev},$$
$$y_{ev} = C_{ev,c}x_{ev}.$$

Stationary battery model:

$$\dot{x}_{bat} = A_{bat,c}x_{bat} + B_{bat,c}u_{bat} + E_{bat,c}d_{bat},$$
$$y_{bat} = C_{bat,c}x_{bat}.$$

Smart home energy system model:

$$\dot{x} = A_c x + B_c u + E_c d,$$
$$y = C_c x.$$

Characterization of a smart home

Our idea of a smart home

1. Heat pump input power is regulated such that indoor temperature is kept between pre-specified intervals
2. EV battery is charged such that a pre-defined driving pattern is possible
3. Stationary battery is discharged to provide power for heat pump, to charge EV battery and to sell energy
4. Stationary battery is charged by power from photo voltaic cells and purchasing power

Models of the devices in the energy system

Steps 1-4 are accomplished using an economic model predictive controller (EMPC) minimizing electricity costs

Linear state space models

Each device is modeled as a continuous-time linear state space model:

$$\dot{x} = A_c x + B_c u + E_c d,$$
$$y = C_c x.$$

Here x is the state-, u is the manipulated-, d is the disturbance- and y is the output-variable.

The heat pump is modeled as a ground source based heat pump for a floor heating system with constant coefficient of performance [1]

The batteries are modeled as simple integrators with transfer losses [2]

R. Halvgaard, N. K. Poulsen, H. Madsen, and J. B. Jørgensen.
Economic model predictive control for building climate control in a smart grid.
In *2012 IEEE PES Innovative Smart Grid Technologies (ISGT)*, pages 1–6, Jan 2012.

Rasmus Halvgaard, Niels Poulsen, Henrik Madsen, John Jørgensen, Francesco Marra, and Daniel Esteban Morales Bondy.
Electric vehicle charge planning using economic model predictive control.
2012 IEEE International Electric Vehicle Conference, IEVC 2012, 03 2012.

State space model of the smart home energy system

Coupling in the state space matrices

Variable definitions:

$$x = \begin{bmatrix} x_{hp}; x_{ev}; x_{bat} \end{bmatrix}, u = \begin{bmatrix} u_{hp}; u_{ev}; u^{+}_{bat}; u^{-}_{bat} \end{bmatrix}, d = \begin{bmatrix} d_{hp}; d_{ev}; d_{bat} \end{bmatrix}$$

$$y = \begin{bmatrix} y_{hp}; y_{ev}; y_{bat} \end{bmatrix}$$

State space matrices:

$$A_c = \begin{bmatrix} A_{hp,c} & 0 & 0 \\ 0 & A_{ev,c} & 0 \\ 0 & 0 & A_{bat,c} \end{bmatrix}, \quad B_c = \begin{bmatrix} B_{hp,c} & 0 & 0 & 0 \\ 0 & B_{ev,c} & 0 & 0 \\ -c_{bat} & -c_{bat} & c_{bat} & -c_{bat} \end{bmatrix}$$

$$E_c = \begin{bmatrix} E_{hp,c} & 0 & 0 \\ 0 & E_{ev,c} & 0 \\ 0 & 0 & c_s \cdot c_{bat} \end{bmatrix}, \quad C_c = \begin{bmatrix} C_{hp,c} & 0 & 0 \\ 0 & C_{ev,c} & 0 \\ 0 & 0 & C_{bat,c} \end{bmatrix}$$

Where d_{bat} = [solar radiation power],

$$c_{bat} = \frac{\text{[charging efficiency]}}{\text{[battery capacity]}}, \quad c_s = [\#\text{photo voltaic cells}] \cdot [\text{cell efficiency}].$$

State space model of the smart home energy system

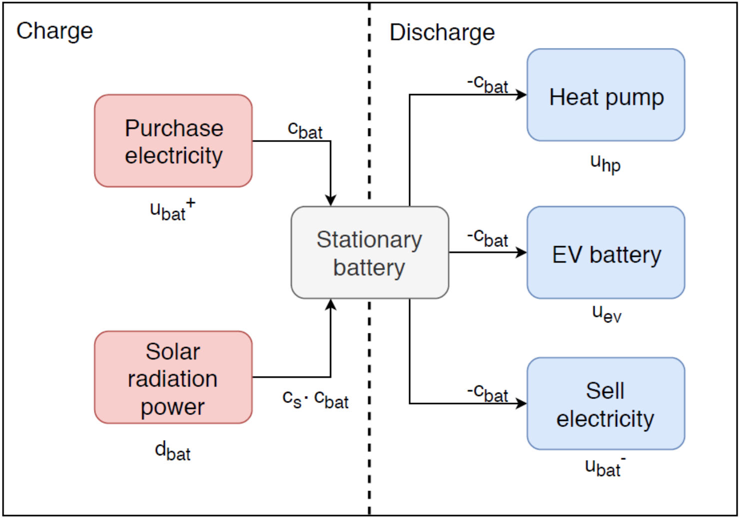


Figure: Overview of the coupling in the smart home energy system.

