

# Unit Commitment: Dealing with the Uncertainties



*Roger J-B Wets*  
*University of California, Davis*  
*Summer 2014 @ Technical U. Denmark*

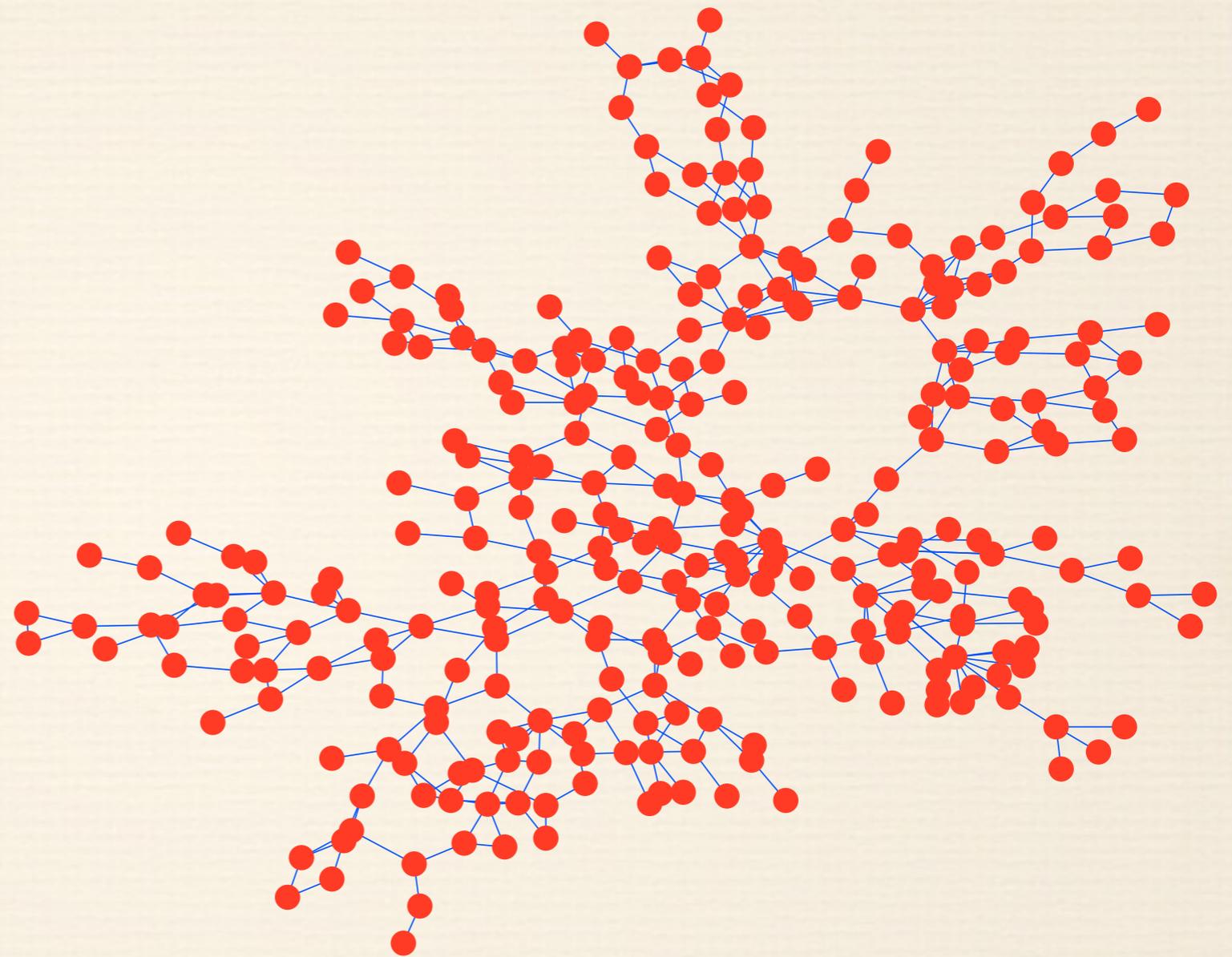
# Unit commitment

## Part I



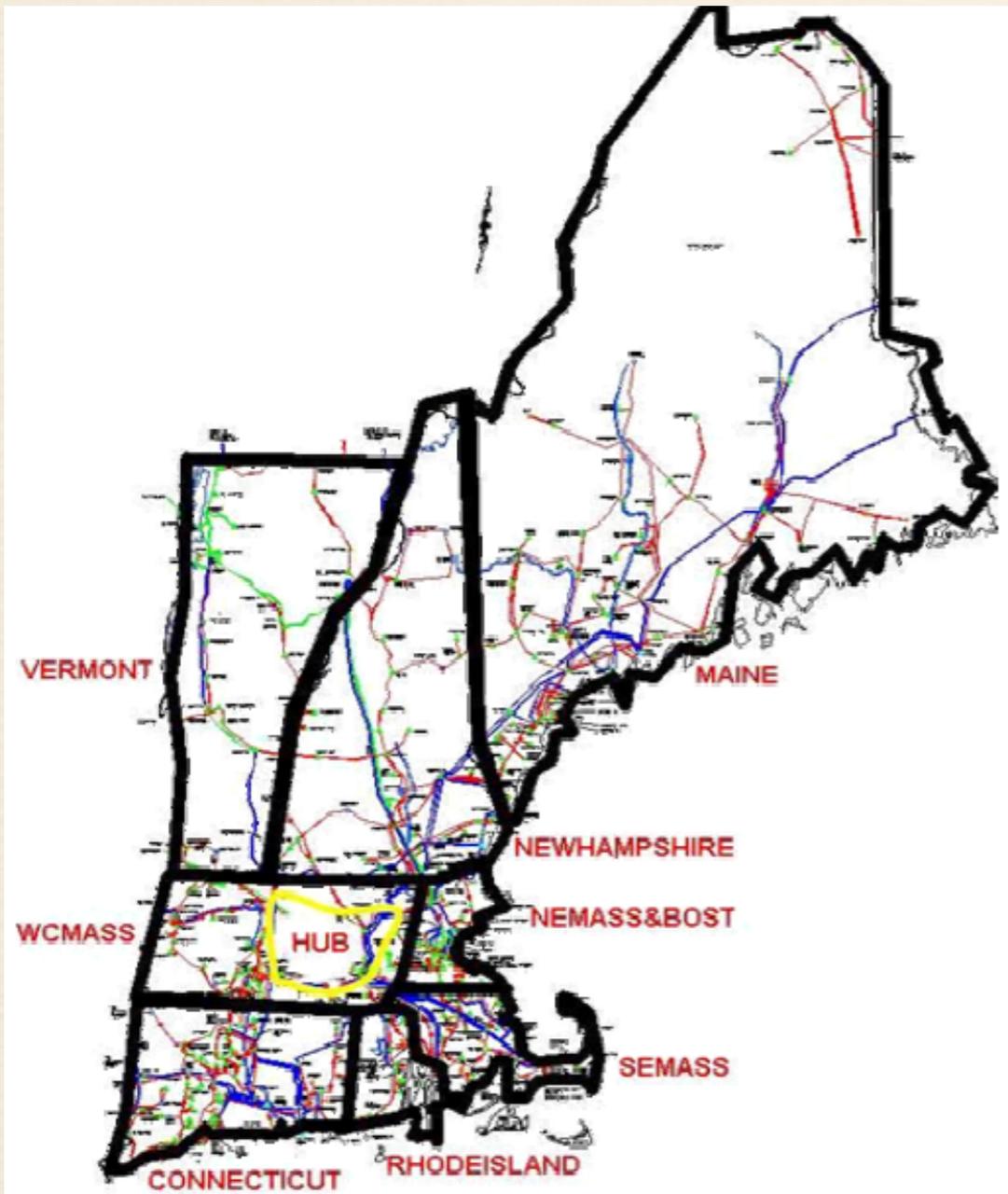
**Progressive Hedging:  
dealing with binary variables**

# Transmission Network



**Figure 1. Topology of the IEEE 300 node system**

# Transmission Network

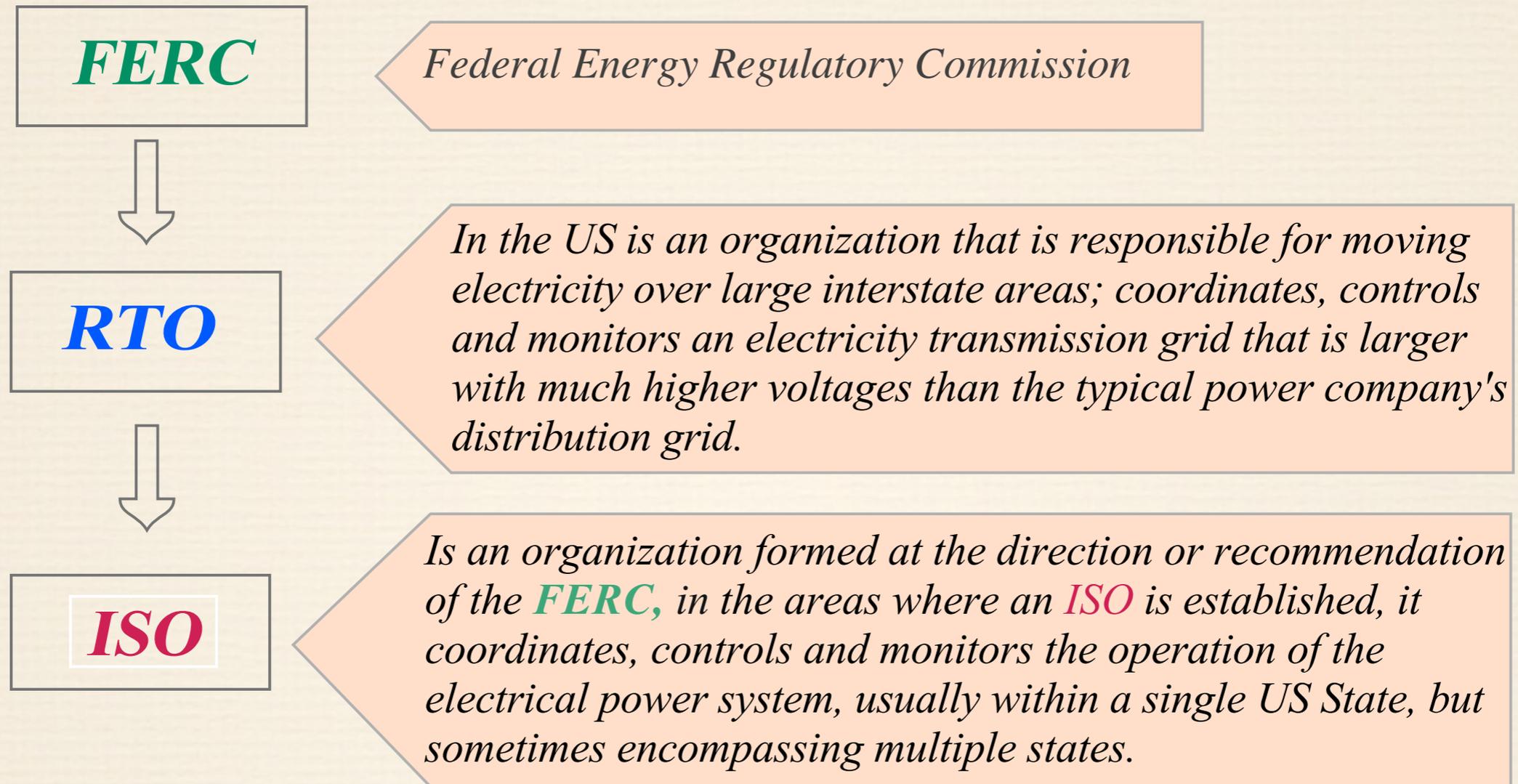


NE-ISO net ~30,000 BUS



Figure 1. Topology of the IEEE 300 node system

# ISO: Independent System Operator

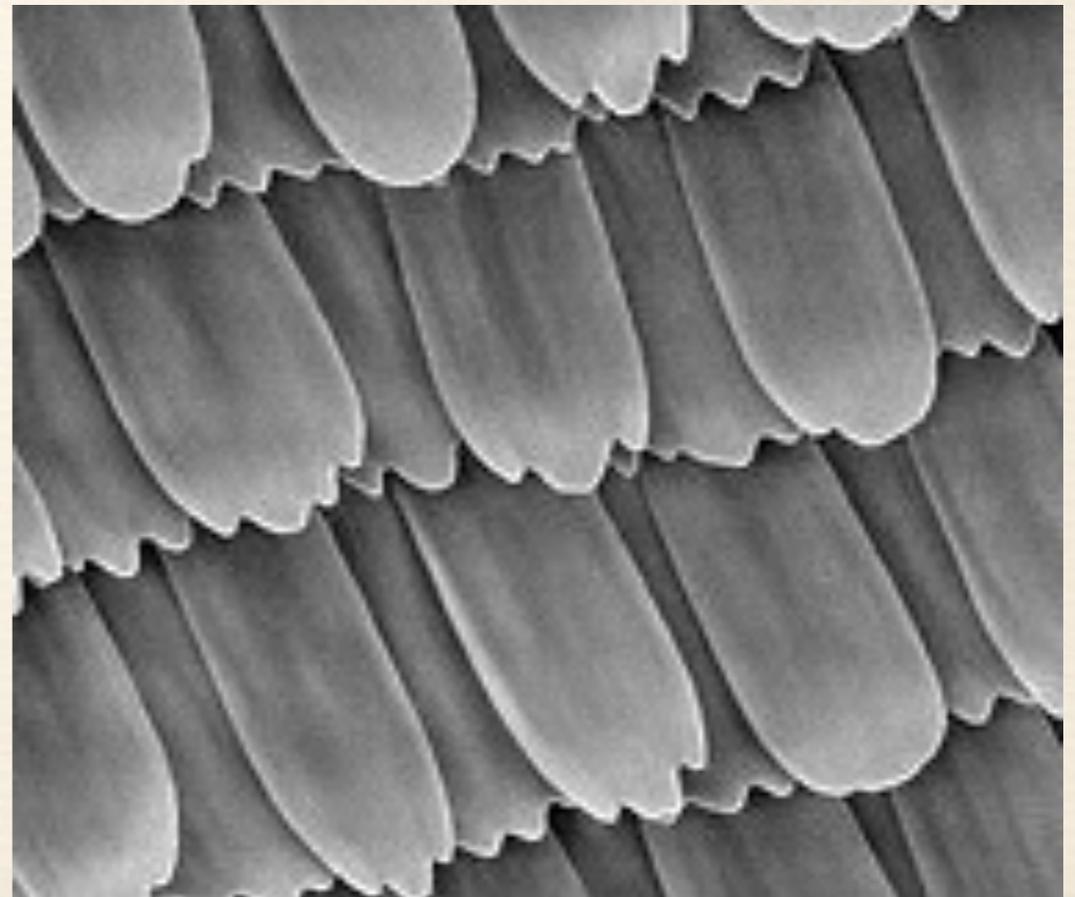


*ISO New England Inc. (**ISO-NE**) is an independent, non-profit RTO, serving Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island and Vermont. Its Board of Directors and its over 400 employees have no financial interest or ties to any company doing business in the region's wholesale electricity marketplace.*



# Uncertainties

- WEATHER: demand & supply (especially renewables)
- industrial-commercial environment (demand)
- seasonal, day of the week, time of the day
- contingencies: transmission lines, generators



# Uncertainties

- WEATHER: demand & supply (especially renewables)
- industrial-commercial environment (demand)
- seasonal, day of the week, time of the day
- contingencies: transmission lines, generators



# Energy Sources

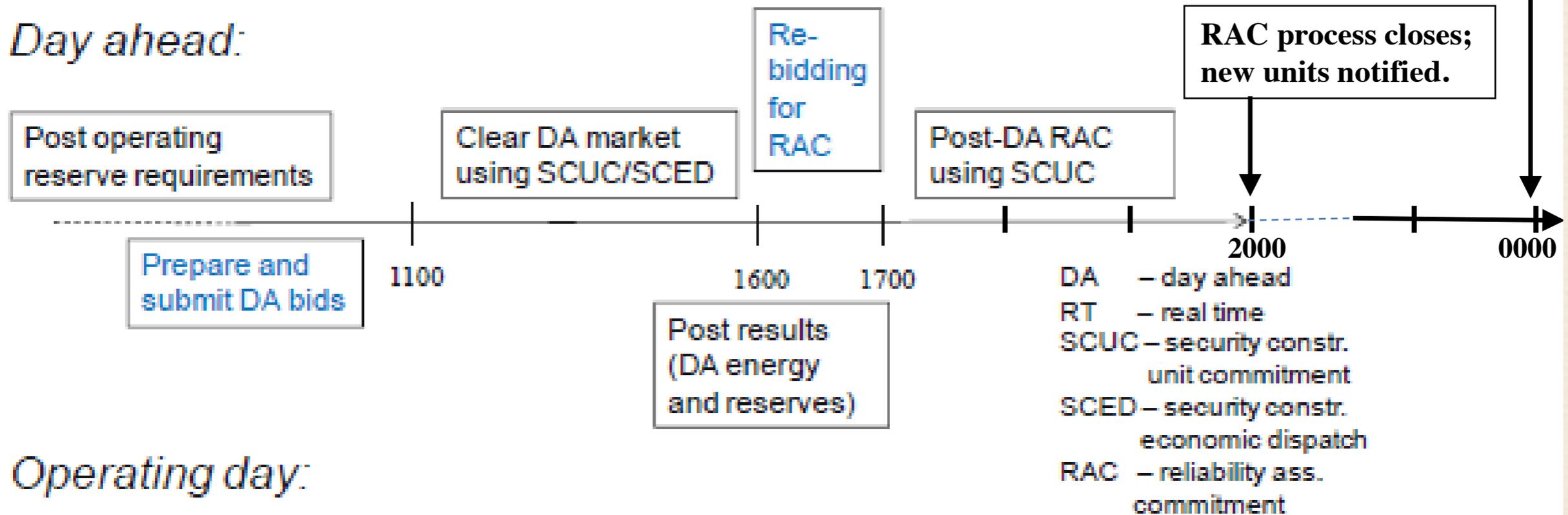


- nuclear energy
- hydro-power
- thermal plants (coal, oil, shale oil, bio, rubbish, ...)
- gas turbines (natural gas, from "cracking")
- renewables (wind, solar, ..., ocean waves)

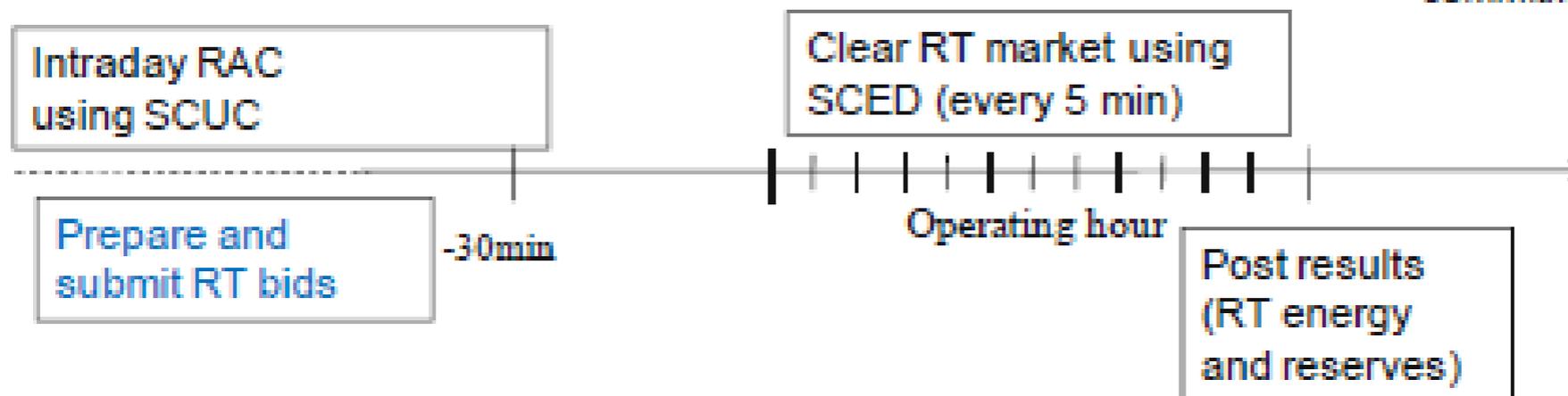
*different characteristics*

# Market time line

## Day ahead:



## Operating day:



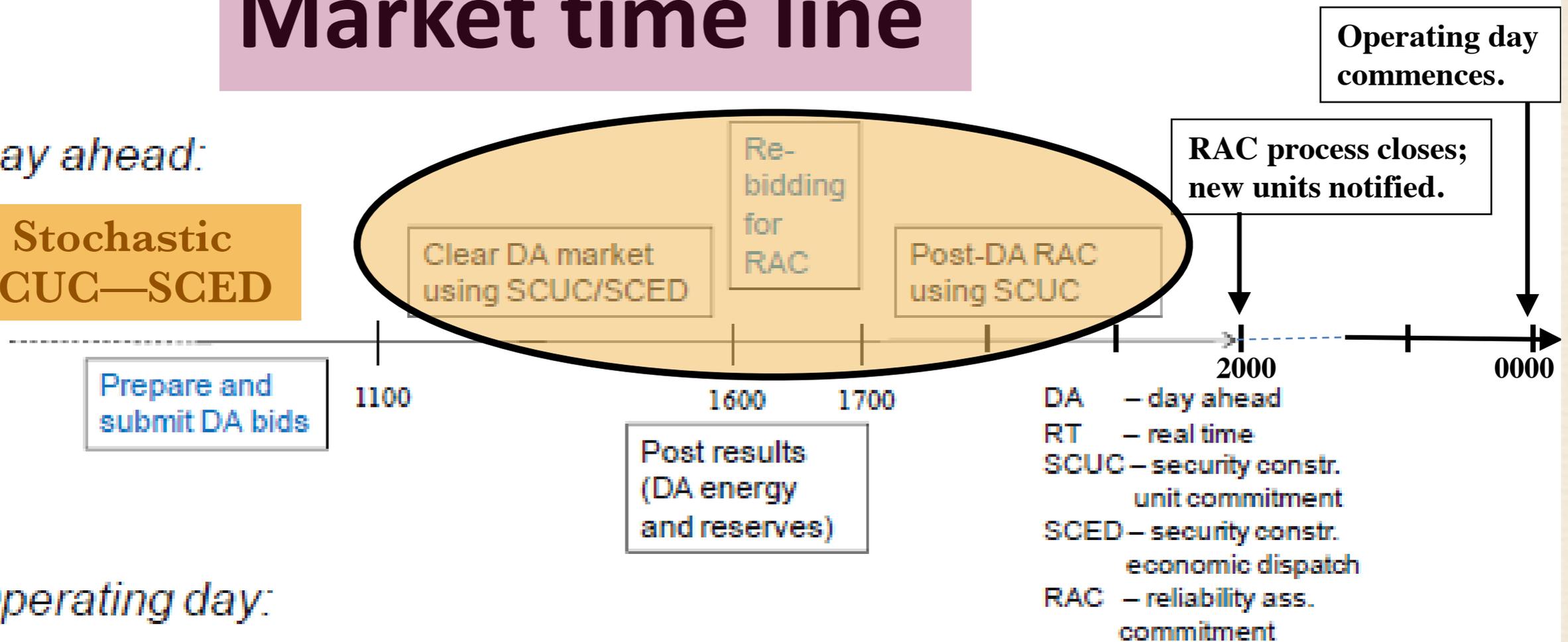
	MISO	NYISO	PJM	ERCOT	CAISO
Market timeline	DA offers due: 11am DA results: 4pm Re-bidding due: 5pm RT offers due: OH -30 min	DA offers due: 5am DA results: 11am RT offers due: OH -75 min	DA offers due: noon DA results: 4pm RT offers due: 6pm DA	DA bids due (reserves): 1pm/4pm DA results (reserves): 1.30pm/6pm RT offers due: OH -60 min	DA offers: 10am DA results: 1pm RT offers: OH -75 min

Ref: A. Botterud, J. Wang, C. Monteiro, and V. Miranda "Wind Power Forecasting and Electricity Market Operations," available at [www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud\\_etal\\_paper.pdf](http://www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud_etal_paper.pdf)

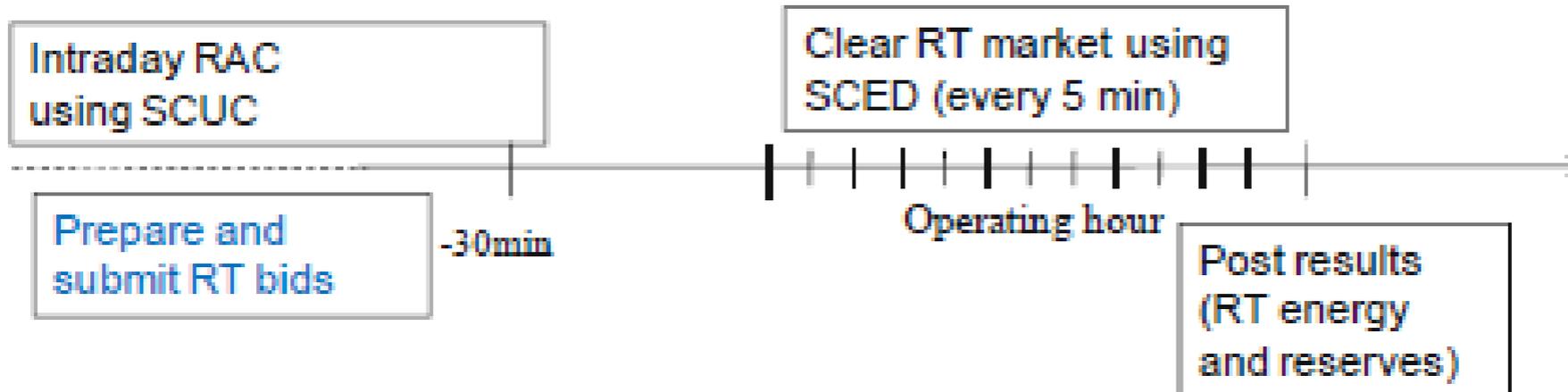
# Market time line

Day ahead:

Stochastic SCUC—SCED



Operating day:



	MISO	NYISO	PJM	ERCOT	CAISO
Market timeline	DA offers due: 11am DA results: 4pm Re-bidding due: 5pm RT offers due: OH -30 min	DA offers due: 5am DA results: 11am RT offers due: OH -75 min	DA offers due: noon DA results: 4pm RT offers due: 6pm DA	DA bids due (reserves): 1pm/4pm DA results (reserves): 1.30pm/6pm RT offers due: OH -60 min	DA offers: 10am DA results: 1pm RT offers: OH -75 min

Ref: A. Botterud, J. Wang, C. Monteiro, and V. Miranda "Wind Power Forecasting and Electricity Market Operations," available at [www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud\\_etal\\_paper.pdf](http://www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud_etal_paper.pdf)

# Market time line

Day ahead:

Stochastic SCUC—SCED

Prepare and submit DA bids

1100

Clear DA market using SCUC/SCED

1600

Re-bidding for RAC

1700

Post-DA RAC using SCUC

RAC process closes; new units notified.

2000

0000

Post results (DA energy and reserves)

DA – day ahead

RT – real time

SCUC – security constr. unit commitment

SCED – security constr. economic dispatch

RAC – reliability ass. commitment

Operating day:

Stochastic SCED2

Intraday RAC using SCUC

Clear RT market using SCED (every 5 min)

Prepare and submit RT bids

-30min

Operating hour

Post results (RT energy and reserves)

	MISO	NYISO	PJM	ERCOT	CAISO
Market timeline	DA offers due: 11am DA results: 4pm Re-bidding due: 5pm RT offers due: OH -30 min	DA offers due: 5am DA results: 11am RT offers due: OH -75 min	DA offers due: noon DA results: 4pm RT offers due: 6pm DA	DA bids due (reserves): 1pm/4pm DA results (reserves): 1.30pm/6pm RT offers due: OH -60 min	DA offers: 10am DA results: 1pm RT offers: OH -75 min

Ref: A. Botterud, J. Wang, C. Monteiro, and V. Miranda "Wind Power Forecasting and Electricity Market Operations," available at [www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud\\_etal\\_paper.pdf](http://www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud_etal_paper.pdf)

# between a rock and a hard place



MIP-CPLEX &  
good **ISP-codes** (S. Sen & Co.)  
can only handle effectively  
problems of moderate size

recall “deadlines”

# Our “ARPA-e Team”

- ❖ Sandia National Labs: Jean-Paul Watson, César Silva-Monroy, Ross Guttromsom (team builder), John Siirola, William Hart, ...
- ❖ Iowa State University: Sarah Ryan, Dinakar Gade, Yonghan Feng, Youngrok Lee
- ❖ University of California, Davis: David Woodruff, Roger J-B Wets, Ignacio Rios, Kai Spürkel, Fabian Rüdel, (+ Chuangyin Dang, Julia Peyre ... later this year)
- ❖ Alstom: Kwok Cheung (+ ...)
- ❖ @ New-England ISO: Eugene Litvinov (& Joe Mercer, William Callan)
- ❖ Unofficial associates: Johannes Royset (NPS), Hoa Chen (UCD) - uncertainty design

# Abstract Unit Commitment

$$\text{Minimize } \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with  
*J generating units*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with  
*K time periods*      *J generating units*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*production cost*

*K time periods*      *J generating units*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*K time periods*      *J generating units*

*production cost*      *startup cost*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with  
*K time periods*      *J generating units*

*production cost*      *startup cost*      *shutdown cost*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with  
*K time periods*      *J generating units*

*production cost*      *startup cost*      *shutdown cost*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

*demand*

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*production cost* *startup cost* *shutdown cost*

*K time periods* *J generating units*

*power output*  $\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$

*demand*

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*production cost* *startup cost* *shutdown cost*

*K time periods* *J generating units*

*power output*  $\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$

*demand*

*max power output*  $\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*K time periods*      *J generating units*

*production cost*      *startup cost*      *shutdown cost*

*power output*  $\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$

*demand*

*max power output*  $\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$

*spinning reserve*

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*K time periods*      *J generating units*

*production cost*      *startup cost*      *shutdown cost*

*power output*  $\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$

*demand*

*max power output*  $\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$

*spinning reserve*

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

# Abstract Unit Commitment

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with

*production cost* *startup cost* *shutdown cost*

*K time periods* *J generating units*

*power output*  $\sum_{j \in J} \underline{p}_j(k) = \underline{D}(k), \quad \forall k \in K$

*demand*

*max power output*  $\sum_{j \in J} \underline{\bar{p}}_j(k) \geq D(k) + R(k), \quad \forall k \in K$

*spinning reserve*

$$p_j(k), \bar{p}_j(k) \in \underline{\Pi}, \quad \forall j \in J, \quad \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
The specific nature of  $\Pi$  is model-dependent.

*"Stochastic Version"*

# Abstract Unit Commitment

*min. expectation  
(actually: risk measure)  
with penalties*

Minimize  $\sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k)$  with  
*K time periods      J generating units*

*production cost    startup cost    shutdown cost*

*power output*  $\sum_{j \in J} \underline{p}_j(k) = \underline{D}(k), \quad \forall k \in K$

*demand*

*adjust node  
balance eq'ns*

*max power output*  $\sum_{j \in J} \underline{\bar{p}}_j(k) \geq D(k) + R(k), \quad \forall k \in K$

*spinning reserve*

$$p_j(k), \bar{p}_j(k) \in \underline{\Pi}, \quad \forall j \in J, \quad \forall k \in K$$

$\Pi$  region of feasible production, all generating units, all time periods.  
 The specific nature of  $\Pi$  is model-dependent.

*"Stochastic Version"*

# Aggregation Principle in Stochastic Optimization

⇒ to Progressive Hedging Algorithm

# Here-&-Now vs. Wait-&-See

- ❖ Basic Process: decision  $\rightarrow$  observation  $\rightarrow$  decision  
 $x^1 \rightarrow \xi \rightarrow x_{\xi}^2$
- ❖ Here-&-Now problem!  $x^1$   
not all contingencies can be “protected” by  
available instruments, i.e.
- ❖ Wait-&-See problem:  
instruments are available to cover all contingencies  
choose  $(x_{\xi}^1, x_{\xi}^2)$  after observing  $\xi$

# Stochastic Optimization: Fundamental Theorem

A here-&-now problem can be transformed in a wait-&-see problem by introducing the

**appropriate `contingencies' costs  
(price of nonanticipativity)**

# Price of Nonanticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

# Price of Nonanticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

## Explicit non-anticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x_\xi^1, x_\xi^2) \} \\ x_\xi^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x_\xi^1), \forall \xi. \end{aligned}$$

# Price of Nonanticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

## Explicit non-anticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x_\xi^1, x_\xi^2) \} \\ x_\xi^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x_\xi^1), \forall \xi. \end{aligned}$$

$$\boxed{x_\xi^1 = \mathbb{E} \{ x_\xi^1 \} \quad \forall \xi}$$

# Price of Nonanticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

## Explicit non-anticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x_\xi^1, x_\xi^2) \} \\ x_\xi^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x_\xi^1), \forall \xi. \end{aligned}$$

$$x_\xi^1 = \mathbb{E} \{ x_\xi^1 \} \quad \forall \xi$$

$w_\xi \perp$  subspace of constant fcns

*multipliers*

$$\Rightarrow \mathbb{E} \{ w_\xi \} = 0$$

# Price of Nonanticipativity

## Explicit non-anticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x_\xi^1, x_\xi^2) \} \\ x_\xi^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x_\xi^1), \forall \xi. \end{aligned}$$

$$\boxed{x_\xi^1 = \mathbb{E} \{ x_\xi^1 \} \quad \forall \xi}$$

$w_\xi \perp$  subspace of constant fcns

*multipliers*

$$\Rightarrow \mathbb{E} \{ w_\xi \} = 0$$

$$\min \mathbb{E} \left\{ f_0(\xi, x_\xi^1, x_\xi^2) + \langle w_\xi, x_\xi^1 \rangle - \langle w_\xi, \mathbb{E} \{ x_\xi^1 \} \rangle \right\}$$

$$\text{such that } x_\xi^1 \in C_1, \quad x_\xi^2 \in C_2(\xi, x_\xi^1)$$

# Price of Nonanticipativity

## Explicit non-anticipativity

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x^1, x_\xi^2) \} \\ x^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x^1), \forall \xi. \end{aligned}$$

$$\begin{aligned} \min \mathbb{E} \{ f_0(\xi, x_\xi^1, x_\xi^2) \} \\ x_\xi^1 \in C^1 \subset \mathbb{R}^n, \\ x_\xi^2 \in C^2(\xi, x_\xi^1), \forall \xi. \end{aligned}$$

$$\boxed{x_\xi^1 = \mathbb{E} \{ x_\xi^1 \} \quad \forall \xi}$$

$w_\xi \perp$  subspace of constant fcns

*multipliers*

$$\Rightarrow \mathbb{E} \{ w_\xi \} = 0$$

$$\min \mathbb{E} \left\{ f_0(\xi, x_\xi^1, x_\xi^2) + \langle w_\xi, x_\xi^1 \rangle - \langle w_\xi, \mathbb{E} \{ x_\xi^1 \} \rangle \right\}$$

$$\text{such that } x_\xi^1 \in C_1, \quad x_\xi^2 \in C_2(\xi, x_\xi^1)$$

# Progressive Hedging Algorithm

0.  $w_\xi^0$  such that  $\mathbb{E}\{w_\xi^0\} = 0$ ,  $\nu = 0$ . Pick  $\rho > 0$

1. for all  $\xi$ :

$$(x_\xi^{1,\nu}, x_\xi^{2,\nu}) \in \arg \min f_0(\xi; x^1, x^2) - \langle w_\xi^\nu, x^1 \rangle$$

$$x^1 \in C^1 \subset \mathbb{R}^{n_1}, x^2 \in C^2(\xi, x^1) \subset \mathbb{R}^{n_2}$$

2.  $\bar{x}^{1,\nu} = \mathbb{E}\{x_\xi^{1,\nu}\}$ . Stop if  $|x_\xi^{1,\nu} - \bar{x}^{1,\nu}| = 0$  (approx.)

otherwise  $w_\xi^{\nu+1} = w_\xi^\nu + \rho[x_\xi^{1,\nu} - \bar{x}^{1,\nu}]$ , return to 1. with  $\nu = \nu + 1$

# Progressive Hedging Algorithm

0.  $w_\xi^0$  such that  $\mathbb{E}\{w_\xi^0\} = 0$ ,  $\nu = 0$ . Pick  $\rho > 0$

1. for all  $\xi$ :

$$(x_\xi^{1,\nu}, x_\xi^{2,\nu}) \in \arg \min f_0(\xi; x^1, x^2) - \langle w_\xi^\nu, x^1 \rangle$$

$$x^1 \in C^1 \subset \mathbb{R}^{n_1}, x^2 \in C^2(\xi, x^1) \subset \mathbb{R}^{n_2}$$

2.  $\bar{x}^{1,\nu} = \mathbb{E}\{x_\xi^{1,\nu}\}$ . Stop if  $|x_\xi^{1,\nu} - \bar{x}^{1,\nu}| = 0$  (approx.)

otherwise  $w_\xi^{\nu+1} = w_\xi^\nu + \rho[x_\xi^{1,\nu} - \bar{x}^{1,\nu}]$ , return to 1. with  $\nu = \nu + 1$

Convergence: add a proximal term

$$f_0(\xi; x^1, x^2) - \langle w_\xi^\nu, x^1 \rangle - \frac{\rho}{2} |x^1 - \bar{x}^{1,\nu}|^2$$

linear rate in  $(x^{1,\nu}, w^\nu)$  ... eminently parallelizable

# PH: Implementation issues

implementation: choice of  $\rho$  ... scenario ( $\times$ ),  $i$ th-decision ( $i$ ) dependent

(heuristic) extension to problems with integer variables

non-convexities: e.g. ground-water remediation with non-linear PDE recourse

asynchronous

partitioning (= different information feeds)

$$\min \mathbb{E} \{ f(\xi, x) \} , \quad f(\xi, x) = f_0(x) + \iota_{C(\xi, x)}(x)$$

$S = \{\Xi_1, \Xi_2, \dots, \Xi_K\}$  a partitioning of  $\Xi$ ,  $p_k = P(\Xi_k)$

$$\mathbb{E} \{ f(\xi, x) \} = \sum_n p_n \mathbb{E} \{ f(\xi, x) | \Xi_n \} \quad (\text{Bundling})$$

defining  $g(k, x) = \mathbb{E} \{ f_0(\xi, x) | \Xi_n \}$  if  $x \in C_k = \bigcap_{\xi \in \Xi_k} C_\xi$

$$\text{solve the problem as: } \min \sum_{n=1}^N p_k g(k, x)$$

Bundling

# PH: binary variables

$\min \langle c, x \rangle + \sum_{\xi \in \Xi} p_{\xi} \langle q_{\xi}, y_{\xi} \rangle$  such that

$x \in C_1, y_{\xi} \in C_2(\xi, x) \forall \xi \in \Xi$

binary (integer) variables: some  $x$ 's, some  $y_{\xi}$ 's.

# PH: binary variables

$\min \langle c, x \rangle + \sum_{\xi \in \Xi} p_{\xi} \langle q_{\xi}, y_{\xi} \rangle$  such that

$x \in C_1, y_{\xi} \in C_2(\xi, x) \forall \xi \in \Xi$

binary (integer) variables: some  $x$ 's, some  $y_{\xi}$ 's.

Choice of  $\rho \rightarrow \rho_j$  depending on  $c_j, |x_j|, \dots$  and augmentation

Variable Fixing, in particular binaries,  $x_j(s) = \text{constant}$  ( $k$  iterations)

Variable Slamming: aggressive variable fixing  $x_j(s) \approx \text{constant}$  (&  $c_j x_j(s)$ )

“Sufficient” variable convergence  $\sim$  for small values of  $c_j x_j(s)$

Termination criterion: variable slamming when  $x_j^{\nu}(\xi) - x_j^{\nu+1}(\xi)$  small

Detecting cycling behavior: (simple) hashing scheme

# PH: binary variables

$\min \langle c, x \rangle + \sum_{\xi \in \Xi} p_{\xi} \langle q_{\xi}, y_{\xi} \rangle$  such that

$x \in C_1, y_{\xi} \in C_2(\xi, x) \forall \xi \in \Xi$

binary (integer) variables: some  $x$ 's, some  $y_{\xi}$ 's.

Choice of  $\rho \rightarrow \rho_j$  depending on  $c_j, |x_j|, \dots$  and augmentation

Variable Fixing, in particular binaries,  $x_j(s) = \text{constant}$  ( $k$  iterations)

Variable Slamming: aggressive variable fixing  $x_j(s) \approx \text{constant}$  (&  $c_j x_j(s)$ )

“Sufficient” variable convergence  $\sim$  for small values of  $c_j x_j(s)$

Termination criterion: variable slamming when  $x_j^{\nu}(\xi) - x_j^{\nu+1}(\xi)$  small

Detecting cycling behavior: (simple) hashing scheme

*Enough variables fixed  $\Rightarrow$  clean up with CPLEX-MIP*

# Error Bounds

$f(\xi, x) = f_0(\xi, x^1, x^2)$  if  $x^1 \in C^1, x^2 \in C^2(\xi, x^1)$ ;  $+\infty$  else

Stochastic Program ( $P$ ):

$$\min_{x \in \mathcal{M}} \mathbb{E} \{ f(\xi, x) \} \text{ such that } x^1 \equiv \mathbb{E} \{ x_\xi^1 \}$$

Dual Program ( $D$ )

$$\max_{w \in \mathcal{M}^*} \mathbb{E} \{ -f^*(\xi, w_\xi) \} \text{ such that } \mathbb{E} \{ w_\xi \} = 0$$

weak duality holds:  $\inf P \geq \sup D \Rightarrow$  for any feasible  $\hat{w}$

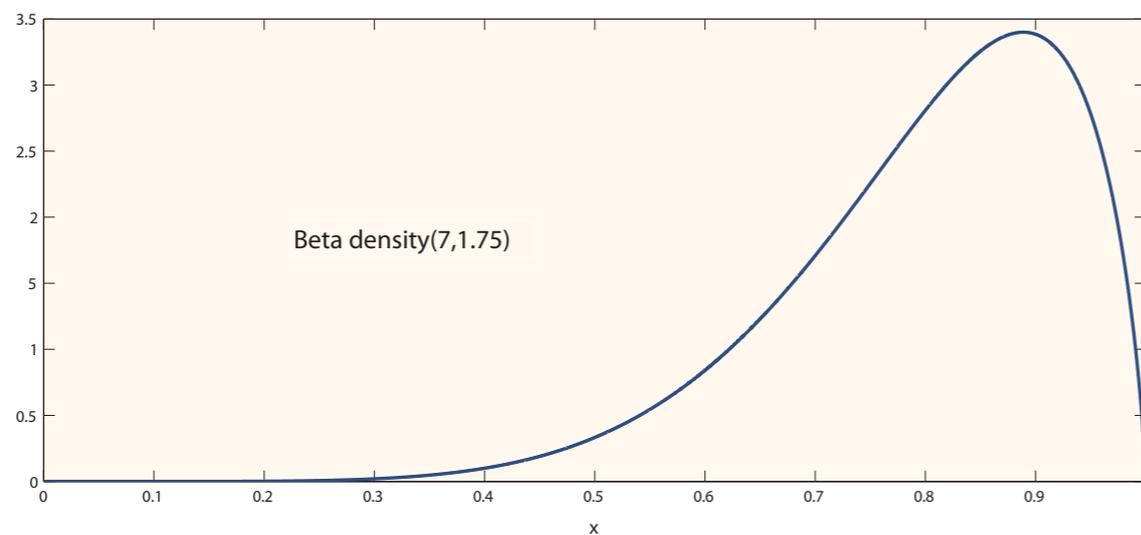
$$-f^*(\xi, \hat{w}_\xi) = \min_x \left[ f(\xi, x) + \langle \hat{w}_\xi, x^1 \rangle, x \in \mathbb{R}^{n_1+n_2} \right]$$

yields a lower bound for ( $P$ ), better if  $\hat{w}_\xi$  is near-optimal

$\Rightarrow$  rely on  $w^*$  of PH-algorithm to generate lower bound.