Economic MPC

DTU

Optimization problem

Objective function: $$\min_{u,v} \ \varphi_1 = \sum_{k=0}^{N-1} c_{u,k} u_k + \rho_v v_{k+1}$$

Discretized state space model: $$s.t. \ x_{k+1} = A x_k + B u_k + E d_k \qquad k \quad N$$
$$y_k = C x_k \qquad k \quad N^+$$

Input constraints: $$u_{\min,k} \leq u_k \leq u_{\max,k} \qquad k \quad N$$
$$\Delta u_{\min,k} \leq \Delta u_k \leq \Delta u_{\max,k} \qquad k \quad N$$

Soft output constraints: $$y_{\min,k} \leq y_k + v_k \qquad k \quad N^+$$
$$y_{\max,k} \geq y_k - v_k \qquad k \quad N^+$$

Slack variable constraint: $$0 \leq v_k \qquad k \quad N^+$$

With $N = \{0, ..., N - 1\}$ and $N^+ = \{1, ..., N\}$, where N is the prediction horizon, $c_{u,k}$ are electricity prices and ρ_v is a penalty parameter.

Challenge for the heat pump

Choice of the penalty parameter

When the penalty parameter ρ_v is chosen too small the controller will not turn on the heat pump

When ρ_v is chosen too large the controller will avoid violating the lower temperature constraint leading to higher electricity costs

Solution approach

We introduce multi-level soft constraints such that temperature violations are tolerated to some degree:

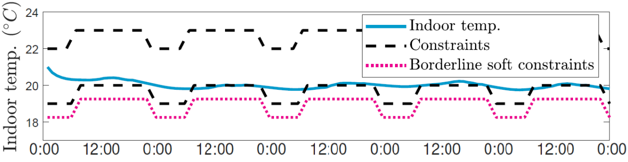


Figure: Illustration of multi-level soft constraints. Violations below the borderline are penalized harder than above.

Enhance heat pump performance

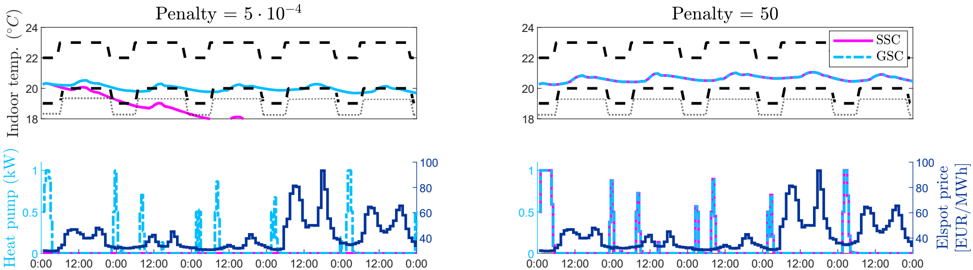


Figure: Heat pump performance for different penalty parameters. SSC = standard soft constraints, MLSC = multi-level soft constraints.

Challenge for short prediction horizons

Challenge

When using a short prediction horizon there is a chance that the controller does not turn the device on, which can lead to higher electricity costs later

Solution: Cost-to-go term

The solution is to account for the value of stored energy in the end of the prediction horizon:

$$\varphi_2 = \varphi_1 + [\text{cost-to-go term}]$$

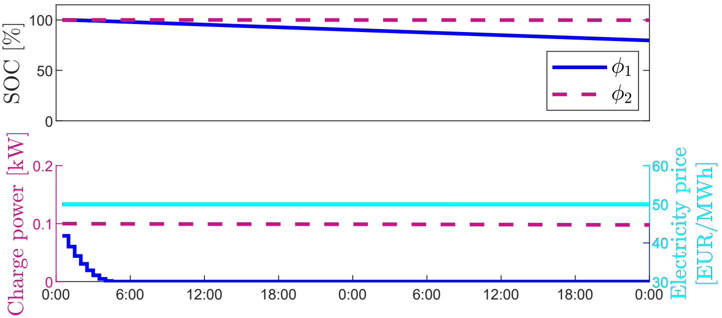


Figure: Battery performance in presence of a constant discharge of 0.1 kW. φ_1 = standard objective function, φ_2 = objective function with cost-to-go term.