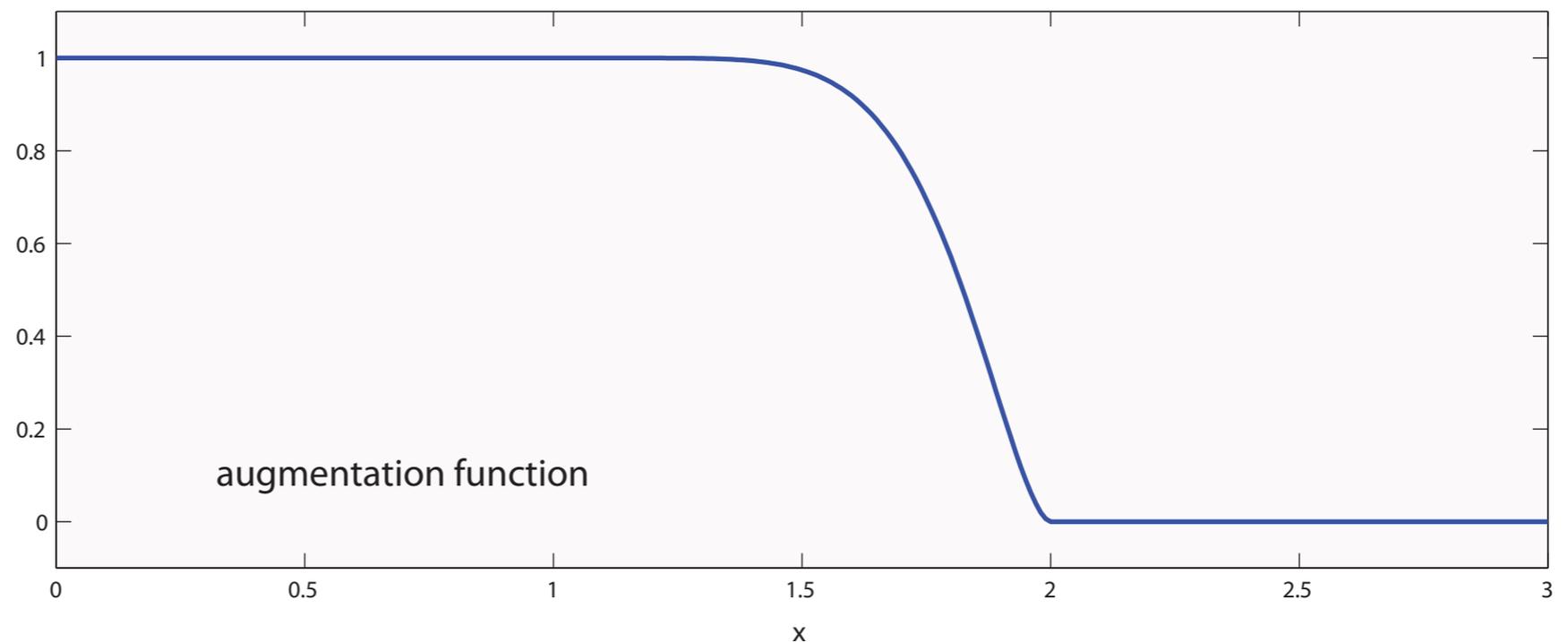
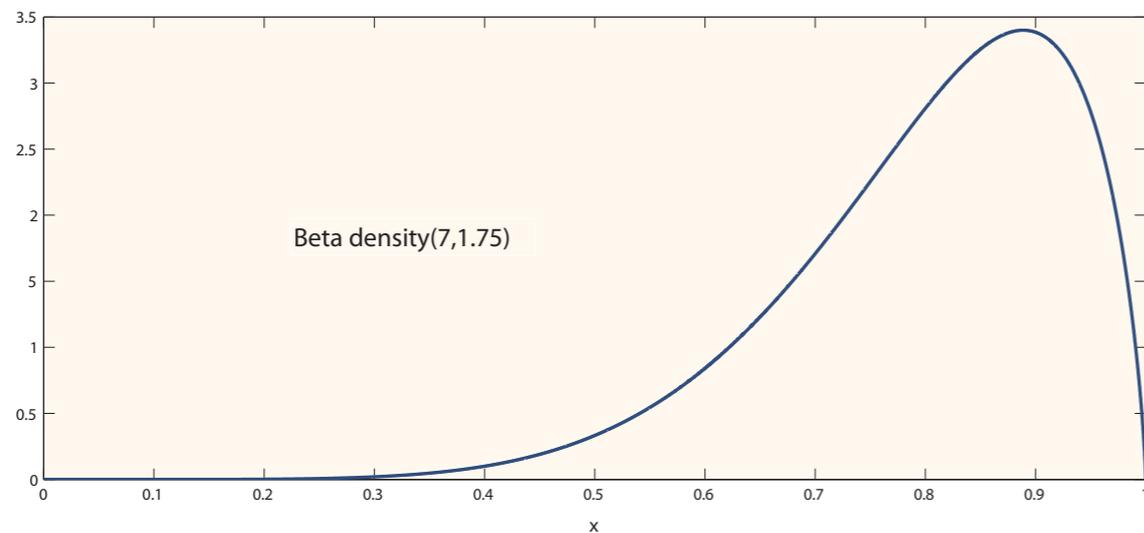
# Augmentation function

$$m(\Delta, \lambda_{\max}; z, \lambda) = \int_0^{\Delta} \psi(z - s, \lambda, \lambda_{\max}) \varphi(\Delta; s) ds, \quad z \in [0, \lambda_{\max}]$$



# Augmentation function

$$m(\Delta, \lambda_{\max}; z, \lambda) = \int_0^{\Delta} \psi(z - s, \lambda, \lambda_{\max}) \varphi(\Delta; s) ds, \quad z \in [0, \lambda_{\max}]$$



# Unit commitment

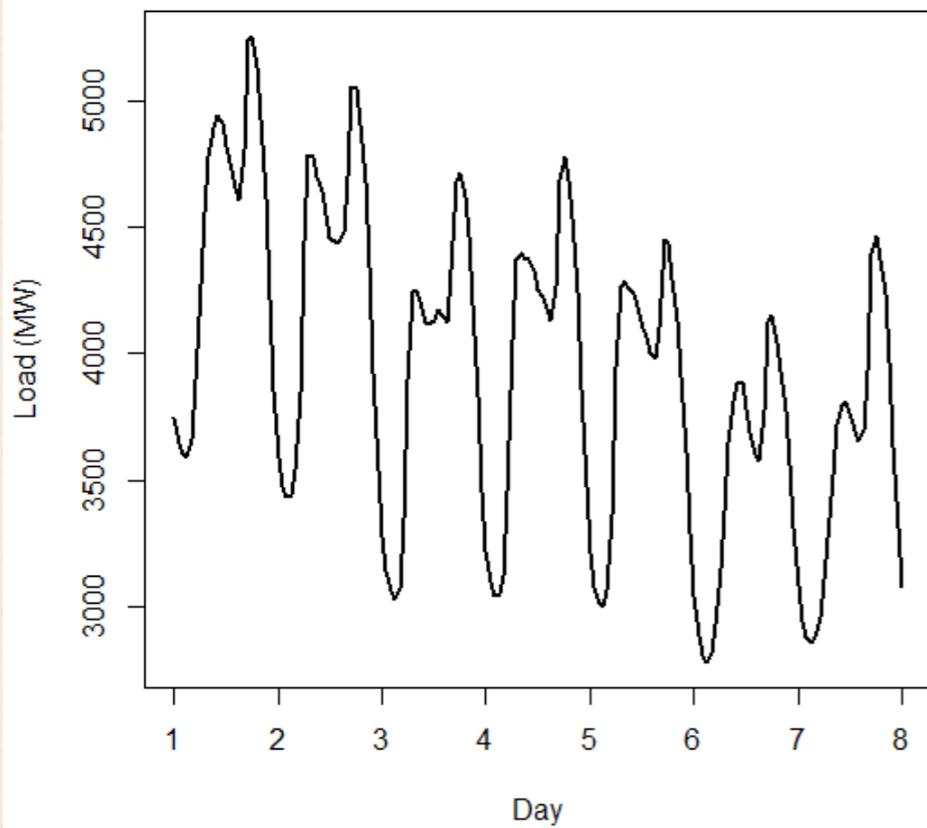
## Part II



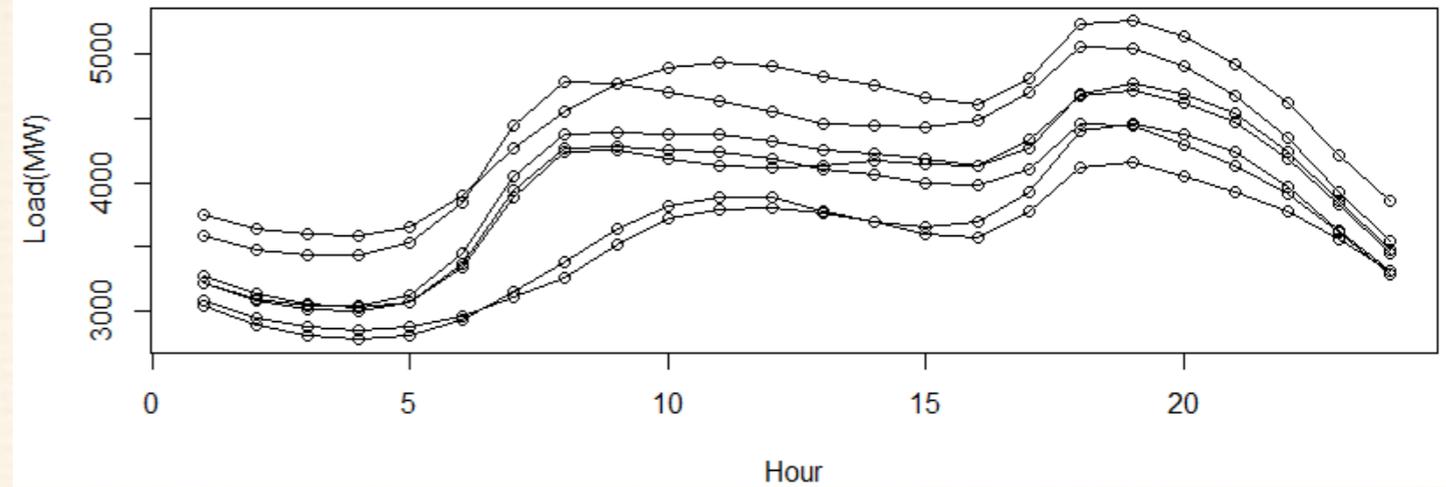
Dealing with  
load (demand) uncertainty

# CT load exploratory process

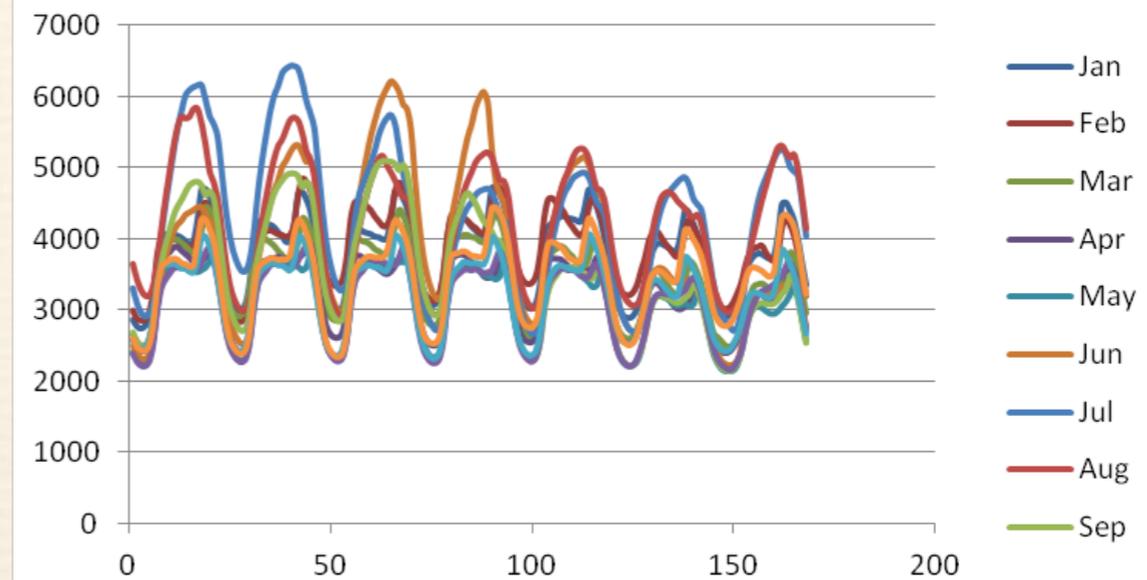
Hourly Load in A Week



Seasonal Trend of Hourly Load in Days

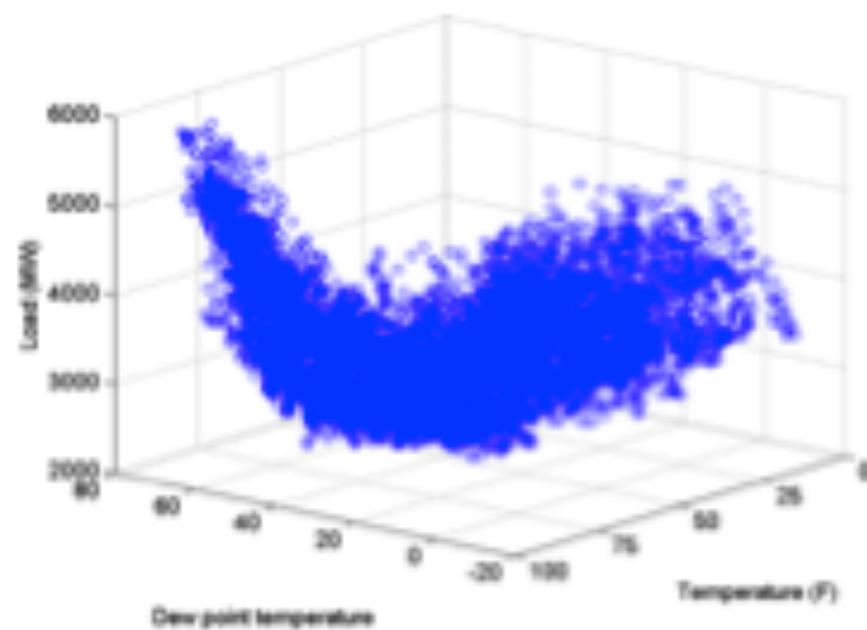


Weekly Pattern by Month, CT 2011

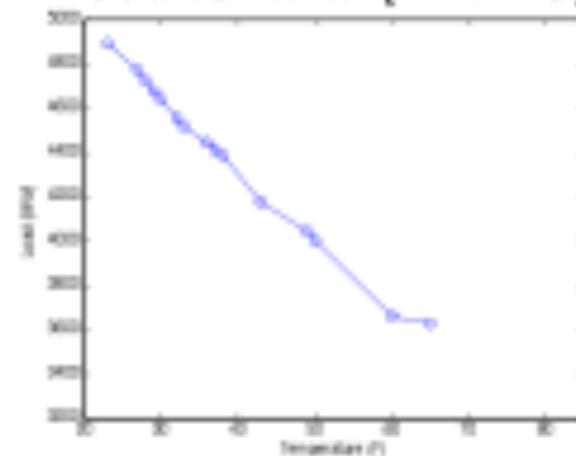


# CT load vs. weather variables

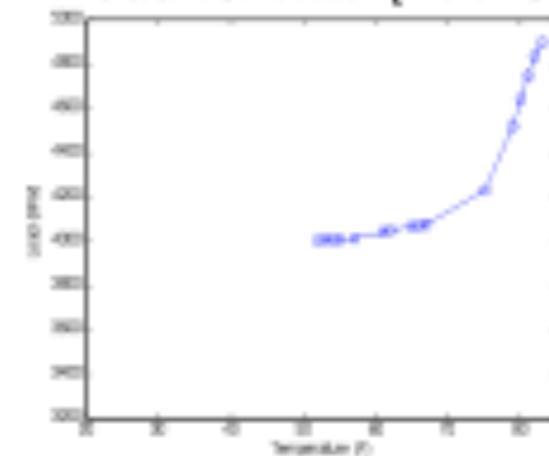
Load vs. temperature (TMP) and dew point temperature (DPT)



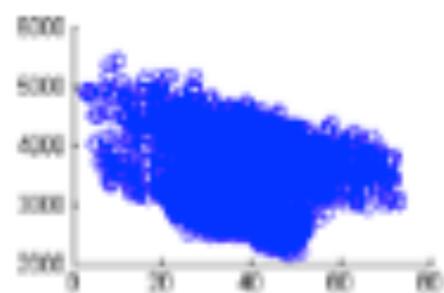
Load vs. TMP (DPT=19)



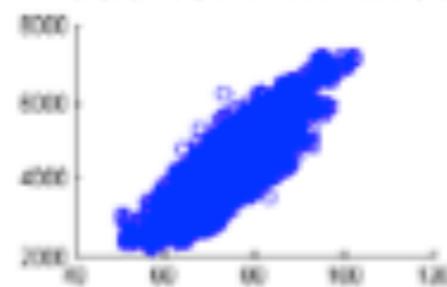
Load vs. TMP (DPT=52)



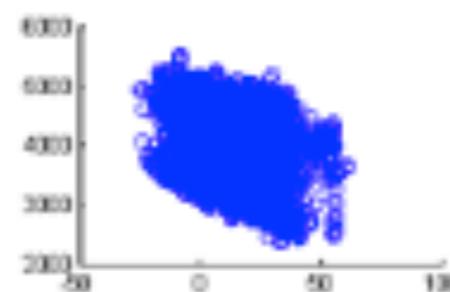
Load vs. TMP in Mar



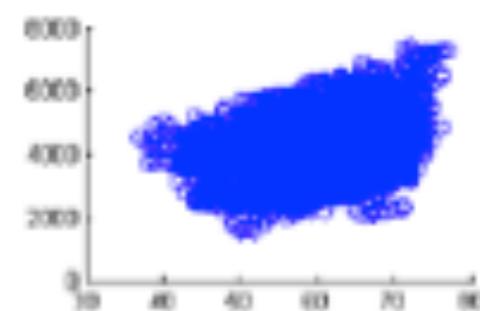
Load vs. TMP in Jul

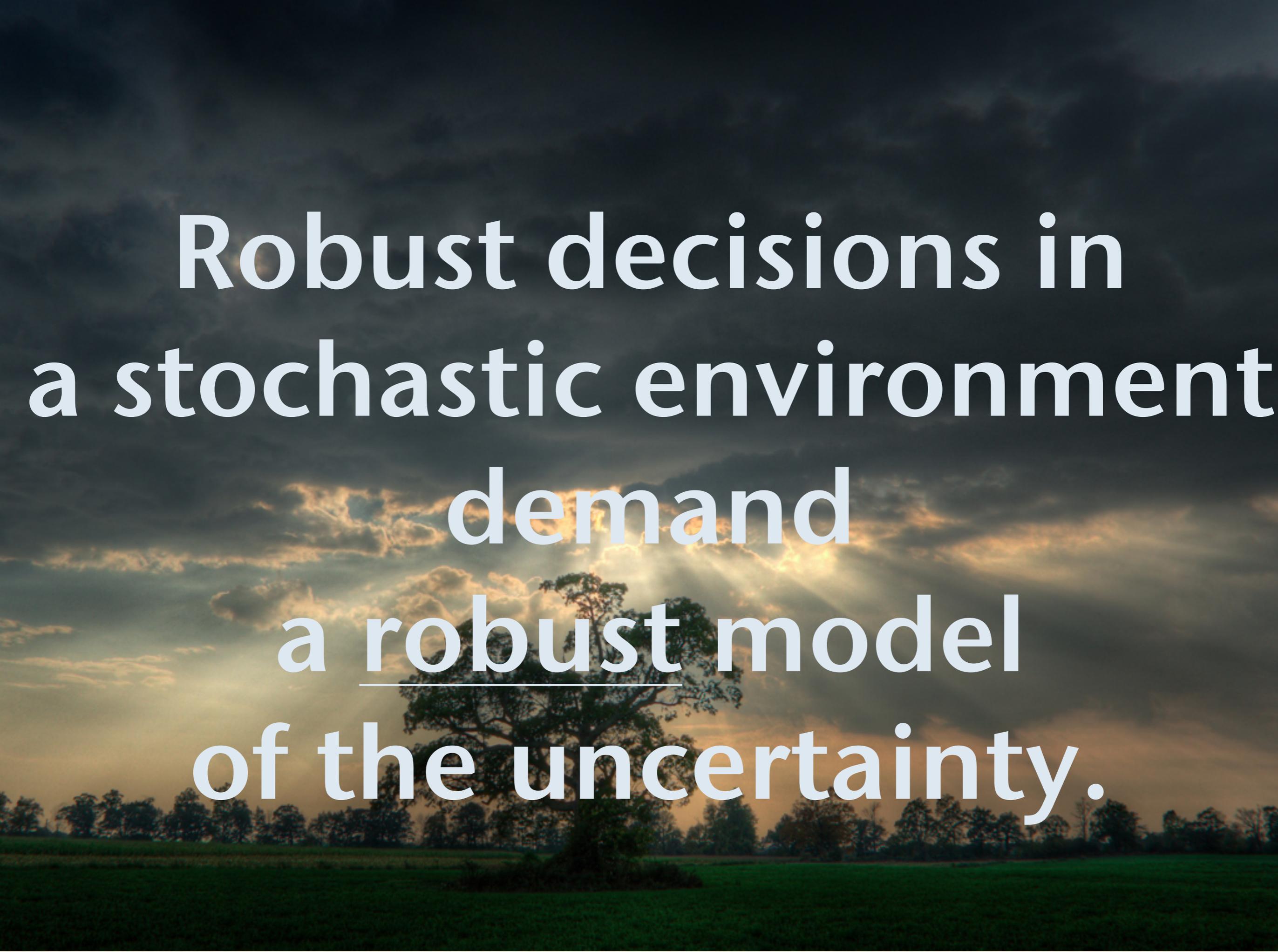


Load vs. DPT in Jan



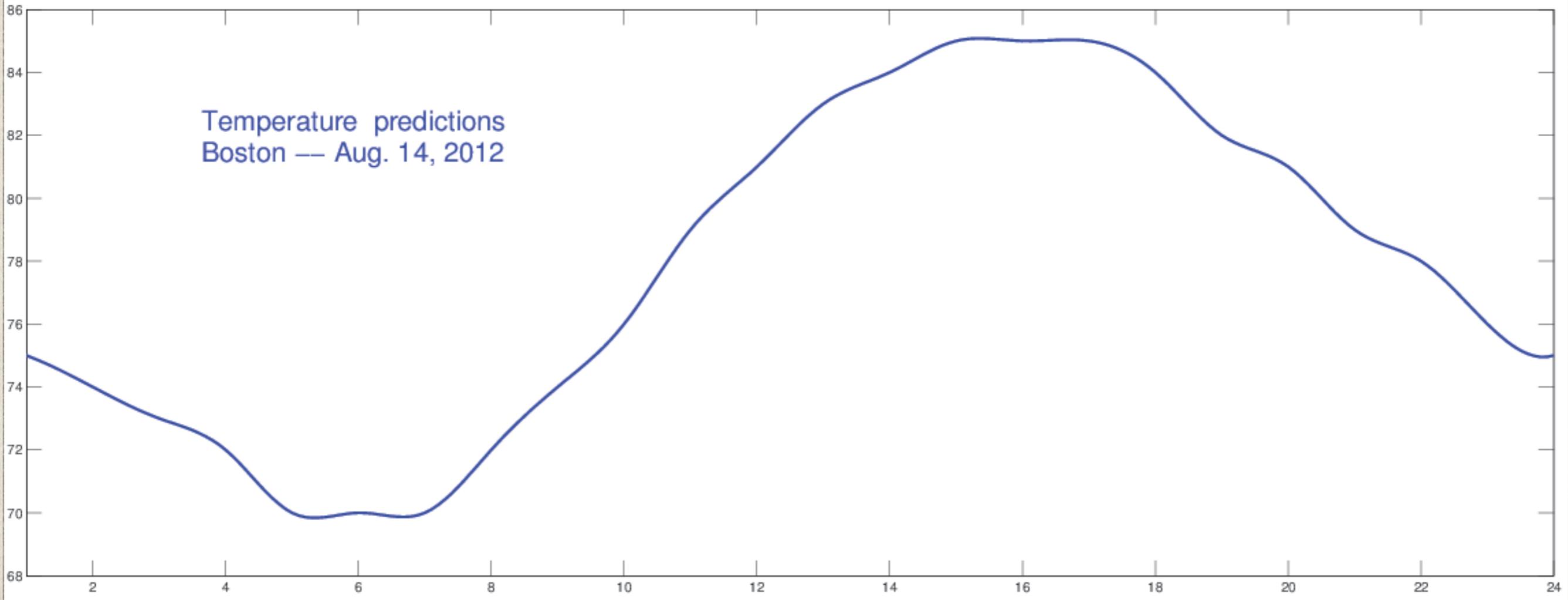
Load vs. DPT in Aug



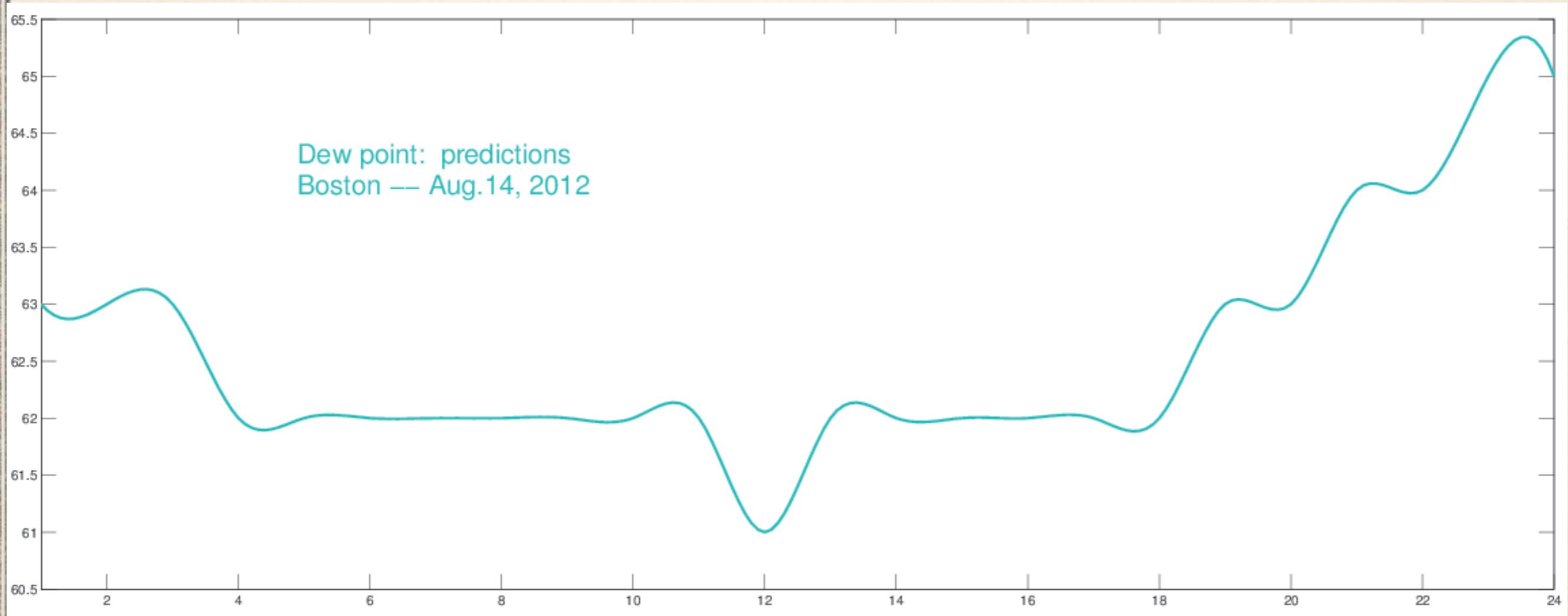


**Robust decisions in  
a stochastic environment  
demand  
a robust model  
of the uncertainty.**

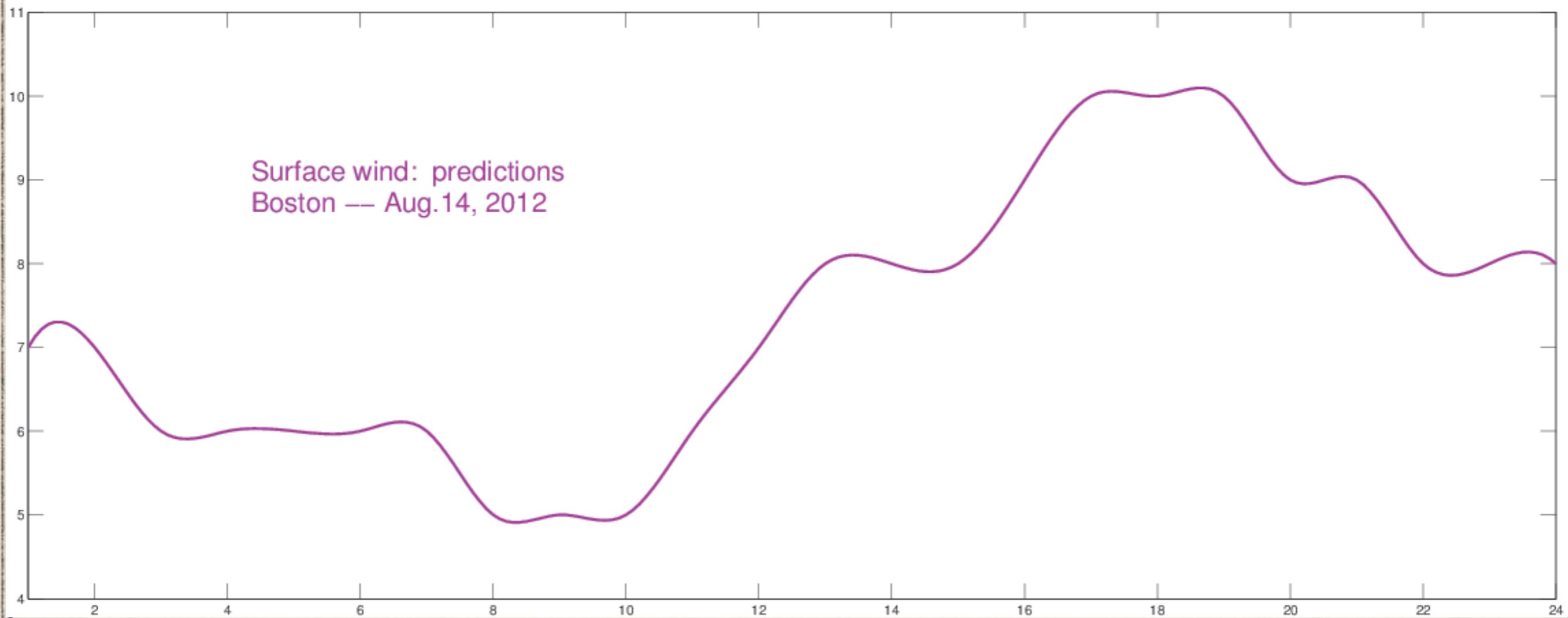
# from predictions on day D-1 to load forecasts on day D



# from predictions on day D-1 to load forecasts on day D



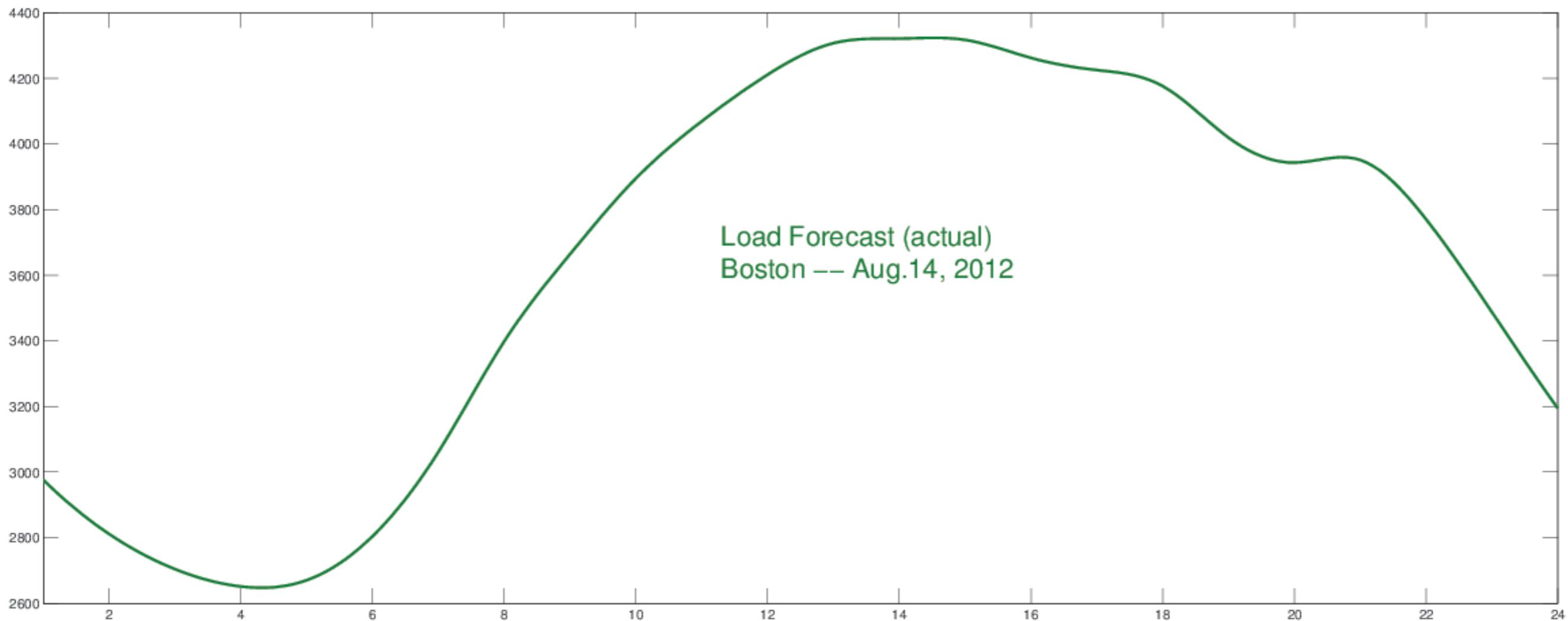
# from predictions on day D-1 to load forecasts on day D



# ... to load on day D

to be delivered-load:  $l(t)$

$$= \text{fcn}(\text{temp}(\tau \leq t), \text{dewpt}(\tau \leq t), \text{clcover}(\tau \leq t), \text{wind}(\tau \leq t)), t \leq 24$$



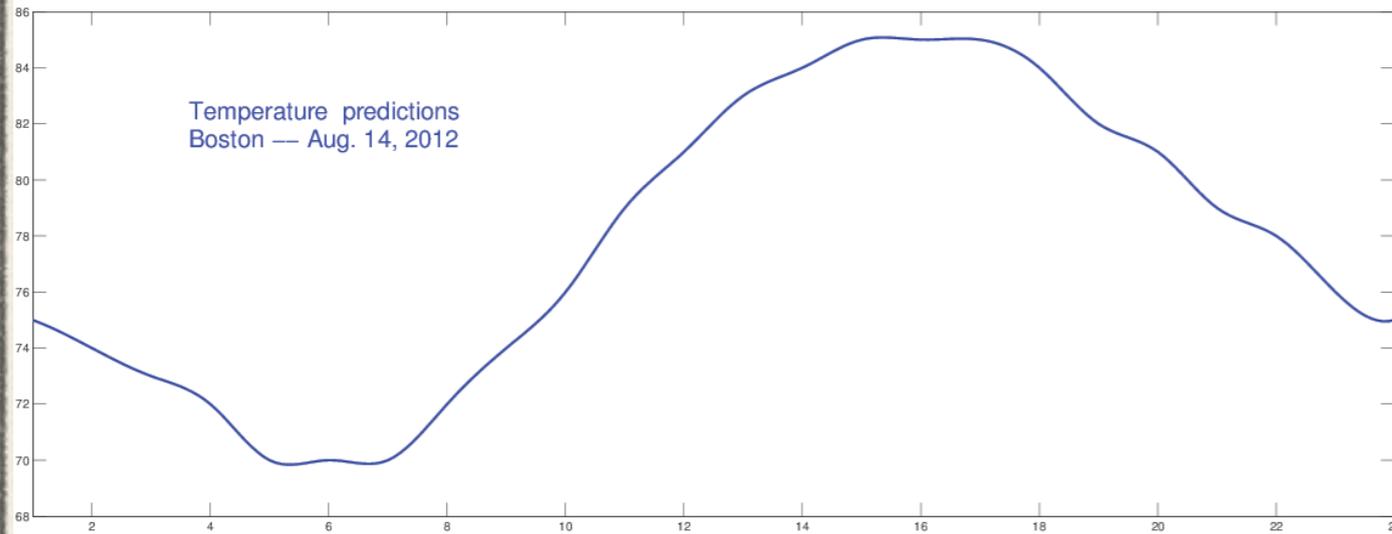
# ... to load on day D

to be delivered-load:  $l(t)$

$$= \text{fcn}(\text{temp}(\tau \leq t), \text{dewpt}(\tau \leq t), \text{clcover}(\tau \leq t), \text{wind}(\tau \leq t)), t \leq 24$$

**BUT THAT WOULDN'T CAPTURE THE UNCERTAINTY!  
ONE WOULD EXPECT:**

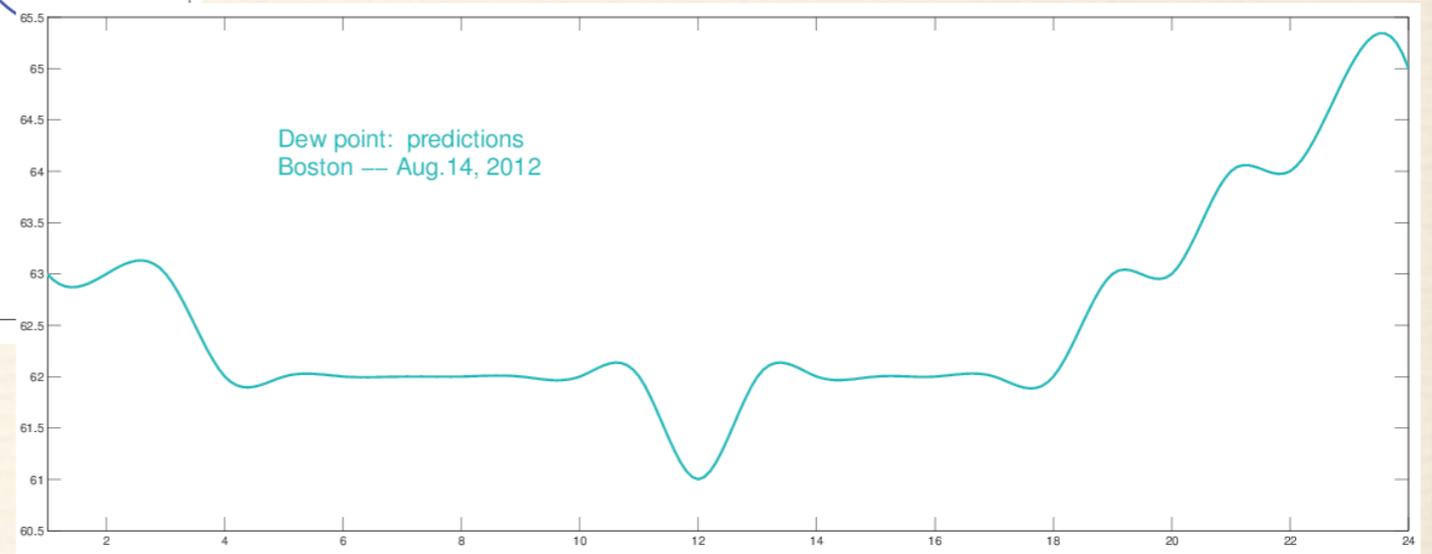
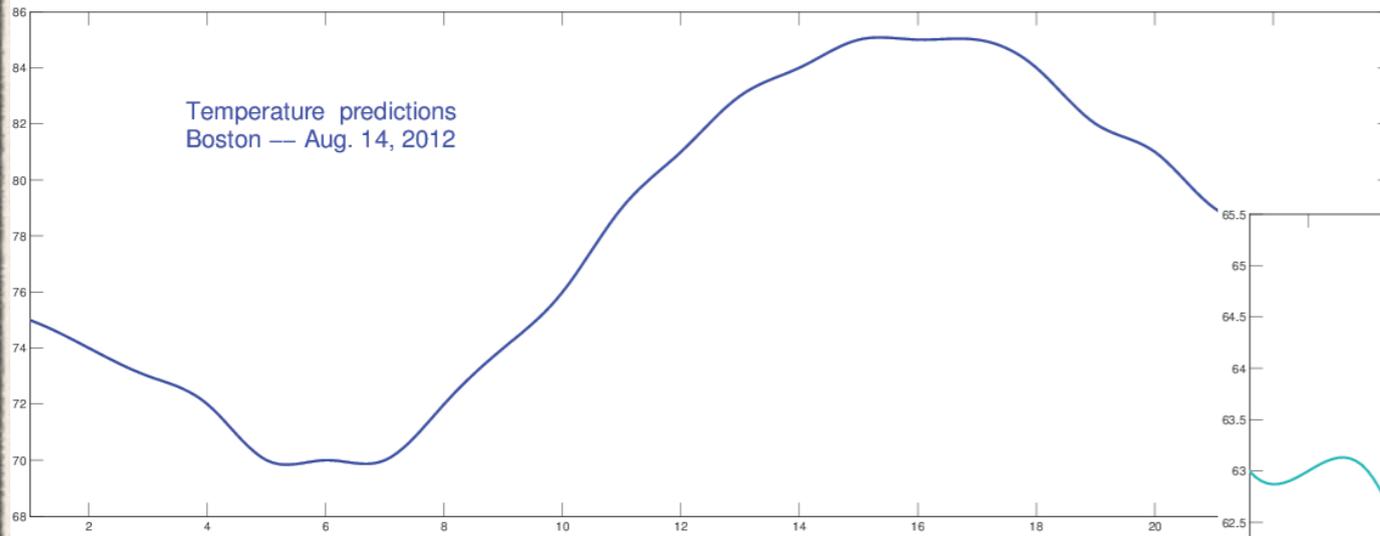
# from predictions on day D-1 to load forecasts on day D