Performance of the smart home energy system

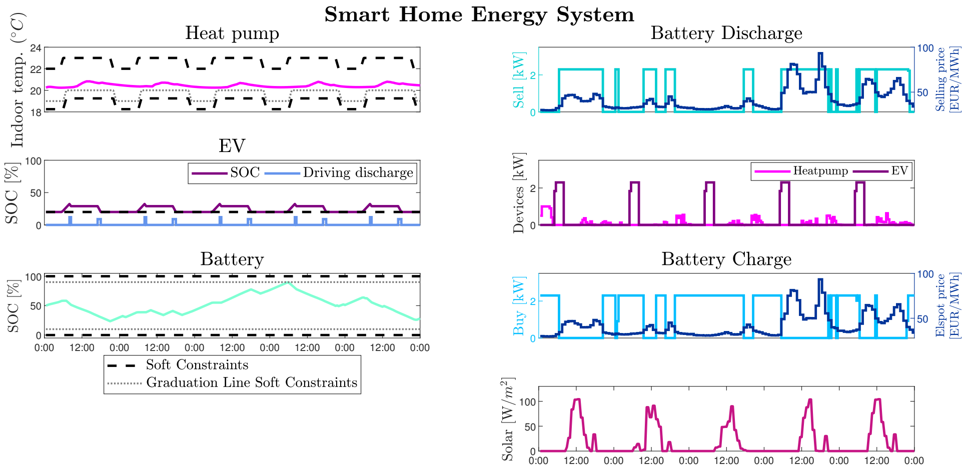


Figure: Illustration of EMPC performance.

Contributions

We formulated:

A simple state space model of a smart home energy system

Source: https://www.tesla.com/solarroof

We controlled the smart home energy system by an economic MPC, where we used

Multi-level soft constraints for more intuitive tuning

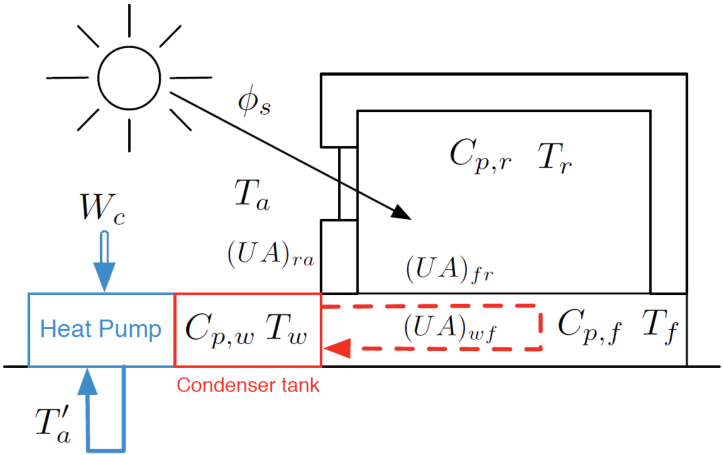
An objective function with a cost-to-go term to account for the value of stored energy in the end of the prediction horizon

Thank you
for your attention!

Contact information:
Hjørdis Amanda Schlüter, hjsc@dtu.dk
John Bagterp Jørgensen, jbjo@dtu.dk

Additional slides

Heat pump model



Additional slides

Heat pump model

Energy balances:

$$
\begin{aligned}
C_{p,r}\dot{T}_r &= R_{fr} - R_{ra} + (1-p)\varphi_s, \\
C_{p,f}\dot{T}_f &= R_{wf} - R_{fr} + p\varphi_s, \\
C_{p,w}\dot{T}_w &= \eta W_c - R_{wf},
\end{aligned}
$$

Heat transfer rates:

$$
\begin{aligned}
R_{wf} &= (UA)_{wf}(T_w - T_f), \\
R_{fr} &= (UA)_{fr}(T_f - T_r), \\
R_{ra} &= (UA)_{ra}(T_r - T_a),
\end{aligned}
$$

Additional slides

Heat pump model

Variable	Unit	Description
T_r	$^\circ$C	Room air temperature
T_f	$^\circ$C	Floor temperature
T_w	$^\circ$C	Water temperature in the floor heating pipes
T_a	$^\circ$C	Ambient temperature
T_a	$^\circ$C	Ground temperature
W_c	W	Heat pump compressor input power
φ_s	W	Solar radiation power: Solar radiation (W/m^2) times effective window area (2.9 m^2)

Additional slides

Battery model

$$\dot{\zeta} = \frac{1}{Q_n}\left(\eta^+ P_c^+ - \eta^- P_c^-\right).$$

	Description
ζ	State of Charge (SOC)
P_c^+	Charge power
P_c^-	Discharge power
η^+	Charging efficiency
η^-	Discharging efficiency
Q_n	Nominal battery capacity

Additional slides

Multi-level soft constraints formulation

$$\varphi = \sum_{k=0}^{N-1} (c_{u,k} u_k + \rho_v^1 v_{k+1}^1 + \rho_v^2 v_{k+1}^2)$$

$$y_{\min,k} \leq y_k + v_k^1 + v_k^2,$$

$$y_{\max,k} \geq y_k - v_k^1,$$

$$0 \leq v_k^1 \leq v_{\max}^1,$$

$$0 \leq v_k^2.$$

Additional slides

Value of stored energy at the end of the prediction horizon

Formulation:

$$\varphi_2 = \varphi_1 - (\bar{c}_N x_N - \bar{c}_0 x_0),$$

where $\bar{c}_0$ and $\bar{c}_N$ is the value of stored energy at the start and the end of the prediction horizon.