**THE DATA**

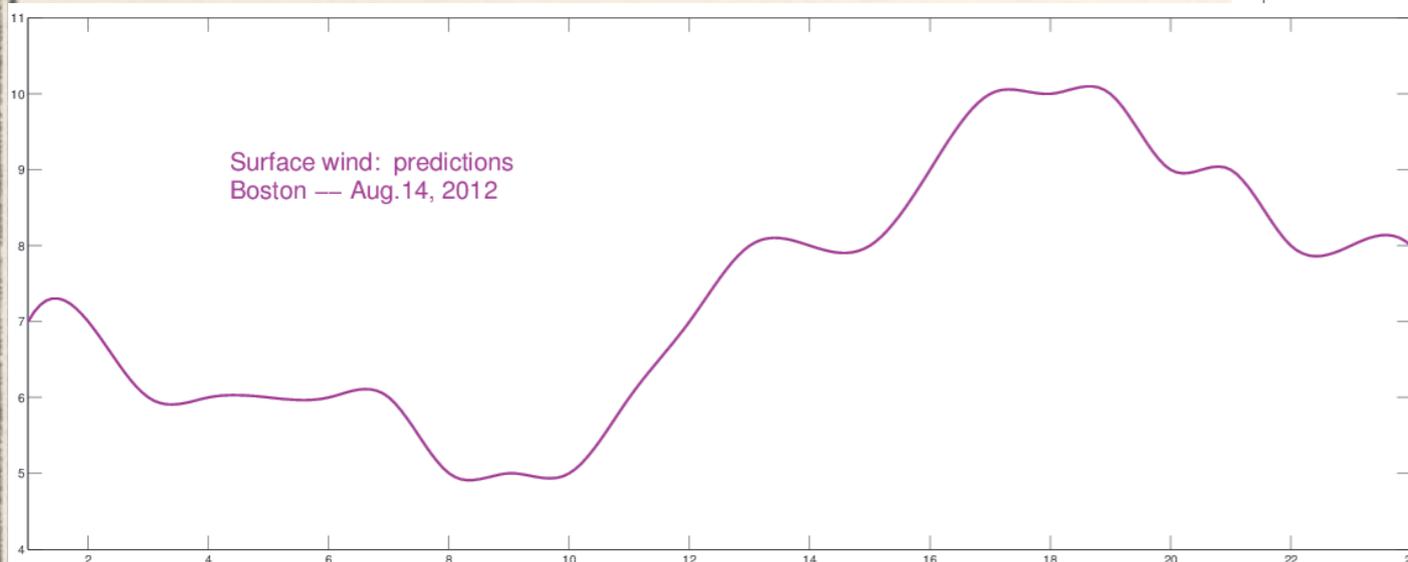
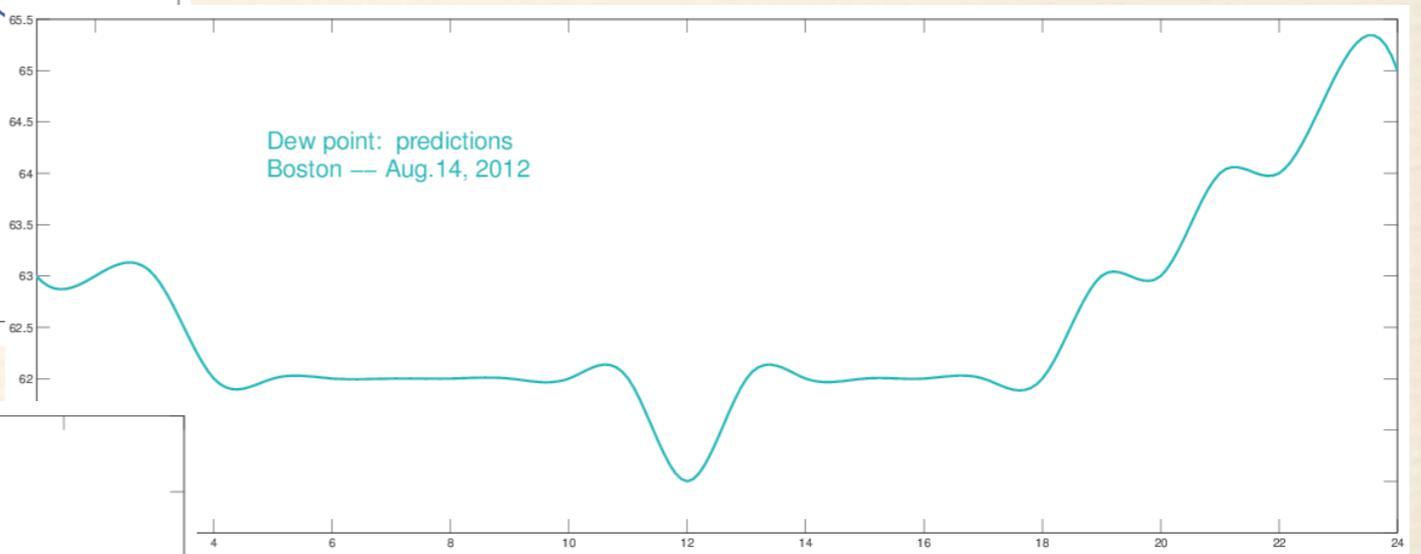
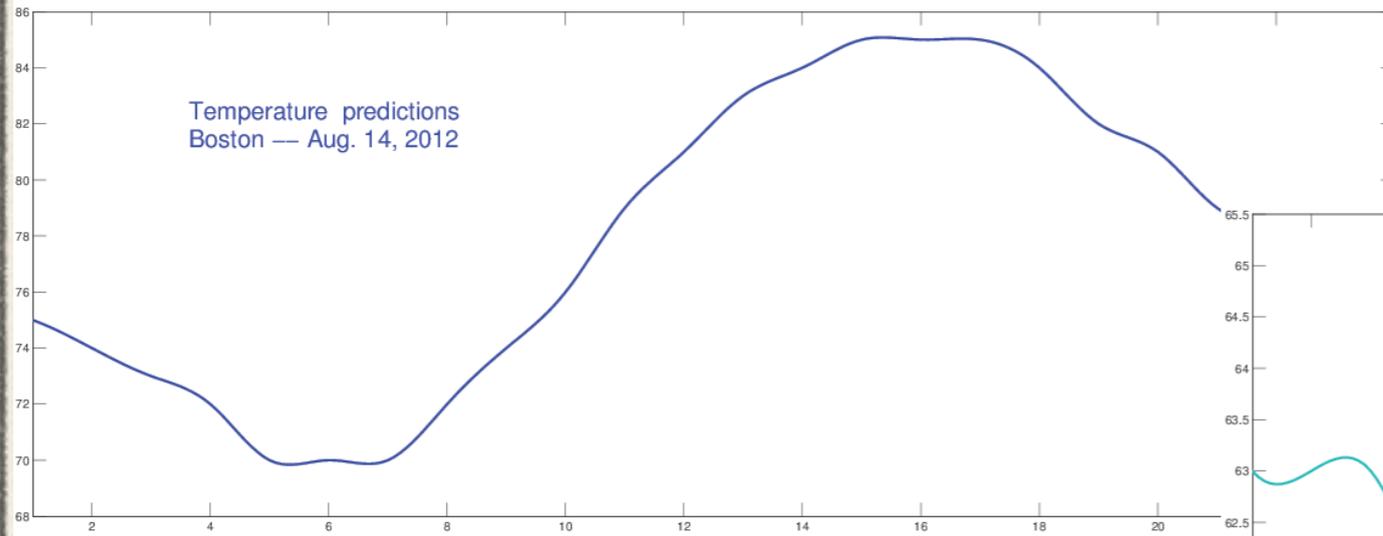
# from predictions on day D-1 to load forecasts on day D

## THE DATA



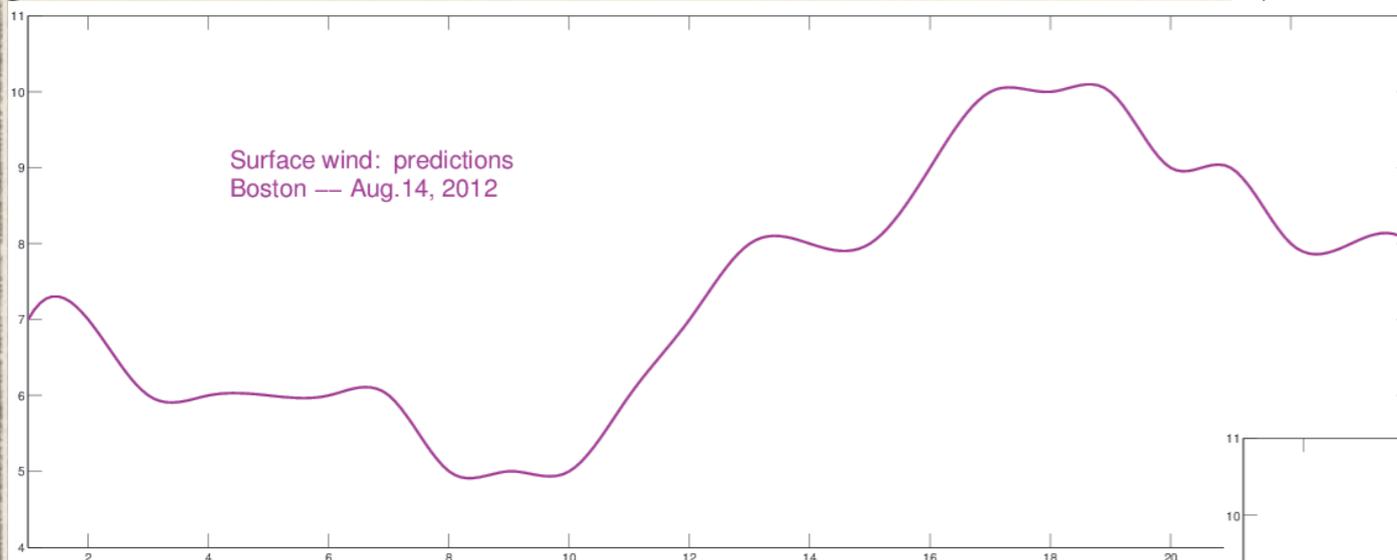
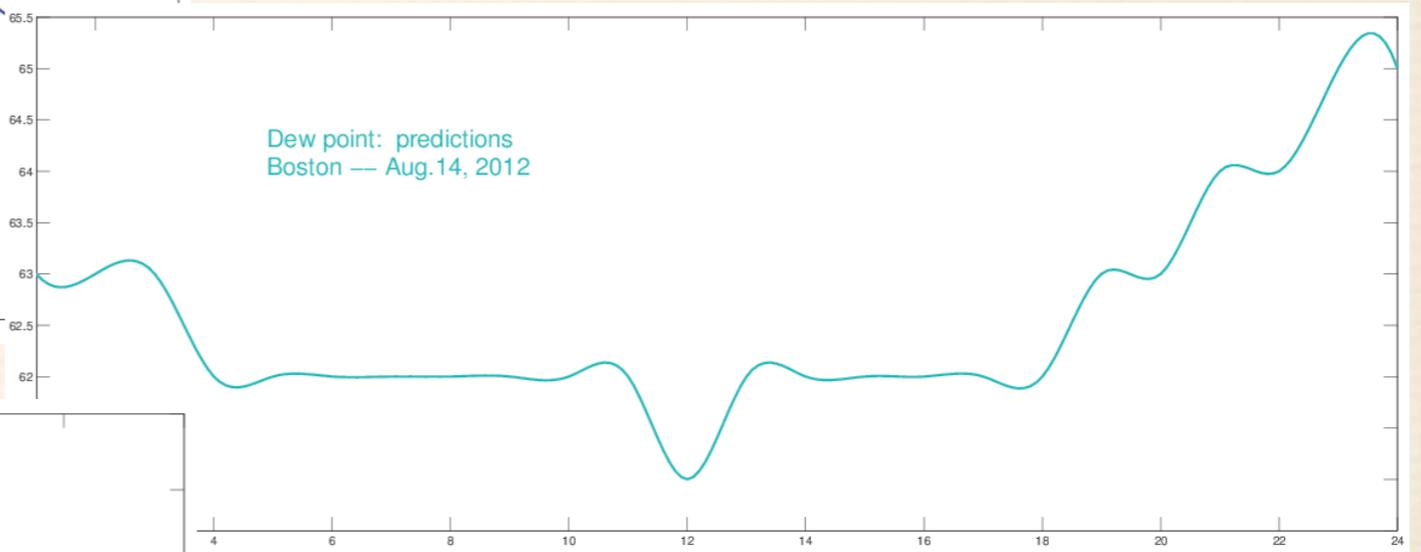
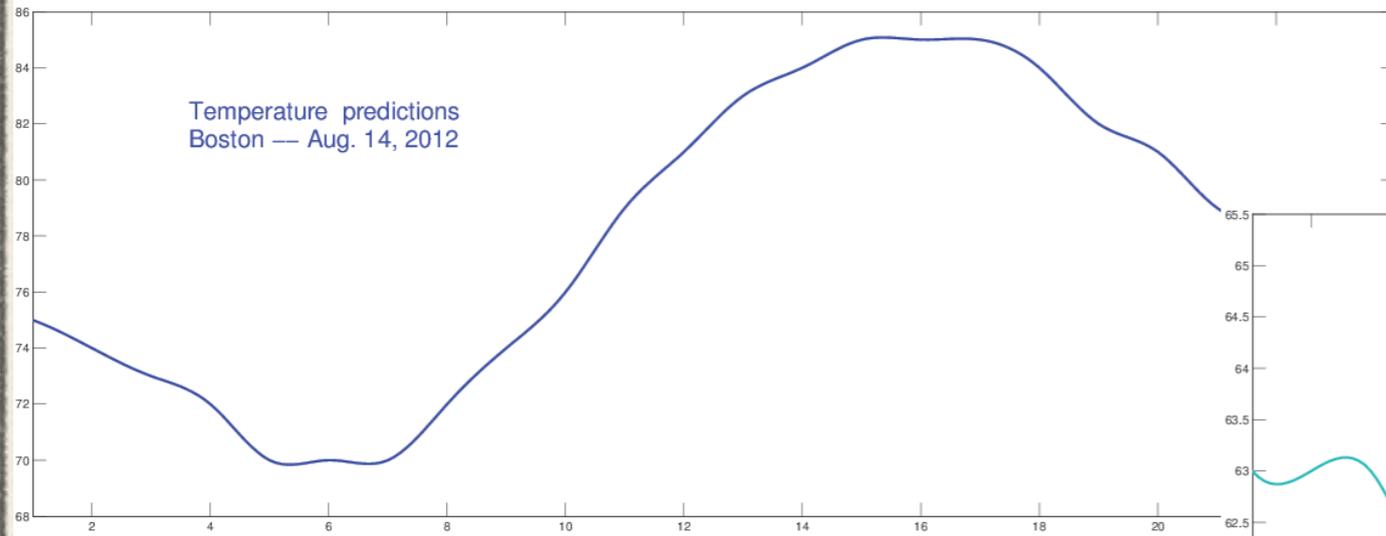
# from predictions on day D-1 to load forecasts on day D

## THE DATA



# from predictions on day D-1 to load forecasts on day D

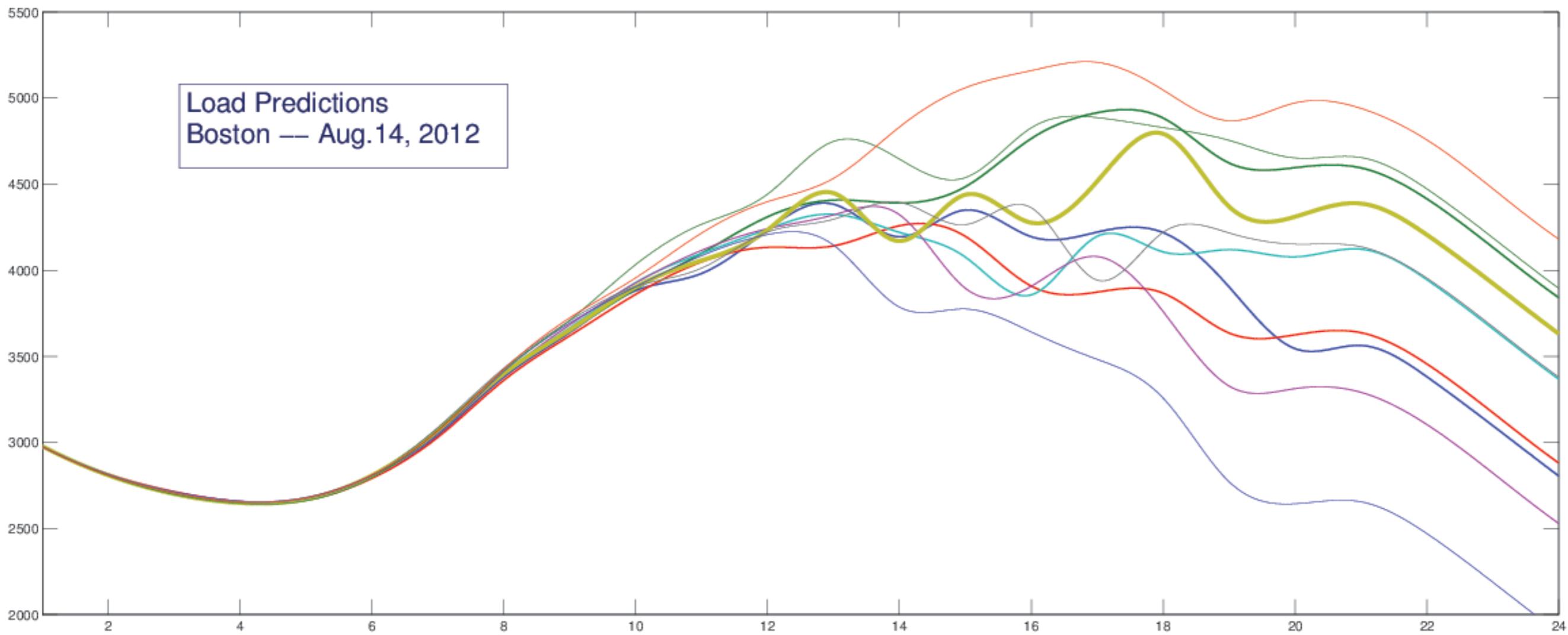
## THE DATA



NOAA: ACTUALS

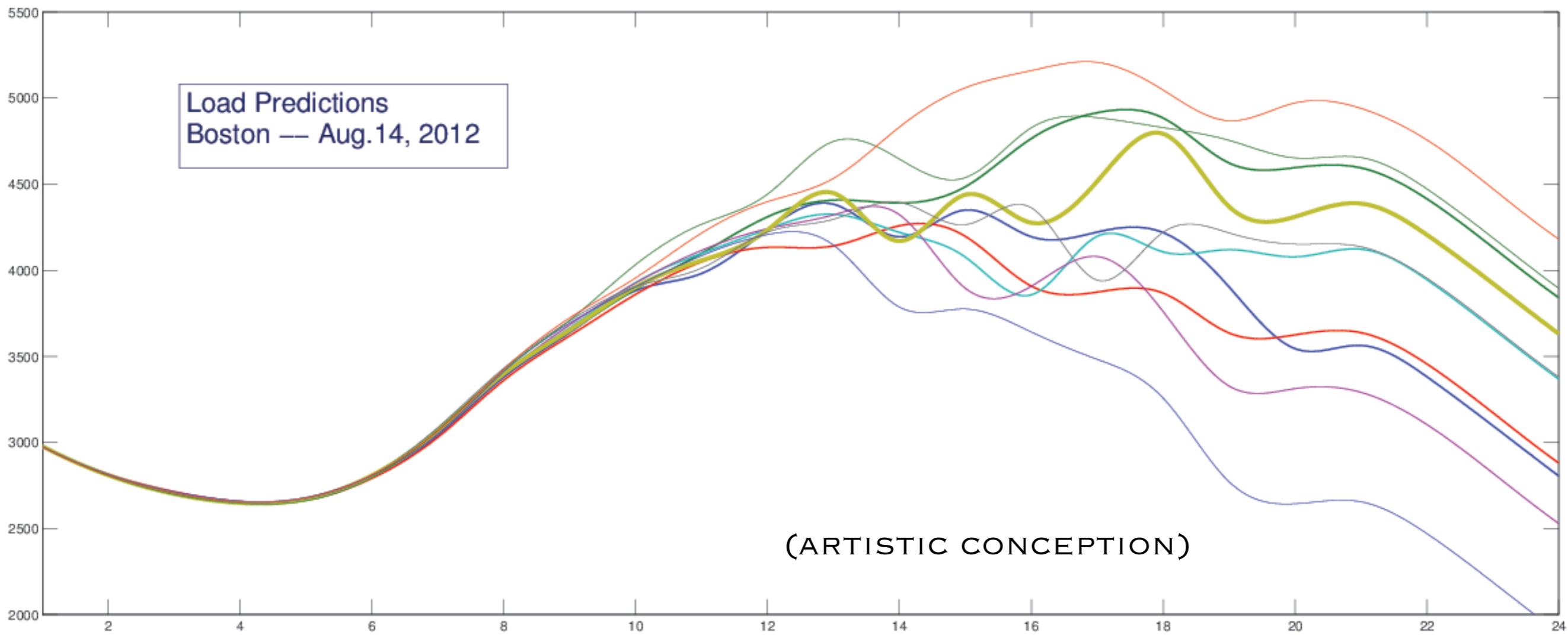
# “Realistic” Forecasts

Load Predictions  
Boston — Aug. 14, 2012



# “Realistic” Forecasts

Load Predictions  
Boston — Aug. 14, 2012



(ARTISTIC CONCEPTION)

# Troubling Issues









Stochastic Load Process

⇒ Scenarios



# Stochastic Load Process

⇒⇒ Scenarios



a) segmentation: season + day characteristics

# Stochastic Load Process

## ⇒ Scenarios



- a) segmentation: season + day characteristics
- b) functional regression for given segment

# Stochastic Load Process

## ⇒ Scenarios



- a) segmentation: season + day characteristics
- b) functional regression for given segment
- c) hourly distribution of errors per segment

# Stochastic Load Process

## ⇒ Scenarios



- a) segmentation: season + day characteristics
- b) functional regression for given segment
- c) hourly distribution of errors per segment

**HOW THIS IS CARRIED OUT ?**

# Stochastic Load Process

## ⇒ Scenarios



- a) segmentation: season + day characteristics
- b) functional regression for given segment
- c) hourly distribution of errors per segment
- HOW THIS IS CARRIED OUT ?**
- d) conditional distribution of errors => process

# Stochastic Load Process

## ⇒ Scenarios

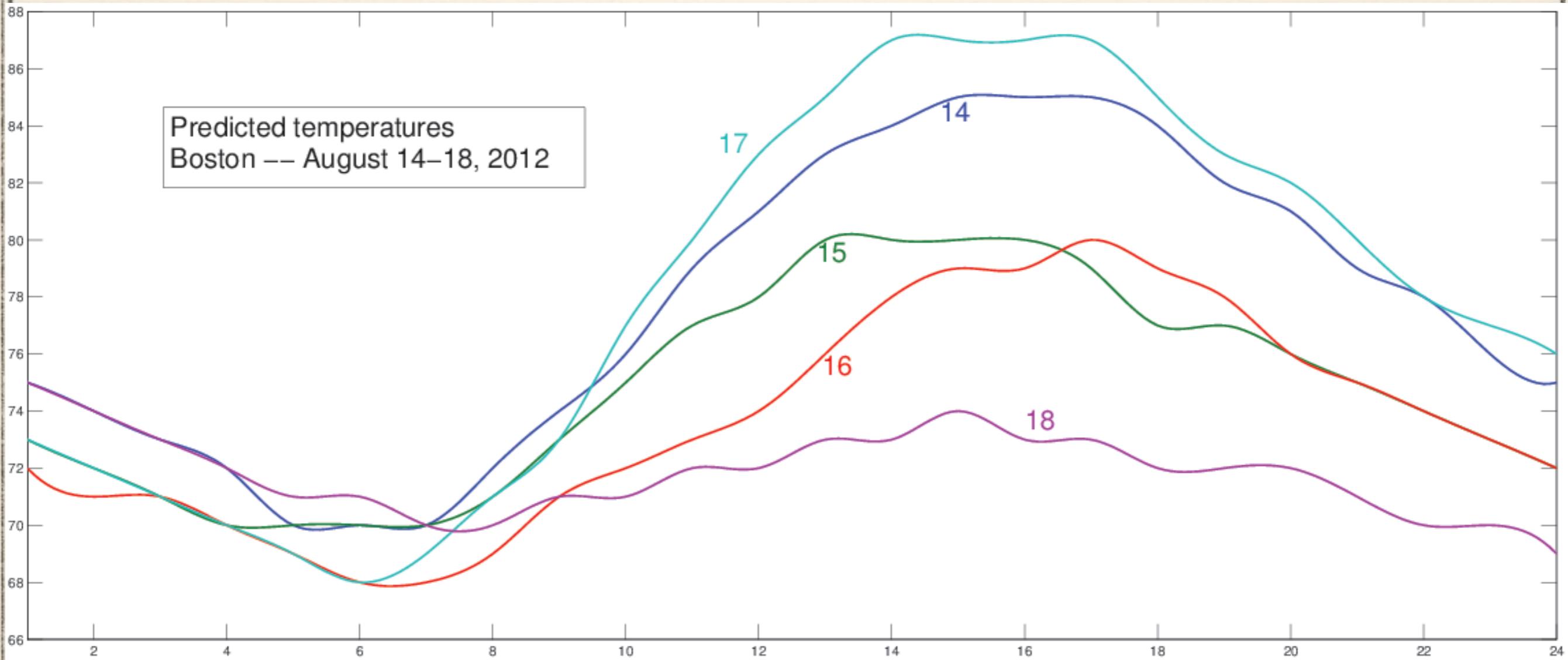


- a) segmentation: season + day characteristics
- b) functional regression for given segment
- c) hourly distribution of errors per segment
- HOW THIS IS CARRIED OUT ?**
- d) conditional distribution of errors => process
- e) discretization of the process => scenarios

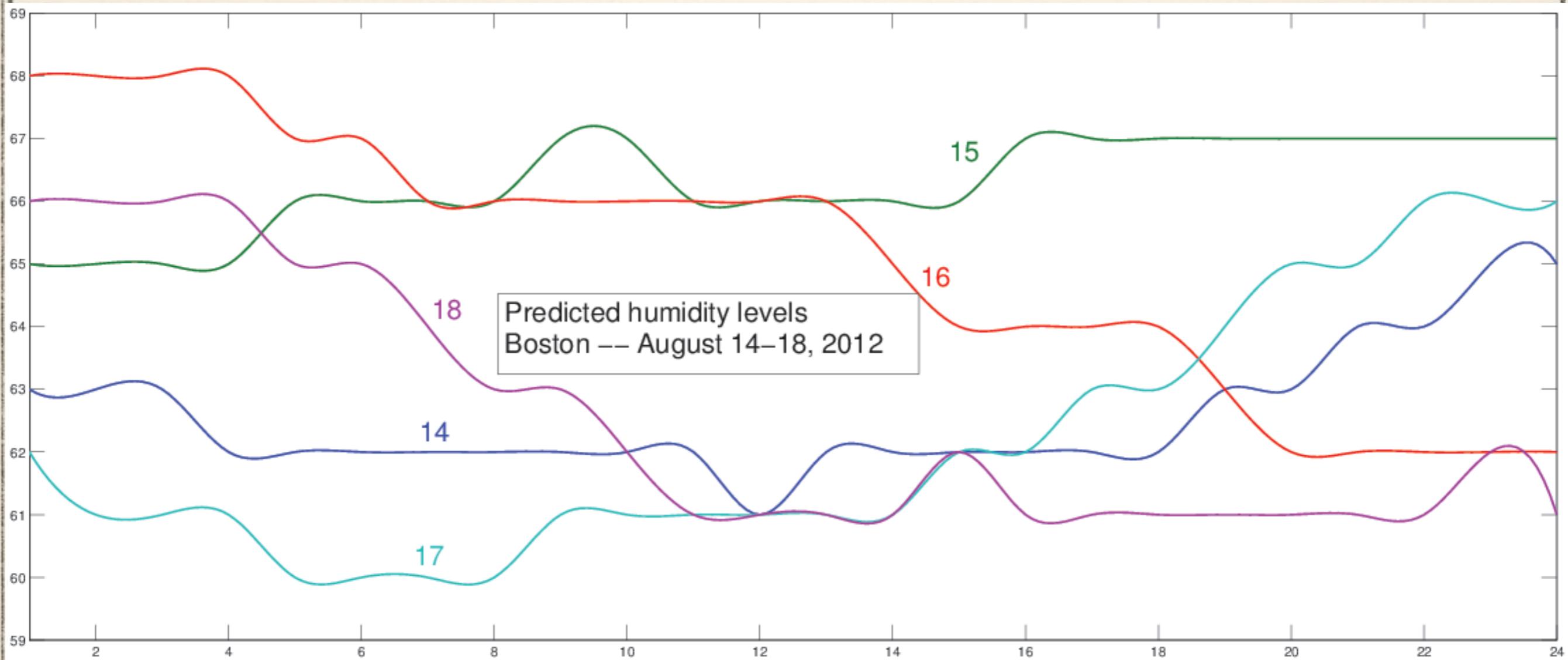
# Segmentation

- ~ similars, analogs ( $\pm$  standard) to enrich data: Wednesday rule, zone rule?
- seasons: (factor analysis, 'heuristics')
  - $\pm$  spring & fall : temperature
  - winter: temperature & cloud cover
  - summer: temperature & dew point
- wind power (at present): handled independently based on 3 TIER analogs total load  $\approx$  load scenario - wind power scenario

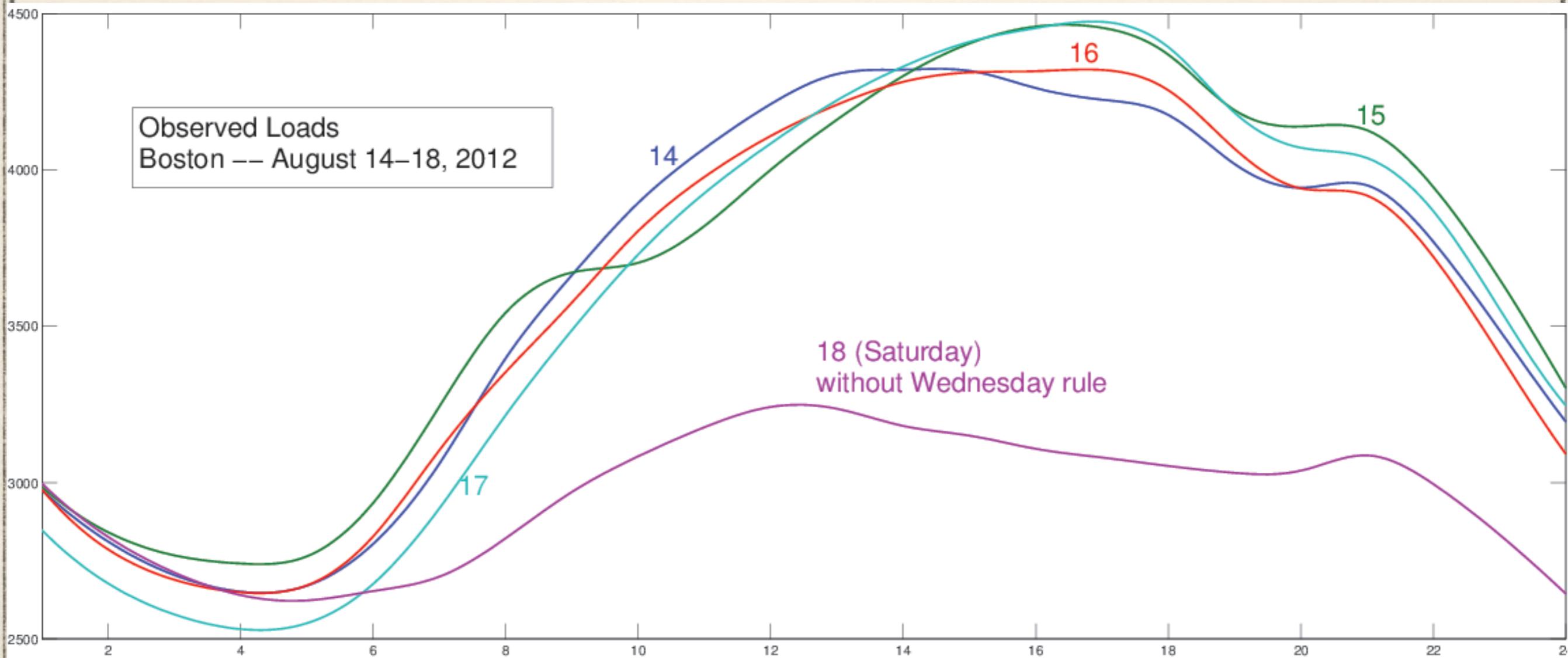
# Summer segment “#1”

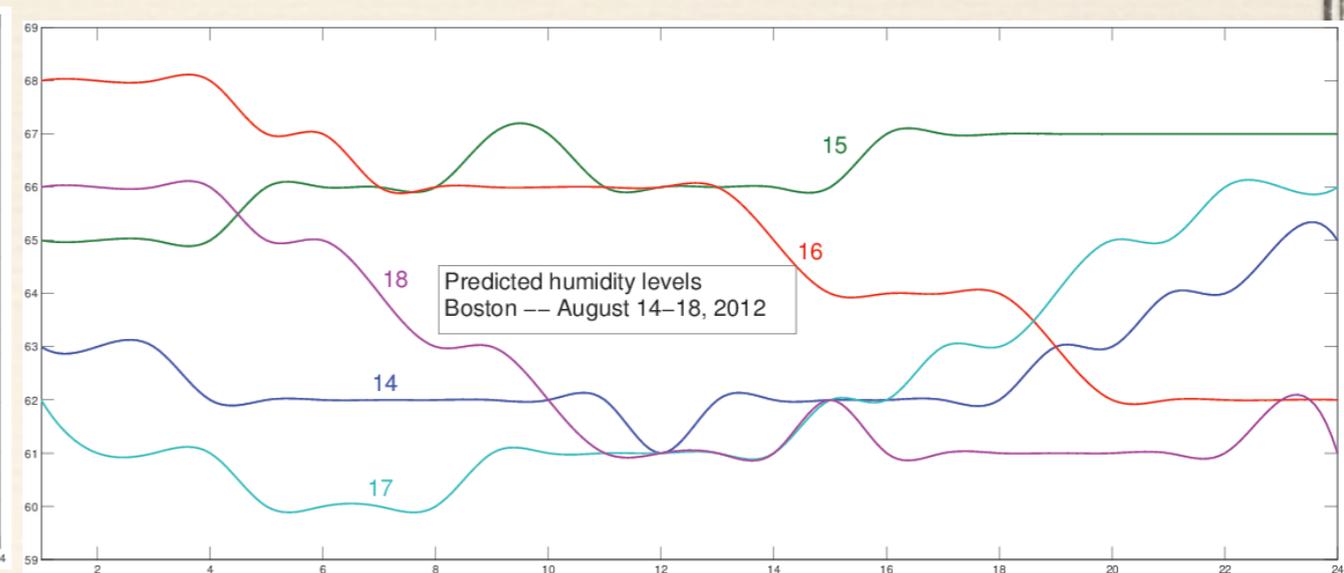
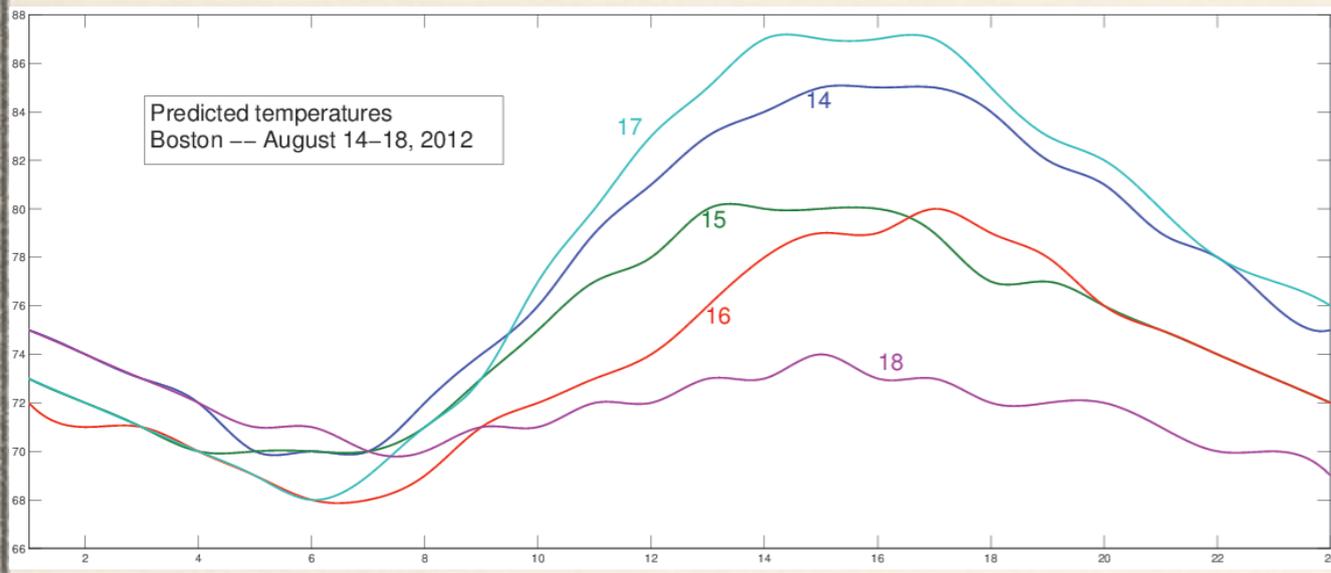


# Summer segment “#1”



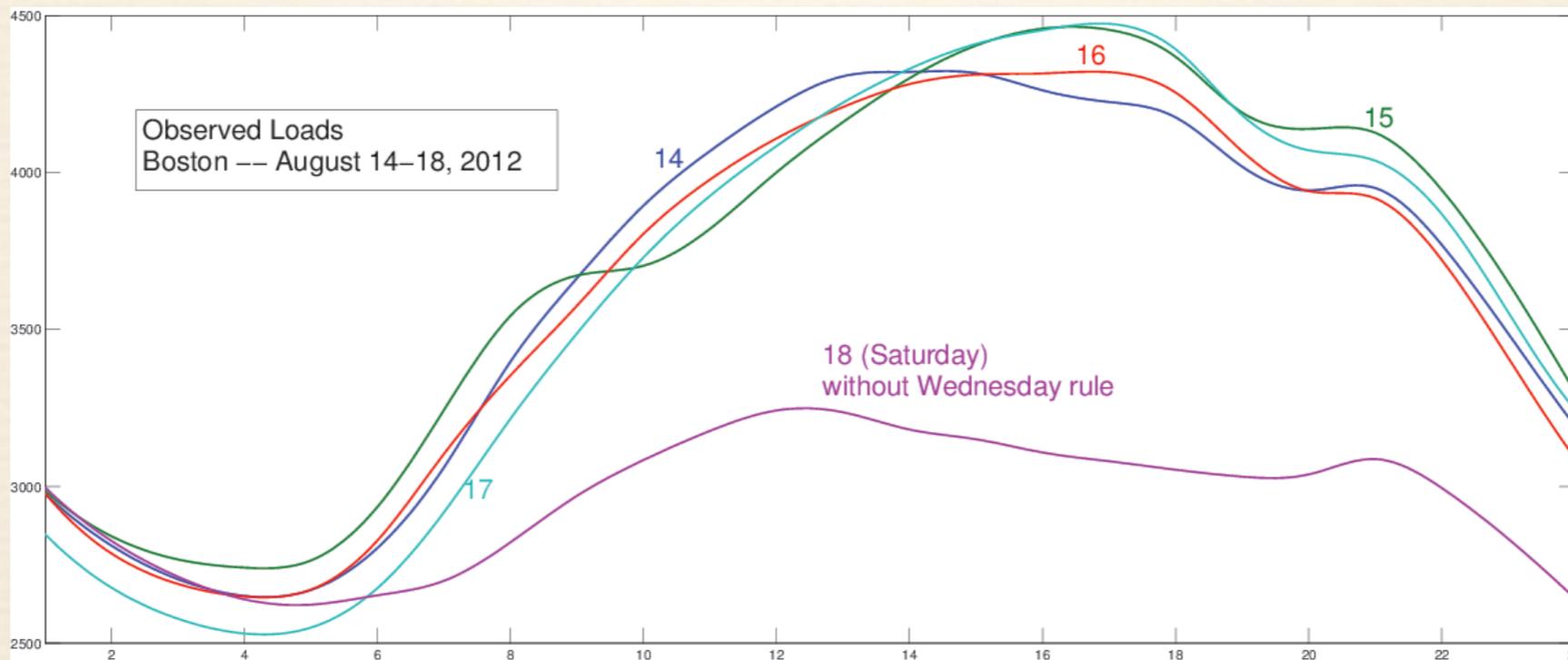
# Summer segment “#1”

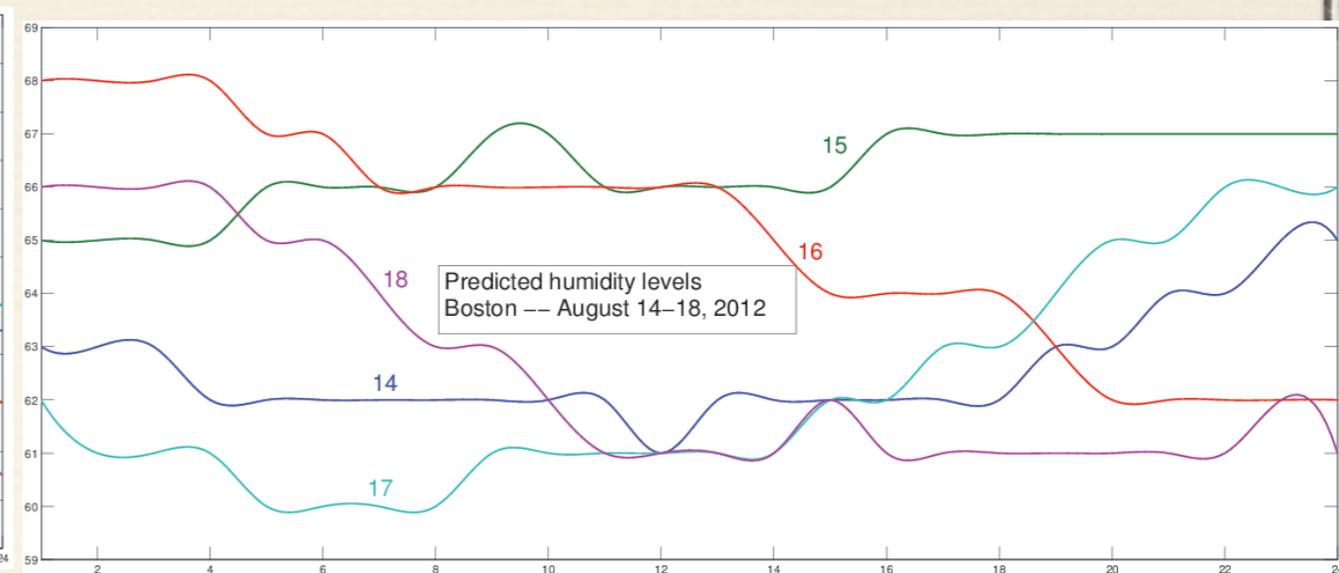
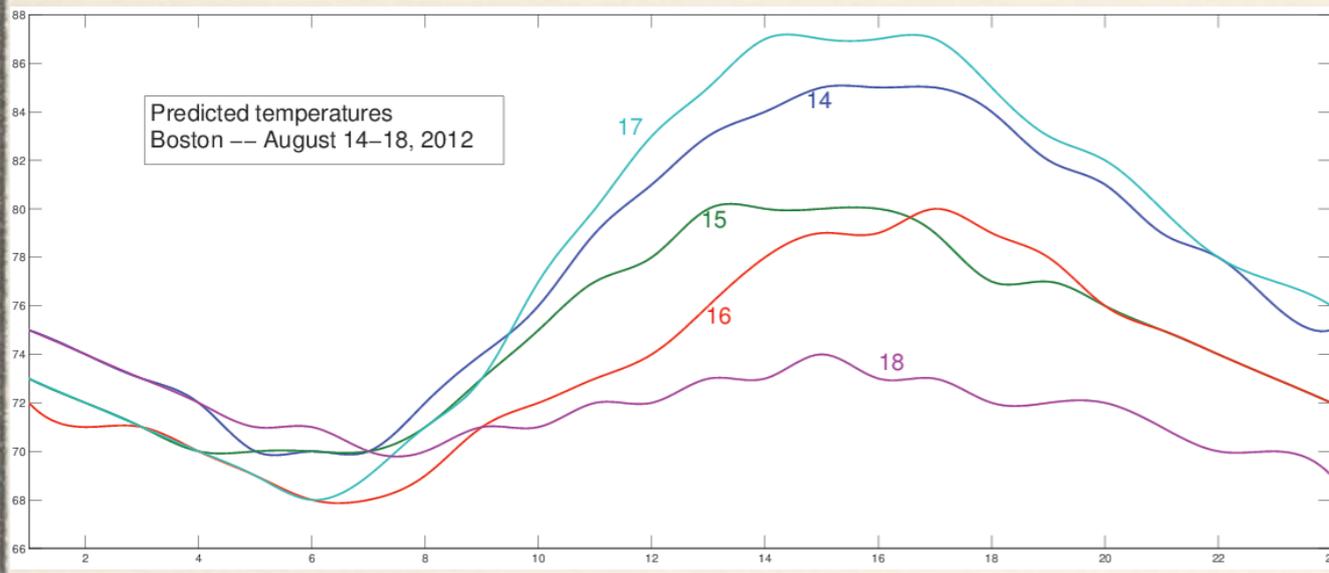




from day  $d-1 \Rightarrow$  possible load on day  $d \quad d = 14, \dots, 18$

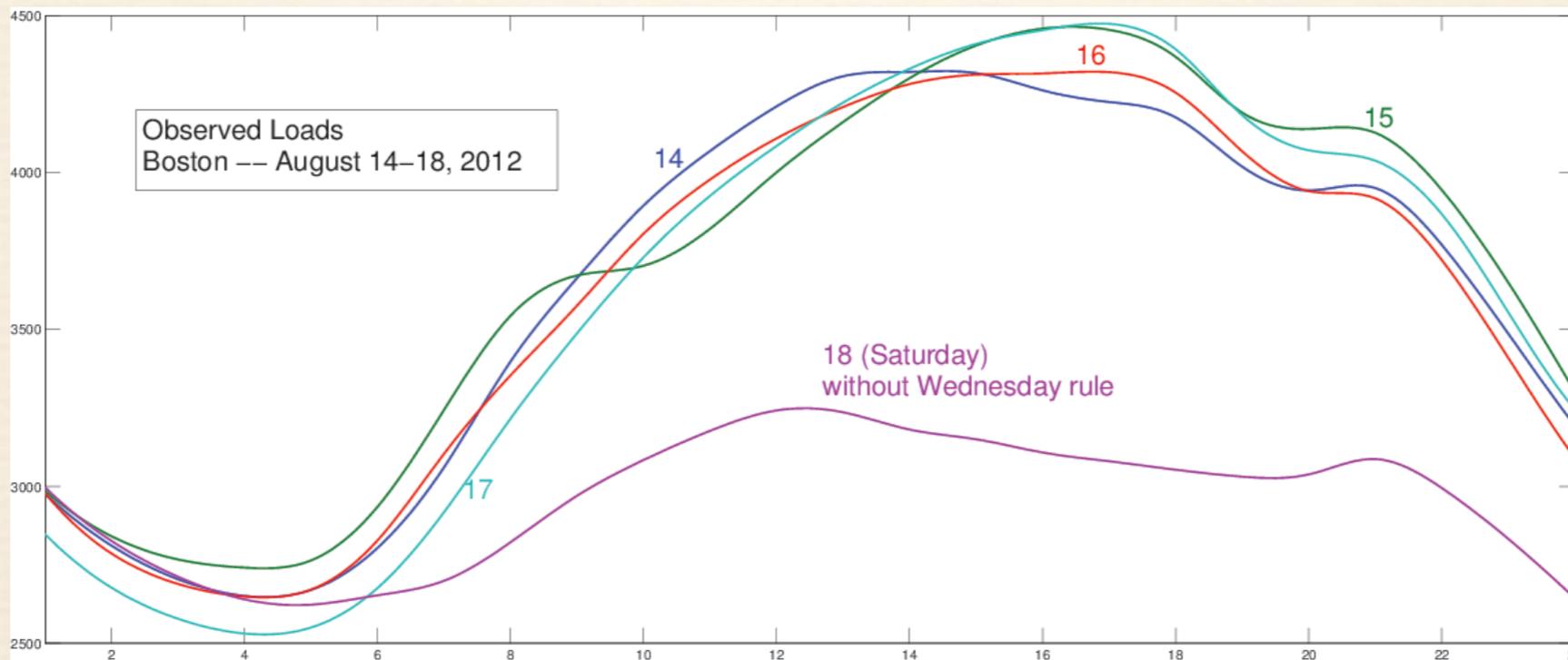
1. regression(temp. curve, humid. curve)  $\Rightarrow$  'expected' load curve
2. get distribution of the errors (hourly, .... at any time)





from day  $d-1 \Rightarrow$  possible load on day  $d \quad d = 14, \dots, 18$

1. regression(temp. curve, humid. curve)  $\Rightarrow$  'expected' load curve
2. get distribution of the errors (hourly, .... at any time)



# The Regression Problem

find a function  $r$  that minimizes errors (with respect to  $\|\square\|$ )

$$\sum_{\text{days } d \text{ in segment}} \sum_{\text{hours } h \text{ in day}} \left\| r\left(\text{tmp}_{d,h}, \text{hum}_{d,h}\right) - \text{load}_{d,h} \right\|$$

an infinite dimensional problem!

Our approach: rely on 2-dimensional epi-splines ("innovation")

- epi-splines approximate with arbitrary accuracy 'any' function
- epi-splines are completely determined by a finite # of parameters
- allows (via constraints) to include 'soft' (non-data) information

# The Errors Distributions

Given segment # and associated  $r$ , for fixed hour  $h$

$$e_{d,h} = \text{load}_{d,h} - r\left((tmp_{d,h}, hum_{d,h})\right), d \in \text{segment \#}$$

$\Rightarrow$  estimate the density  $f_h$  of the errors (at  $h$  in segment #)

yields an overall estimate of the 'volatility' (in fact, more)

another infinite dimensional problem & data might be scarce

Our approach: estimation via exponential epi-spline (novel):

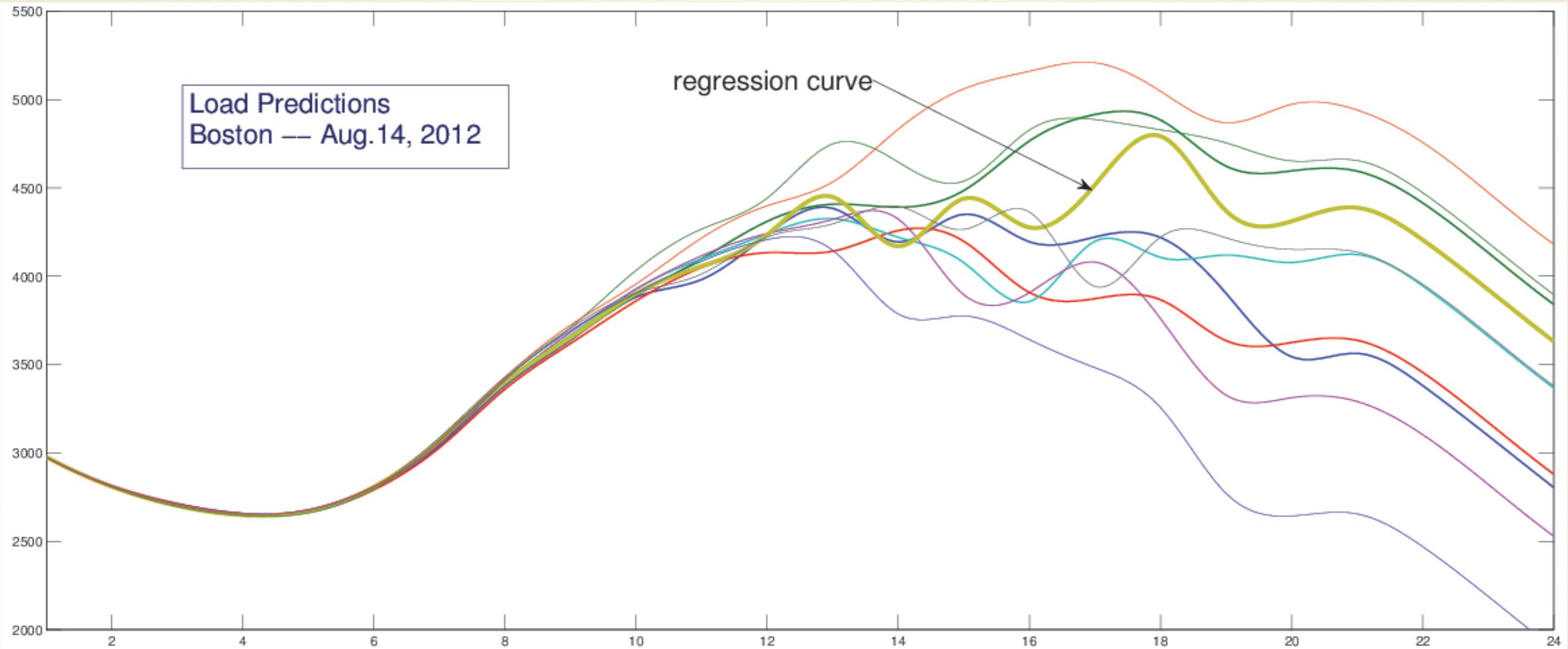
-  $f_h = \exp(-s_h)$ ,  $s_h$  an epi-spline ( $\Rightarrow f_h \geq 0$ )

- same properties as epi-spline, could include unimodality restriction



... et voilà!

regression curve & sampling from errors distribution



*a.* how many samples?  $10^3, 10^5, \dots$ ?

*b.* conditioning: @ 10 o'clock above or below the regression curve

# Approximation foundation

Function Identification Problem: optim., diff. eqtns, processes, ...

$$(FIP) \text{ find } f \in \arg \min \{ \psi(f) \mid f \in F \subset \mathcal{F} \}$$

here  $\mathcal{F} = \text{lsc-fcns}(\mathbb{R}^n)$  more generally a Polish space

$\mathcal{F}$ -approximation: *aw*-topology  $\sim dl(f, g)$  epi-distance

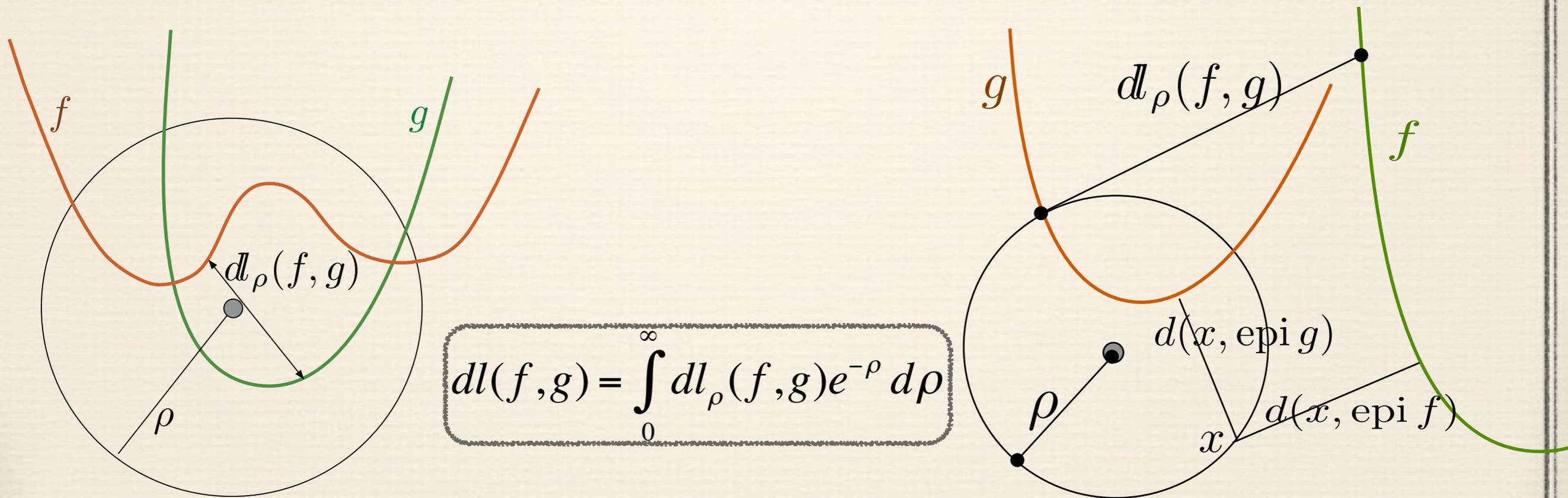
# Approximation foundation

Function Identification Problem: optim., diff. eqtns, processes, ...

$$(FIP) \text{ find } f \in \arg \min \{ \psi(f) \mid f \in F \subset \mathcal{F} \}$$

here  $\mathcal{F} = \text{lsc-fcns}(\mathbb{R}^n)$  more generally a Polish space

$\mathcal{F}$ -approximation: *aw*-topology  $\sim dl(f, g)$  epi-distance



# Epi-splines

$\mathcal{R} = \{R_1, \dots, R_N \text{ open}\}$  **partitions** (no overlap)  $B = \bigcup_{k=1}^N \text{cl } R_k$  (closed)

$\text{poly}^p(\mathbb{R}^n)$  defined by  $n_p \leq (n+p)! / (n! p!)$  real parameters

lsc epi-spline  $s : \mathbb{R}^n \rightarrow \bar{\mathbb{R}}$  with partition  $\mathcal{R}$  of order  $p \in \mathbb{N}_0$  if

on each  $R_k$   $s \in \text{poly}^p(\mathbb{R}^n)$ ,  $s \equiv \infty$  on  $\mathbb{R}^n \setminus B$ ,  $s$  is lsc on  $\mathbb{R}^n$

then  $s \in \text{e-spl}_n^p(\mathcal{R}) \dots \subset \text{lsc-fcns}(B) \subset \text{lsc-fcns}(\mathbb{R}^n)$

# Epi-splines

$\mathcal{R} = \{R_1, \dots, R_N \text{ open}\}$  partitions (no overlap)  $B = \bigcup_{k=1}^N \text{cl } R_k$  (closed)

$\text{poly}^p(\mathbb{R}^n)$  defined by  $n_p \leq (n+p)! / (n! p!)$  real parameters

lsc epi-spline  $s : \mathbb{R}^n \rightarrow \bar{\mathbb{R}}$  with partition  $\mathcal{R}$  of order  $p \in \mathbb{N}_0$  if

on each  $R_k$   $s \in \text{poly}^p(\mathbb{R}^n)$ ,  $s \equiv \infty$  on  $\mathbb{R}^n \setminus B$ ,  $s$  is lsc on  $\mathbb{R}^n$

then  $s \in \text{e-spl}_n^p(\mathcal{R}) \dots \subset \text{lsc-fcns}(B) \subset \text{lsc-fcns}(\mathbb{R}^n)$

◆ 1.  $\forall p \in \mathbb{N}_0$ , infinite refinement  $\{\mathcal{R}^\nu\}_{\nu=1}^\infty$  of closed  $B \subset \mathbb{R}^n$

$\bigcup_{\nu=1}^\infty \text{e-spl}_n^p(\mathcal{R}^\nu)$  is dense in  $\text{lsc-fcns}(B)$

refinements: boxes, simplexes, ...

◆ 2. When  $(FIP^\nu) \xrightarrow{\text{epi}} (FIP)$ ,  $s^k \in \arg \min(FIP^{\nu_k})$ ,  $dl(s^k, f) \rightarrow 0$

then  $f \in \arg \min(FIP)$ .

a few applications

# a few applications

- ❖ curve fitting:  $F$  known properties of the curve

# a few applications

- ❖ curve fitting:  $F$  known properties of the curve
- ❖ financial curves: yield curve, discount factor curve

# a few applications

- ❖ curve fitting:  $F$  known properties of the curve
- ❖ financial curves: yield curve, discount factor curve
- ❖ variogram: geostatistics, deposit dispersion

# a few applications

- ❖ curve fitting:  $F$  known properties of the curve
- ❖ financial curves: yield curve, discount factor curve
- ❖ variogram: geostatistics, deposit dispersion
- ❖ uncertainty quantification in a harmonic excitation

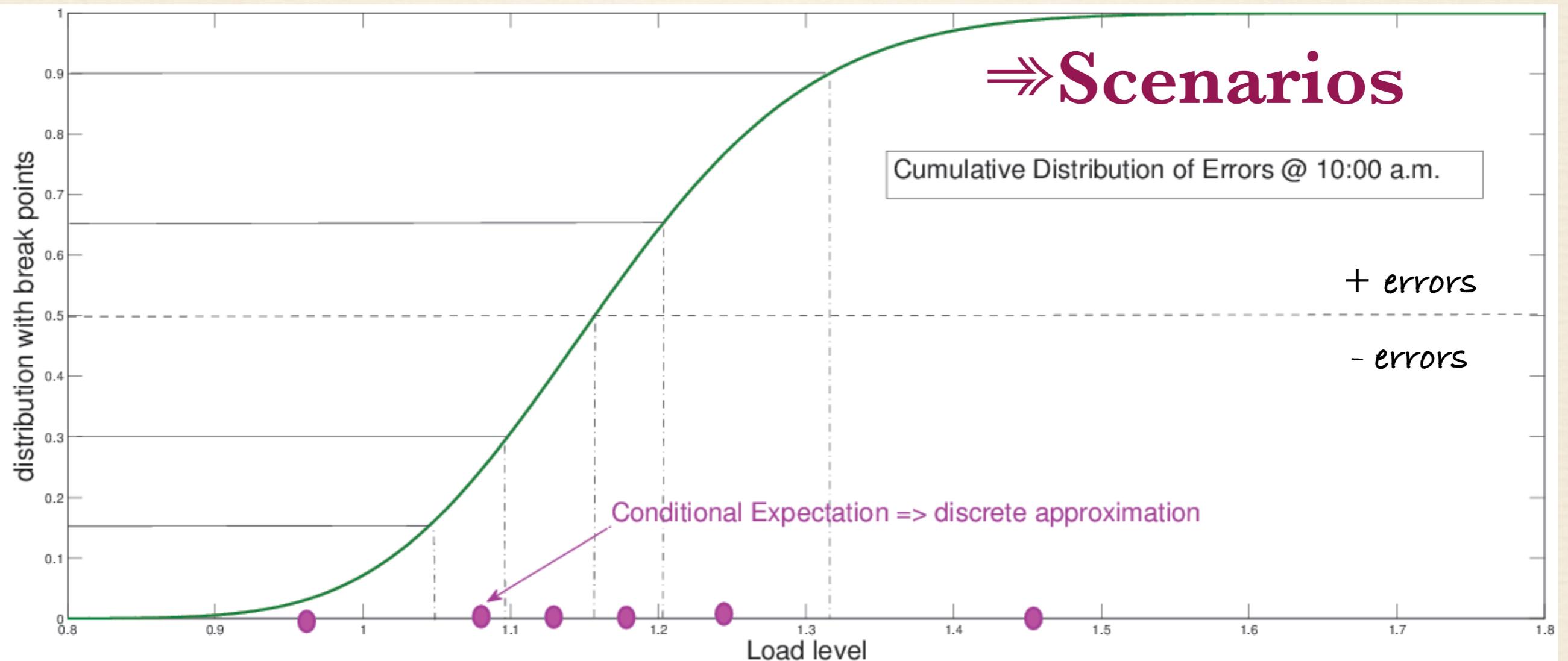
# a few applications

- ❖ curve fitting:  $F$  known properties of the curve
- ❖ financial curves: yield curve, discount factor curve
- ❖ variogram: geostatistics, deposit dispersion
- ❖ uncertainty quantification in a harmonic excitation
- ❖ building stochastic processes: commodities prices, e-loads, ...

# a few applications

- ❖ curve fitting:  $F$  known properties of the curve
- ❖ financial curves: yield curve, discount factor curve
- ❖ variogram: geostatistics, deposit dispersion
- ❖ uncertainty quantification in a harmonic excitation
- ❖ building stochastic processes: commodities prices, e-loads, ...
- ❖ density estimation:  $\psi$  max. likelihood (captures observations),  
 $F$  soft information: support, shape, bounded moments,  
Bayesian, ..

# Conditioning & Discretization



- identify all observed load curves in each sub-segment
- for each sub-segment: re-calculate regression and errors distribution
- repeat for each sub-segment @ (say, 1 p.m.) ⇒ sub-sub-segment

# Unit commitment

## Part II



Dealing with  
renewables-supply uncertainty

# Wind & Solar

- ❖ wind and solar: complementary balance  $\pm$
- ❖ wind scenarios: 3TIER analogues (ARPA-e 😊)
- ❖ scenario building (state-of-the-art)
- ❖ stochastic process building with soft information coming from the dynamics (started)

# Building Wind Scenarios

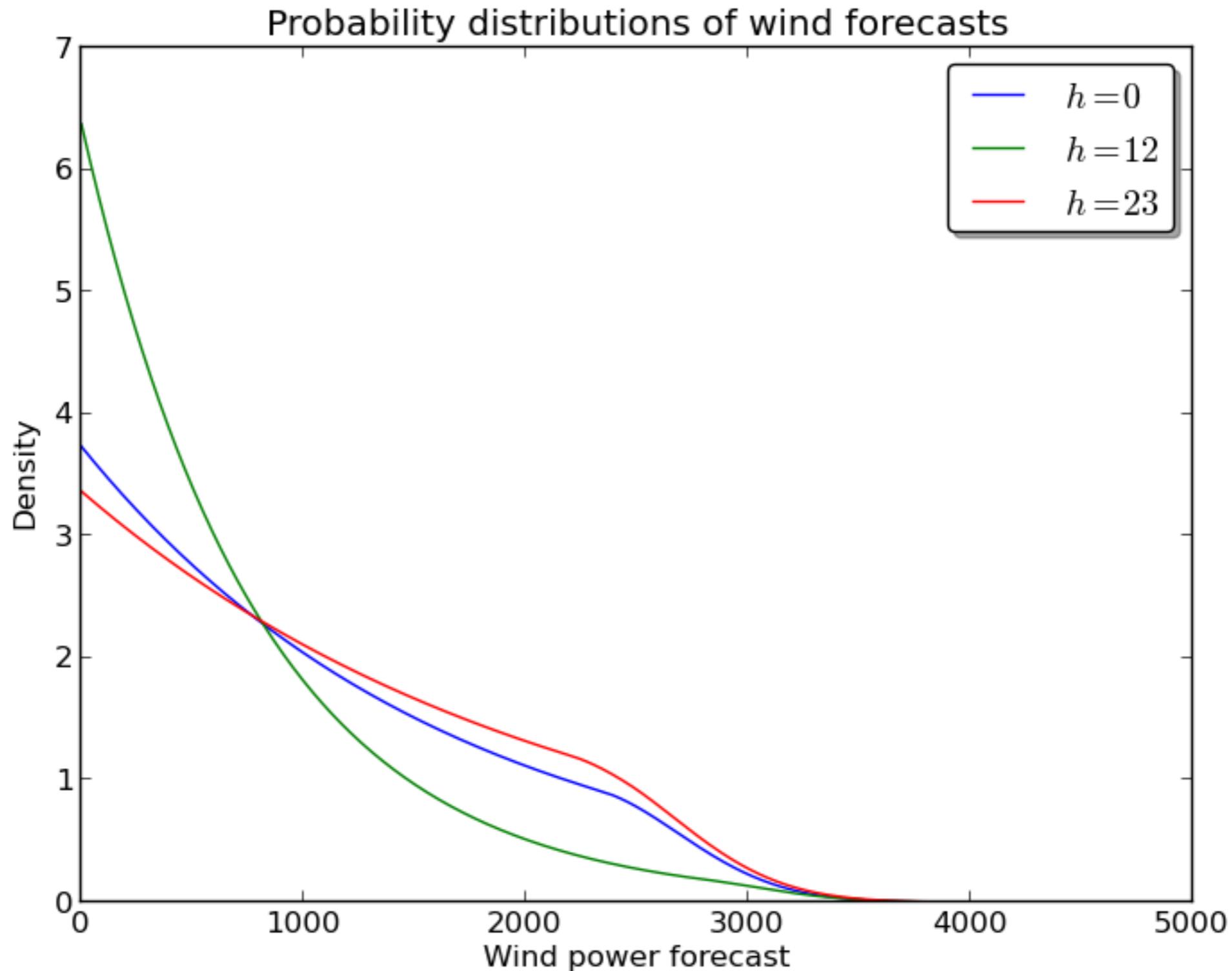
errors:  $e_t^d = a_t^d - f_t^d$  (Bonneville Power Administration)

1. distribution of the forecast for hours of interest
2. segment errors (per day) according to forecast wind power
3. compute conditional error distribution

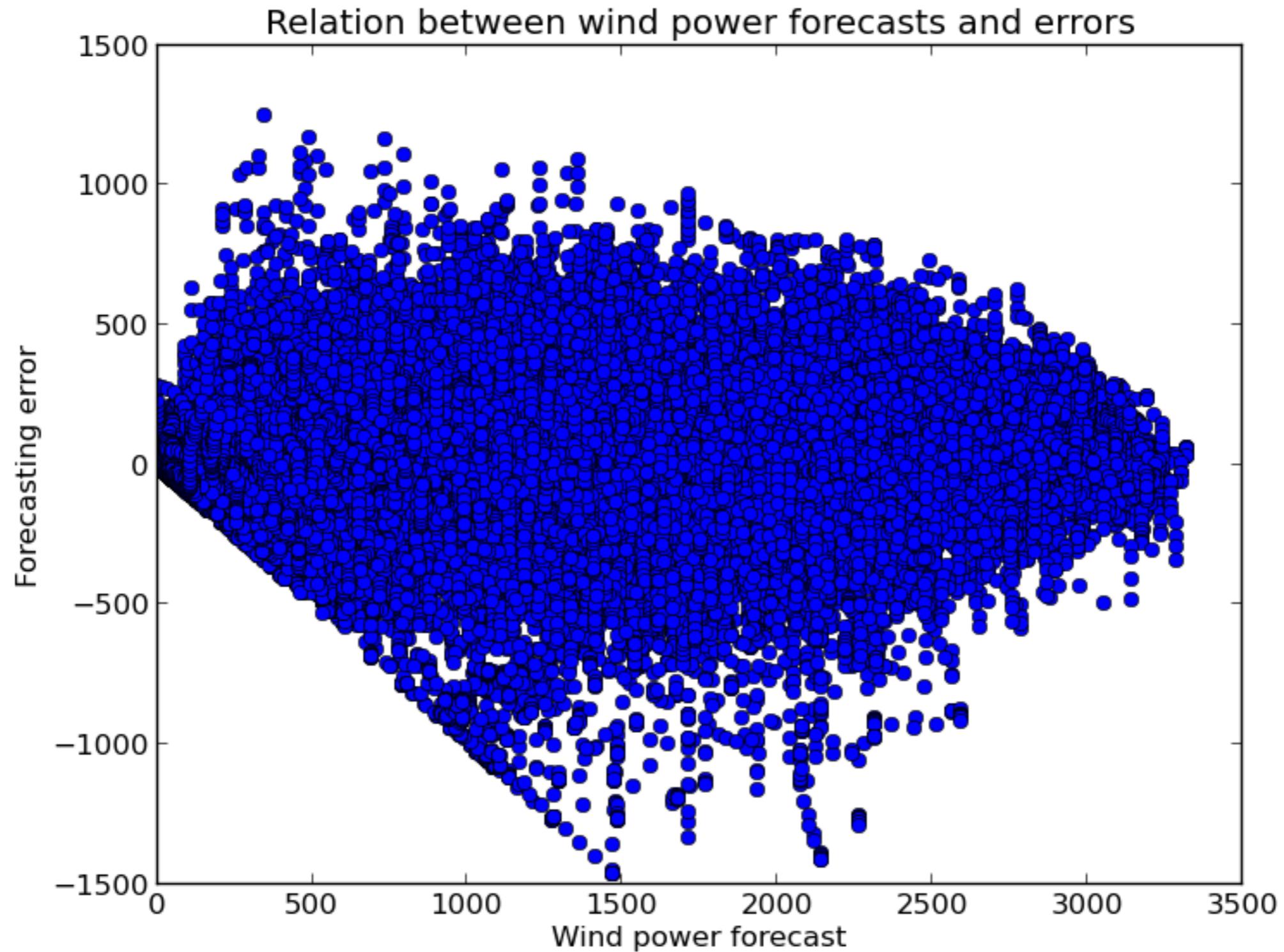
Scenarios: - generate scenario for the errors

- generate scenario paths (discrete process)

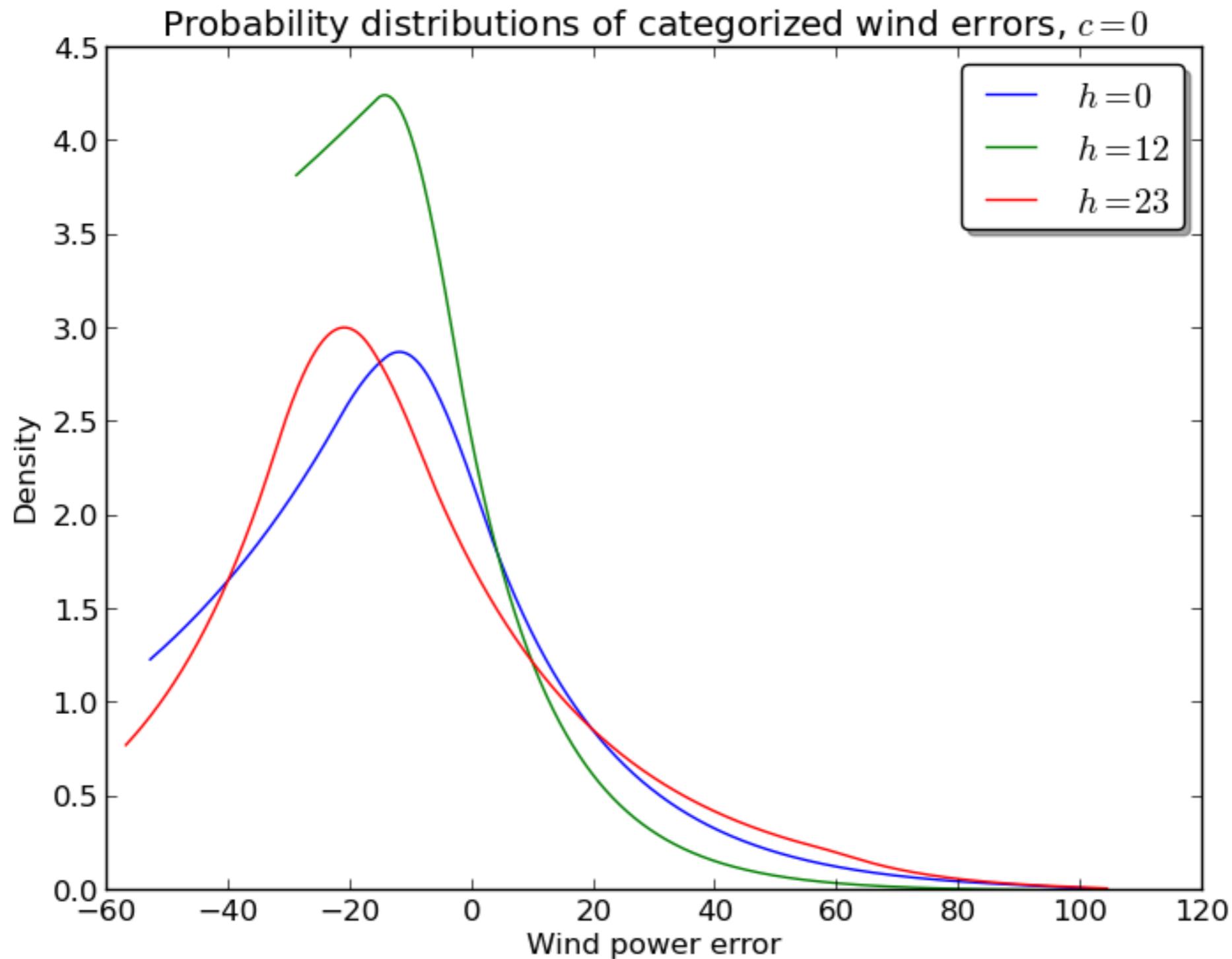
# Building Wind Scenarios



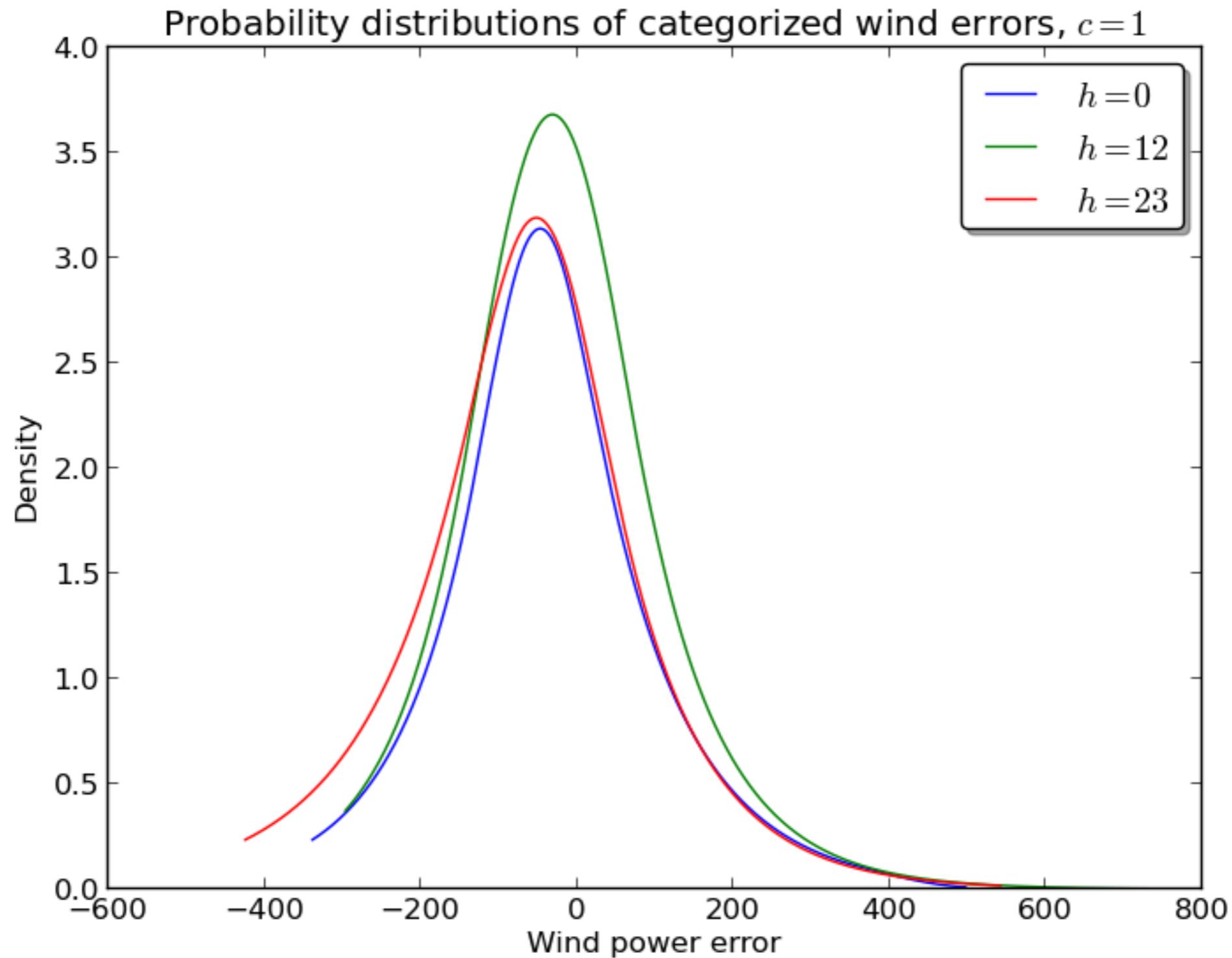
# Building Wind Scenarios



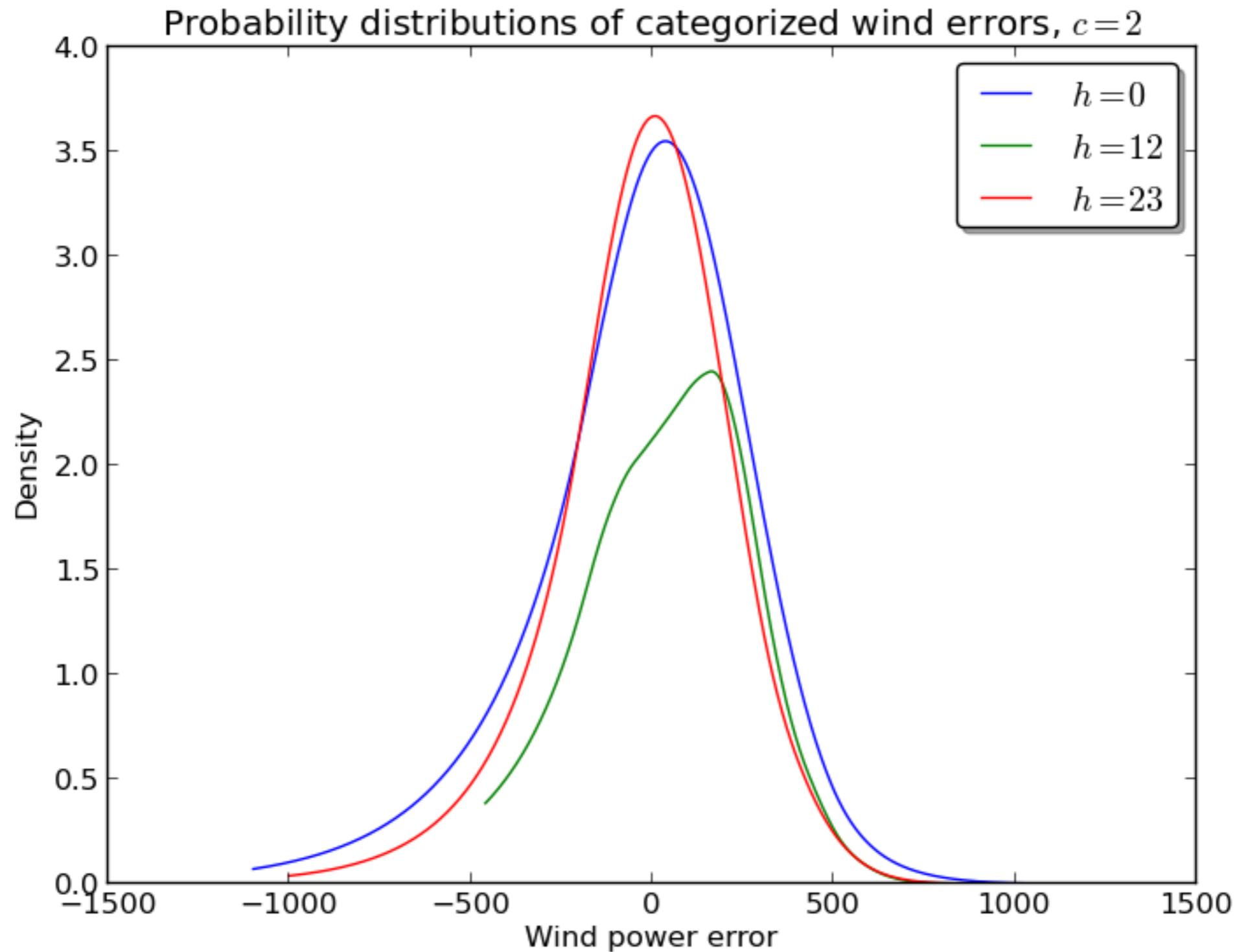
# Building Wind Scenarios



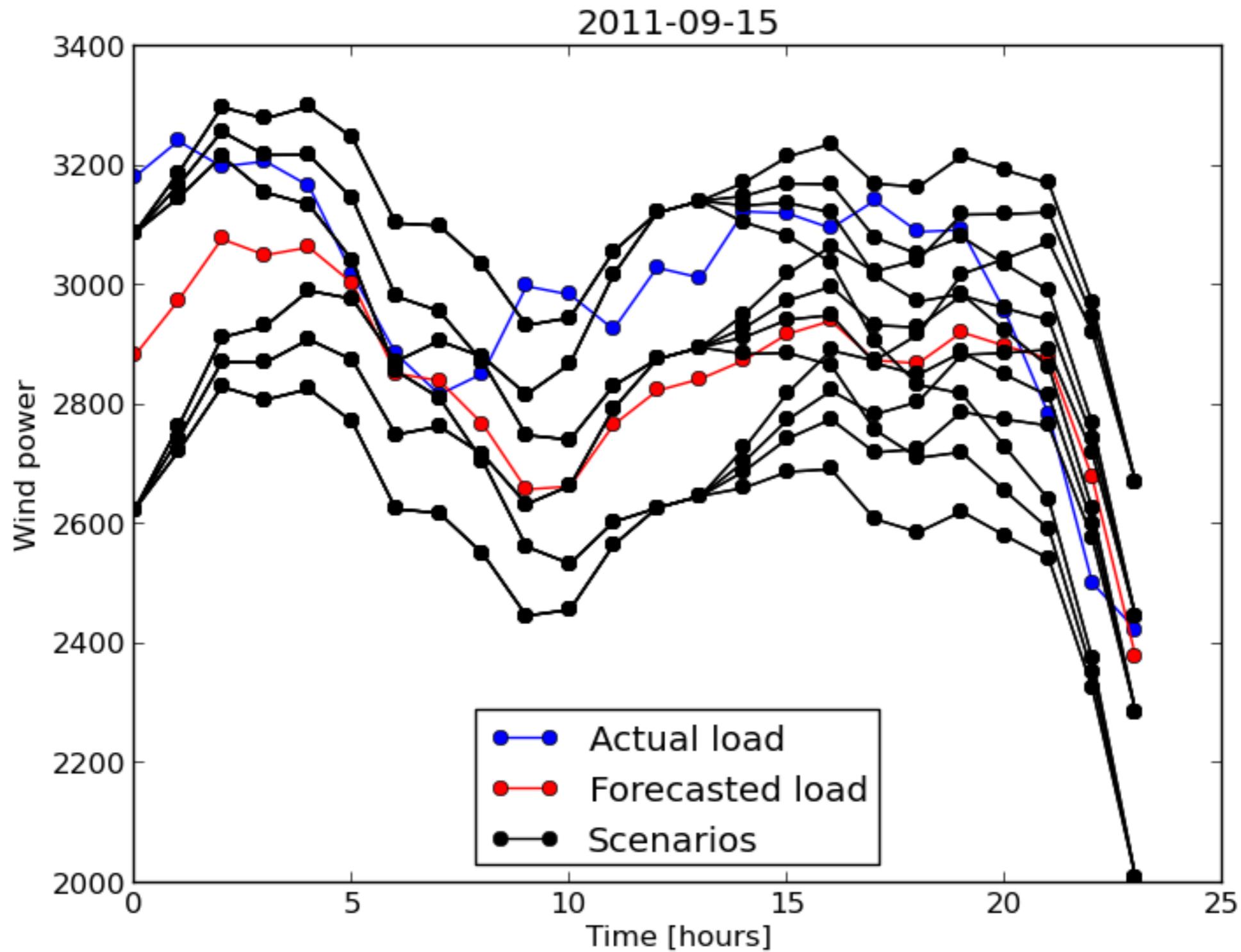
# Building Wind Scenarios



# Building Wind Scenarios



# Building Wind Scenarios



# References



K. Cheung, Y. Feng, D. Gade, Y. Lee, C. Monroy, I. Rios, F. Rüdél, S. Ryan, J.-P. Watson, R. Wets and D. Woodruff. Stochastic Unit Commitment at ISO scale: an ARP Ae projectf. 2014 *IEEE Power & Energy Society General Meeting Proceedings, 2014*

K. Cheung, D. Gade, C. Monroy, S. Ryan, J.-P. Watson, R. Wets and D. Woodruff. Toward scalable stochastic unit commitment - Part 2: Assessing solver performance. *IEEE Transactions on Power Systems* (submitted). 2013

Y. Feng, I. Rios, S. Ryan, K. Spürkel, J.-P. Watson, R. Wets and D. Woodruff. Toward scalable stochastic unit commitment - Part 1: load scenarios generation. *IEEE Transactions on Power Systems* (submitted). 2013

J.-P. Watson and D. Woodruff. Progressive hedging innovations for a class of stochastic mixed integer resource allocation problems, *Computational Management Science*, 2010+

R.T. Rockafellar and R. Wets. Nonanticipativity and L1-martingales in stochastic optimization problems. *Mathematical Programming Study*, 6:170–187, 1976

---

I. Rios, R. Wets and D. Woodruff, Multi-period forecasting with limited information and scenario generation with limited data. 2013 (submitted for publication)

J. Royset and R. Wets. Epi-splines and exponential epi-splines: pliable approximation tools, Tech. Report U. of California, Davis, 2013 (submitted)

J. Royset and R. Wets, Nonparametric density estimation via exponential epi-splines: fusion of soft and hard information. 2013 (submitted)

*Blackout*